



COURSE GUIDE

GRADE 6

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Printed in the U.S.A.

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About Zearn Math

All Children Can Love Learning Math

VISION

Zearn Math was developed by Zearn, a nonprofit educational organization. We believe that an understanding and love of mathematics is critical to helping all children realize their potential and to creating a generation of engaged learners who can change the world.

In 2011, Zearn’s team of educators and engineers created Zearn Math as an evidence-based, student-centered curriculum that balances rich mathematical discourse, collaborative problem solving, and independent thinking and working time. With Zearn Math, students learn math as the progression of connected ideas over the course of the year, grounded in visual problem solving with a concrete-to-pictorial-to-abstract approach.

LEARNING PRINCIPLES

Zearn combines our own digital lessons and assessments with open-source curricula in order to create an engaging, accessible, and coherent K-8 experience for all students. We developed Zearn Math using learning principles drawn from teacher practice, education research, and brain science — as well as extensive data from student engagement with our software over the years, which we use to strengthen our materials and technology each year:

Math is a few big ideas.

We delve deeply into these ideas, connecting and reinforcing them over the course of grades K-8. When kids learn math as a coherent discipline, rather than a series of unrelated tricks, and move from simple to deeper understanding of the structures of math, they build dense connections that can help them tackle any problem (Schmidt et al, 2005).

Math is more than memorization.

We built Zearn Math using the proven Concrete to Pictorial to Abstract (CPA) approach, in which students make sense of math using concrete materials, then pictorial representations, and finally abstract symbols to solve problems (Leong and Cheng, 2015). We also carry visual representations later into each year, grade, and lesson, so students always have something they can reference when they are faced with a problem they don’t know how to solve.

There are lots of “right” ways to solve problems.

We show students many options so they can find a way and move forward, rather than getting stuck and feeling frustrated. We used the Universal Design for Learning (UDL) principles to help teachers teach concepts in multiple ways — with the whole class, in groups with the teacher and their peers, and on their own (CAST, n.d.). This gives students more flexibility in how they learn and in how they demonstrate their understanding.

Mistakes are magic.

We help teachers create inclusive math communities where students feel safe to struggle. Students receive in-the-moment feedback that precisely addresses areas of misconception, both during independent learning and as they explore concepts with their peers. Research on “growth mindsets” shows that children who believe that

making mistakes is part of the learning process enjoy learning more and to be more resilient and self-directed in their learning (Paunesku et al, 2015).

Math is for everyone.

We think every kid should be able to see someone like themselves learning and persevering in math. Scholars have found that members of historically excluded groups may feel “belonging uncertainty,” undermining their motivation to learn (Walton and Cohen, 2007). To counter this, our digital lessons and print-based activities reflect the diversity found in classrooms across the country so that no one group is over- or underrepresented or stereotyped within the context of a particular problem. All students can see someone similar to themselves doing math and persevering through challenges.

About Sixth Grade Math

APPROACH

Guided by our learning principles, Zearn Math’s multimodal learning accommodates and celebrates learning differences and fosters positive math mindsets and social belonging, so all students can love learning math.

With Zearn Math, students will explore the same math concepts in two ways. Alongside their peers and teacher, students will model math with concrete manipulatives, represent their work on paper, discuss their reasoning aloud, and receive direct feedback from their teacher as well as from peers. Independently, students will use self-paced, software-based lessons to explore and practice concepts with concrete and digital manipulatives, interactive videos, and pictorial representations, receiving precise digital feedback at the moment of misconception.

With this approach, mathematics is not a spectator sport. Students spend most of their time in math class doing mathematics, rather than just listening to or watching it. They work on mathematics problems together and independently, often utilizing prior knowledge and skills, with guidance from the teacher and with precise, in-the-moment digital feedback that together ensure all students understand each concept. In the process, they make sense of problems, try different approaches, select and use appropriate tools, notice patterns, explain their ideas and reasoning (and listen to others’), and come to understand that mistakes are a valuable part of the mathematical learning process.

SCOPE AND SEQUENCE

The curriculum map below shows how students will cover a series of big mathematical ideas, color-coded to show the coherent structure of Zearn Math over the course of the grade and in other grades.

WEEK	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36
K	M1 Numbers to 10				M2 2D & 3D Shapes		M3 Comparison of Length, Weight, Capacity, & Numbers to 10					M4 Number Pairs, Addition, & Subtraction to 10					M5 Numbers 10–20; Count to 100 by Ones and Tens				M6 Analyzing, Comparing, & Composing Shapes															
G1	M1 Add & Subtract Small Numbers					M2 Meet Place Value			M3 Measure Length		M4 Add & Subtract Big Numbers				M5 Work with Shapes		M6 Add & Subtract to 100																			
G2	M1 Add & Subtract Friendly Numbers		M2 Explore Length		M3 Counting & Place Value			M4 Add, Subtract, & Solve				M5 Add & Subtract Big Numbers			M6 Equal Groups		M7 Length, Money, & Data			M8 Shapes, Time, & Fractions																
G3	M1 Multiply & Divide Friendly Numbers				M2 Measure It				M3 Multiply & Divide Tricky Numbers			M4 Find the Area		M5 Fractions as Numbers				M6 Display Data		M7 Shapes & Measurement																
G4	M1 Add, Subtract, & Round		M2 Measure & Solve	M3 Multiply & Divide Big Numbers				M4 Construct Lines, Angles, & Shapes		M5 Equivalent Fractions					M6 Decimal Fractions		M7 Multiply & Measure																			
G5	M1 Place Value with Decimal Fractions		M2 Base Ten Operations				M3 Add & Subtract Fractions		M4 Multiply and Divide Fractions & Decimals				M5 Volume, Area, & Shapes		M6 The Coordinate Plane																					
G6	M1 Area and Surface Area		M2 Introducing Ratios		M3 Rates and Percentages		M4 Dividing Fractions		M5 Arithmetic in Base Ten		M6 Expressions and Equations		M7 Rational Numbers		M8 Data Sets and Distributions		M9 Putting It ALL Together																			
G7	M1 Scale Drawings	M2 Introducing Proportional Relationships		M3 Measuring Circles	M4 Proportional Relationships and Percentages		M5 Rational Number Arithmetic		M6 Expressions, Equations, and Inequalities			M7 Angles, Triangles, and Prisms		M8 Probability and Sampling		M9 Putting It ALL Together																				
G8	M1 Rigid Transformations and Congruence		M2 Dilations, Similarity, and Introducing Slope		M3 Linear Relationships		M4 Linear Equations and Linear Systems		M5 Functions and Volume		M6 Associations in Data		M7 Exponents and Scientific Notation		M8 Pythagorean Theorem and Irrational Numbers		M9 Putting It ALL Together																			

Key	Whole Numbers and Operations	Expanding Whole Numbers and Operations to Fractions and Decimals	Algebraic Thinking and Reasoning Leading to Functions	Geometry	Measurement, Statistics and Probability

Grade 6 begins with a unit on reasoning about area and understanding and applying concepts of surface area. It is common to begin the year by reviewing the arithmetic learned in previous grades, but starting instead with a mathematical idea that students haven't seen before sets up opportunities for students to surprise the teacher and themselves with the connections they make. Instead of front-loading review and practice from prior grades, these materials incorporate opportunities to practice elementary arithmetic concepts and skills through warm-ups and in the context of instructional tasks.

One of the design principles of these materials is that students should encounter plenty of examples of a mathematical or statistical idea in various contexts before that idea is named and studied as an object in its own right. For example, in the first unit, students will generalize arithmetic by writing simple expressions like $\frac{1}{2}bh$ and $6s^2$ before they study algebraic expressions as a class of objects in the sixth unit. Sometimes this principle is put into play several units before a concept is developed more fully, and sometimes in the first several lessons of a unit, where students have a chance to explore ideas informally and concretely, building toward a more formal and abstract understanding later in the unit.

In the Mission Overview of each unit there is a section called Progression of Disciplinary Language that explains the disciplinary language functions specific to that unit. This level of detail assists teachers in planning and differentiation.

PACING

The sixth-grade year will include nine units, known in Zearn Math as “Missions” (see details below). Each course of Zearn Math for Middle School contains nine missions: each of the first eight are anchored by a few big ideas in grade-level mathematics, and the optional Mission 9 contains lessons that help students apply and tie together big ideas from the year, with no Independent Digital Lessons.

Teachers should aim to cover four lessons per week (with a fifth “Flex” day or equivalent time reserved for teachers to differentiate instruction and administer assessments), so the full sixth grade curriculum should take about 36 weeks to complete. This pace ensures students have sufficient time each week to work through grade-level content with built-in weekly time to assess and address student needs.

Each Mission will conclude with a paper-based, End-of-Mission assessment; some longer Missions also contain a Mid-Mission assessment. Students also complete two formative assessments on each lesson: a digital Tower of Power assessment and a paper Exit Ticket.

Mission	Title	Lessons	Weeks
1	Area and Surface Area	18	5
2	Introducing Ratios	17	4
3	Unit Rates and Percentages	16	4
4	Dividing Fractions	17	4
5	Arithmetic in Base Ten	14	4
6	Expressions and Equations	16	4
7	Rational Numbers	18	5

Mission	Title	Lessons	Weeks
8	Data Sets and Distributions	18	5
9	Putting it All Together	6*	~1
Total		140	36

*This mission is optional.

Note: Weeks are estimates based on four lessons completed per week. Actual time will vary by student.

A Typical Lesson in Zearn Math

In a typical math block with Zearn, covering one lesson, students will experience:

Warm-Up

Concept Exploration

Collaborative Concept Exploration with teacher and peers,
Independent Digital Lessons to explore concepts further on their own; and

Wrap-Up

Lesson Synthesis led by the teacher and the completion of an individual Exit Ticket.

Every lesson in Zearn Math for Sixth Grade is accompanied by an optional classroom presentation that contains the warm up, collaborative concept exploration, and wrap up activities and helps teachers with the overall flow of the math block. Each presentation is built to show students where in the lesson they should be at any given moment and includes cues that support the teacher's transitions if needed.

WARM-UP

The first event in every lesson is a warm-up, which invites students into the day's lesson with a series of engaging activities. A warm-up either helps students get ready for the day's lesson, or gives students an opportunity to strengthen their number sense or procedural fluency:

- A warm-up that helps students get ready for today's lesson might serve to remind them of a context they have seen before, get them thinking about where the previous lesson left off, or preview a calculation that will happen in the lesson so that the calculation doesn't get in the way of learning new mathematics.
- A warm-up that is meant to strengthen number sense or procedural fluency asks students to do mental arithmetic or reason numerically or algebraically. It gives them a chance to make deeper connections or become more flexible in their thinking.

CONCEPT EXPLORATION

Students have two opportunities to study the same concepts: with their teacher and peers in Collaborative Concept Exploration, and using self-paced Independent Digital Lessons. The intentional balance of learning with teachers and peers and learning independently in digital lessons ensures every student has multiple opportunities to represent, engage with, and express their math reasoning.

Concept exploration includes:

Collaborative Concept Exploration

Built around a series of scaffolded math problems that move students towards the overall goal of the lesson.

Each collaborative classroom activity has three phases:

1. **Launch:** During the launch, the teacher makes sure that students understand the context and what the problem is asking them to do. This is not the same as making sure the students know how to do the problem—part of the work that students should be doing for themselves is figuring out how to solve the problem.
2. **Student Work Time:** The launch for an activity frequently includes suggestions for grouping students. This gives students the opportunity to work individually, with a partner, or in small groups.
3. **Activity Synthesis:** During the activity synthesis, the teacher orchestrates some time for students to synthesize what they have learned. This time is used to ensure that all students have an opportunity to understand the mathematical punch line of the activity and situate the new learning within students' previous understanding.

Independent Digital Lessons

Students complete self-paced, software-based lessons on their own.

Students construct their mathematical thinking using visual models and have opportunities to test and confirm their reasoning, with precise feedback to help them find and correct mistakes. This self-paced learning fosters students' sense of ownership over their math learning and boosts students' math mindsets because all students are able to take the amount of time they need to problem solve, review content, or receive scaffolded support. The video player can be paused or rewound at any time.

(For more on what students will experience during Independent Digital Lessons, see “Planning for a Lesson” in the “Implementing Zearn Math” section below. For more on addressing struggle, see “Addressing unfinished learning” in the “Assessments and Reports” section below.)

WRAP-UP

After the concept exploration phase of the lesson, students move to the final phase of the lesson, the wrap-up. This phase provides students with time to synthesize what they just learned during the concept exploration and gives teachers a chance to formatively assess whether students grasped the big idea of the lesson, which can then inform the next lesson.

The wrap-up includes two opportunities to formatively assess student understanding:

Lesson Synthesis

Students incorporate new insights into big-picture understanding and teachers get a sense of students' understanding. Each lesson includes suggested discussion starters that teachers can use to guide students in a conversation to process the lesson. Teachers may pose questions verbally and call on volunteers to respond, and could ask students to add a new component to a persistent display like a word wall. Also, this is the final moment of discourse for every lesson and unfinished learning may be evident by what students are saying or not saying. However, teachers should not use this moment to try and reteach the entire lesson, but rather use this moment as one of many formative assessments provided in each lesson; they can combine this data with their review of student work on the lesson's Exit Ticket and the Tower Alerts report to determine the effectiveness of the lesson. *(For more on how to appropriately diagnose and respond to unfinished learning, see the corresponding section below in "Assessments and Reports.")*

Exit Ticket

Students to demonstrate their understanding of the content of the lesson.

To get the most authentic and helpful data possible, students should complete the un-scaffolded practice problems on the paper Exit Ticket independently to the best of their ability. Teachers can use Exit Tickets as formative assessments to identify students who may need extra help with a particular concept and provide appropriate support, and/or combine this information with observations from the Tower Alerts report to determine the extent of any misconception. *(For more, see "Assessments and Reports" below.)*

Finally, note that each Zearn Math for Middle School lesson includes an associated set of practice problems—found in the Optional Practice Materials packet—including a few problems from that day's lesson along with a mix of topics from previous lessons. Teachers may assign some or all practice problems during class, or for homework, or not at all; they may also decide to collect and score those problems, or to provide students with answers ahead of time for self-assessment, or to score them together as a class.

Implementing Zearn Math

PLANNING FOR A MISSION

The story of Zearn Math for Sixth Grade is told in nine Missions. Each course of Zearn Math for Middle School contains nine missions: each of the first eight are anchored by a few big ideas in grade-level mathematics, and the optional Mission 9 contains lessons that help students apply and tie together big ideas from the year. Concepts are taught through the concrete-to-pictorial-to-abstract progression within each Mission and throughout the year to allow students ample time to continue to build their developing understanding. The table of contents in each Mission book shows you how the lessons of the Mission are divided into topics as well as the placement of the Mission-level, paper-based assessments.

To plan for a Mission:

Familiarize yourself with the mathematics of the Mission and how students will progress toward understanding the big ideas of the Mission.

- Read through the Mission Overview in the Zearn Math Teacher Edition, taking note of the progression of the mission, key representations and strategies used, and any moments that may be challenging for students. The Mission Overviews provide an opportunity to deeply learn the big ideas of the Mission.
- Complete the Mission-level paper-based assessments and study the assessment rubrics provided, including the exemplar student work.
- Take note of any lessons that are marked optional. [Mission 9 is optional and contains no Independent Digital Lessons.] You can omit these lessons to allow for a Flex Day each week and still complete all grade-level content. *(For more on Flex Days, see below section on “Planning for a week.”)*

Work through a selection of the Independent Digital Lessons that students will complete during the Mission, including the Guided and Independent Practice portions of the lessons. This will help you deepen your understanding of how the previously studied concepts act as on-ramps into the new learning, as well as how the ideas will scaffold across the Mission. *(For more on preparing for individual lessons, see the “Planning for a lesson” section below.)*

At the start of each Mission:

Check Zearn Class Reports to review students’ progress and assign all students to the first Independent Digital Lesson of the Mission to ensure that they will have a chance to cover all the big mathematical ideas of the Mission in two ways, both digitally and in person with their class. *For more, see “Assessments and Reports” below.*

Send home the Family Materials packet, which discusses key learning objectives and include sample problems for parents/caregivers to try at home with their students.

Finally, as you prepare for your first Mission, keep in mind that parents/caregivers may want to know how best to support their students. To equip them, go online to visit the Zearn Math Parent and Caregiver Support page of the Zearn web site, available in both English and Spanish.¹ Ideas include:

- Sending home our Learning with Zearn overview flyer.
- Hosting a parent orientation or back to school night using our Zearn Math Parent Presentations.

Throughout the year, send home the Family Materials packet, which discusses key learning objectives and include sample problems for parents/caregivers to try at home with their students. In addition, the Student Report for their child and the assessment rubrics may provide additional insight to their student’s progress and areas of misconception or struggle.

PLANNING YOUR WEEK

We designed Zearn Math to include four “Core Days” when students learn grade-level content, as well as one “Flex Day” that you can tailor to meet students’ needs. This weekly schedule ensures students have sufficient time each

¹ The Zearn Help Center can be found at <http://help.zearn.org>; the Zearn Math Parent and Caregiver Support page of the Zearn web site can be found at <https://about.zearn.org/math-resources/parent-caregiver-support>.

week to work through grade-level content while also giving you additional time to address unfinished learning and/or misconceptions that might be hindering student progress.

Core Days

If you are using this Zearn Math recommended weekly schedule, we recommend omitting the optional lessons in each Mission. On Core Days, you should plan for a balance of learning across multiple formats. We designed this lesson structure to fit into a 50-minute math block, but all times are suggestions that can be modified to accommodate different schedules:

- Warm-Up with the whole class may take up to 5 minutes.
- Concept Exploration includes about 20 minutes of Collaborative Concept Exploration and 20 minutes of Independent Digital Lessons, for a total of 40 minutes. These can be adjusted based on available time, as long as students have daily opportunities to learn and practice in a variety of instructional settings with a variety of different learners. Some teachers choose to have students rotate through these two “stations” while others have students complete Independent Digital Lessons during other flexible time during the day. A few lessons do not include an Independent Digital Lesson. Guidance on how best to handle each situation is detailed in your Zearn Math Teacher Edition.
- Wrap-Up, including lesson synthesis and an Exit Ticket, takes about 5 minutes.

Flex Day

In addition to 4 Core Days, if you are using the Zearn Math recommended weekly schedule and omitting optional lessons, you will have time for a Flex Day each week. Consider how you might use this time to continue addressing individual student needs you may have noticed during your Core Day lessons or while reviewing Zearn class and student reports:

Check Zearn class and student reports to determine what student needs you might address during this time:

- The Pace report indicates which students may need more time to complete Independent Digital Lessons.
- The Tower Alerts report indicates which students are struggling with particular concepts.

Decide how you will address the needs of different students during Flex Day or Flex time:

- Use the Pace Report to identify any students who have completed fewer than four Independent Digital Lessons that week. Some students may need more time to finish these digital lessons. If these students are making progress and simply need more time, allow these students to spend time during Flex Days finishing their Independent Digital Lessons so they can meet their goals.
- Use the Tower Alerts report to identify groups of students struggling with the same concepts or misconceptions. You could teach these groups mini lessons using the Optional Activities found in the Enrichment section of the Optional Practice Materials packet. These optional activities are included for topics where experience shows students often need some additional time to work with the ideas. These activities are marked as optional because no new mathematics is covered, so if a teacher were to skip them, no new topics would be missed.
- If the Tower Alerts report identifies individual students struggling with a particular concept or misconception, you could “bookmark” foundational content for them to complete. *(For more, see “Addressing unfinished learning” in the “Assessments and Reports” section below.)*

- For students who are completing four Independent Digital Lessons each week and demonstrating full understanding on assessments, you can use Flex time to provide opportunities for additional challenge and growth. Zearn Math provides teachers with curricular materials for extending learning that are aligned to students' current grade-level work, including:
 - **Digital Bonuses:** Digital Bonuses are challenging problems students can work on after they complete an Independent Digital Lesson. These problems enrich and extend their learning by going deeper into grade-level mathematics, often making connections between the topic at hand and other concepts. Digital Bonuses do not appear automatically in the Student Feed, so you can direct students to navigate to them from their Badges.
 - **Enrichment Problems:** Also included in the Enrichment section of the Optional Practice Materials packet are Extension Problems, which go deeper into grade-level mathematics and often make connections between the topic at hand and other concepts at grade level or that are outside of the standard K-12 curriculum. They are not routine or procedural, and intended to be used on an opt-in basis by students if they finish the main class activity early or want to do more mathematics on their own. It is not expected that an entire class engages in extension problems and it is not expected that any student works on all of them, which makes them well suited for Flex Days.

When it is time for a Mission-level assessment, plan to have students complete these assessments during Flex time. For more, see “Assessments and Reports” below.

PLANNING FOR A LESSON

You can use your Zearn Math Teacher Edition daily to plan and implement each lesson of the Mission. To prepare for teaching each lesson within a Mission, we recommend that teachers:

Read through the lesson

Gain an understanding of how the big mathematical idea of the lesson unfolds across the lesson, taking note of the discussion guidance and how it helps move students toward the lesson's objective. Each lesson in the Mission has a narrative that contains a description of the mathematical content of the lesson and its place in the learning sequence, the meaning of any new terms introduced in the lesson, and how the mathematical practices come into play, as appropriate. Activities within lessons also have a narrative, which explains the mathematical purpose of the activity and its place in the learning sequence, what students are doing during the activity, what teacher needs to look for while students are working on an activity to orchestrate an effective synthesis, and connections to the mathematical practices when appropriate.

Complete the entire Independent Digital Lesson that students will complete.

As you do this, focus on examining the learning progression and students move toward developing a full understanding of the big mathematical idea of the Mission. We recommend intentionally making mistakes throughout the digital experience to see how students will be supported when making their own mistakes. For students, each lesson includes:

- **Fluency:** Students warm up with a short fluency activity that supports the big ideas developed in that mission. These activities activate prior knowledge of fractions, operations, and mental math strategies that students will need throughout the mission Regular practice through these activities helps all students access

grade level math by enabling them to shift working memory from calculations, to new concept development. (Fluency will be available in pilot form beginning in the 22-23 ASY)

- **Guided Practice:** Students experience one of three different Guided Practice activities—Math Chat, Learning Lab, or Z-Squad. Each activity creates a rich learning environment for students through interactive and multisensory videos featuring real, on-screen teachers and digital manipulatives. Students are prompted to complete problems in their paper Student Notes to transfer their software-based learning, reflect on the key idea in the lesson, and strengthen knowledge retention.
- **Independent Practice (Tower of Power):** Students demonstrate their understanding of the content of an Independent Digital Lesson and unlock the next one by completing all problems correctly in the Tower of Power.
 - If students make a mistake in a Tower of Power problem, a “Boost” breaks down the question into smaller steps with more supportive manipulatives to allow students to understand and correct their mistakes. Students then have a chance to demonstrate their learning with a new problem.
 - If students continue to struggle in the Tower of Power after multiple attempts, their teacher receives an alert in the Tower Alerts Report, enabling them to provide tailored, differentiated support for that student.

Determine specific instructional routines you will use:

Zearn Math lesson plans often include information about instructional routines that may be suited to teaching a particular lesson. The kind of instruction appropriate in any particular lesson depends on the learning goals of that lesson. Some lessons may be devoted to developing a concept, others to mastering a procedural skill, and others to applying mathematics to a real-world problem. For example, four instructional routines frequently used in warm-ups are Number Talks, Notice and Wonder, Which One Doesn’t Belong, and True or False. In addition to the mathematical purposes, these routines serve the additional purpose of strengthening students’ skills in listening and speaking about mathematics. *(For a full list, including when and how to use instructional routines, see Appendix I: Instructional Routines.)*

Consider which strategies you will use to create access for all learners:

You will want to plan intentionally for how to meet the specific and varied needs of your students, including those with unfinished learning, those with disabilities, and multilingual learners, using data from Zearn reports and your own observations from the classroom. Supplemental instructional strategies, labeled “Support for English Language Learners” and “Support for Students with Disabilities,” are included in each lesson and are designed to increase access and eliminate barriers. *For more on the supports for multilingual learners, visit Appendix II: Access for Multilingual Learners. For more on the design principles for the supports for students with disabilities, and the cognitive functioning areas they address, visit Appendix III: Access for Students with Disabilities. For more information and ideas, see the next section on “Supporting Diverse Learners” including both students with disabilities and multilingual learners.)*

After reviewing the lesson materials and completing the Independent Digital Lesson, you can annotate your Zearn Math Teacher Edition using the “Your Notes” section in the margin of each lesson. Annotations may include:

- Key ideas from each moment of the lesson;
- Probing questions you may ask to move students towards understanding the takeaway during the synthesis portion of each activity; and
- Explicit connections to prior activities and/or lessons.

Make sure to have on hand all of the required materials to successfully enact the lesson.

Required materials and required preparation are listed alongside the learning goals on the first page of each lesson. *(For a full list of required materials for the course, see “Required Materials” section below.)*

PREPARING STUDENTS FOR INDEPENDENT DIGITAL LESSONS

To ensure students are ready to complete Independent Digital Lessons, you will want to do the following with them. You should only have to do this during your first week of instruction, unless you observe that a refresher is needed:

Complete 2–3 Independent Digital Lessons together as a whole class.

You can model a lesson from your account by using the “Try lesson as a student” feature on your Zearn Math Welcome Page and project or share your screen so students can follow along. Be sure to:

- **Walk through each component of a digital lesson.** For sixth grade, these are:
 - Fluency (available in pilot form beginning in the 22-23 ASY)
 - Guided Practice
 - Independent Practice
- **Model completing paper Student Notes** when prompted in the Guided Practice section.
- **Discuss strategies for persevering through challenges** like working through a Boost within a Tower of Power, referencing Student Notes, revisiting the Guided Practice, and even guessing if needed and letting the digital lesson provide help. Remind students that some struggle is both expected and useful, and that you will not be helping them to complete these lessons. Instead, they should try their best to resolve challenges on their own with the support of the software’s built-in scaffolds, and you will frequently check Pace Report and Tower Alerts Report to identify any students who may be struggling unproductively.
- **Show students how to work on their “Next Up” activity.** Students work through Independent Digital Lessons at their own pace and are always assigned to one of these activities as their “Next Up” assignment. Students can only access the next digital activity in the sequence once they complete their currently assigned activity.
- **Show students the accessibility features, including:**
 - **Closed captioning:** Closed captioning for all interactive student videos is available for all Missions for all grades. Closed captioning allows students to turn on an English text transcription of all dialogue and other relevant audio information in the Zearn Math video player. This accessibility feature is particularly useful for deaf and hard-of-hearing students, as well as multilingual learners.
 - **Audio support:** All instructional prompts students see in Independent Digital Lessons have audio support through either recorded audio or Zearn Math’s text-to-speech feature. Students can click on the audio button next to text questions or prompts to hear the words spoken aloud. All math expressions in Zearn Math software-based lessons are read correctly with Zearn Math’s text-to-speech tool. Additional audio support, if needed, can be accessed using supported browser text-to-speech tools. These accessibility features are particularly important for students with cognitive impairments, students with learning differences, young students, and multilingual learners.

- **Zoomability:** Students may resize digital pages up to 200% through browser settings to view images or text closer up without losing any content. This accessibility feature is particularly important for students with visual impairments and students using devices with small screens.
- **On-screen keypad:** As students work through Independent Digital Lessons, they have the option to use an on-screen keypad, rather than a computer keyboard, to type and submit answers. This accessibility feature is particularly important for tablet users and young students who may not know how to use a computer keyboard.

You may also use this time to introduce students to the Math Library, noting that you might direct them here throughout the year for additional assignments. *(For more on the Math Library, see the “Addressing unfinished learning” section of “Assessments and Reports” below.)*

To find many additional resources that can help you and your students prepare for software-based lessons, go online to visit the Zearn Help Center.² There, you can find not only a getting-started checklist, a recommended schedule, and technology requirements, but also ideas for how to set up strong classroom systems and routines that will help students learn how to use Zearn Math and how to build the mindsets, habits, and confidence in math.

² <https://help.zearn.org>

Supporting Diverse Learners

COMMITMENT TO ACCESSIBILITY

Zearn believes that with proper structures, accommodations, and support, all children can learn mathematics. As such, we designed Zearn Math to be accessible for all students, using the Universal Design for Learning (UDL) principles to maximize access and engagement for all students. We have also added supports and structures throughout to help teachers accommodate the needs of diverse learners. *(For more, see above sections on “Planning for a lesson” as well as “Preparing students for Independent Digital Lessons” under “Implementing Zearn Math.”)*

DESIGN FEATURES THAT SUPPORT ALL LEARNERS

To support a diverse range of learners – including students with unfinished learning, multilingual learners, and students with disabilities – Zearn Math design features include:

Consistent lesson structures:

The structure of every lesson is the same: Warm-Up, Concept Exploration, Wrap-Up. By keeping the components of each lesson similar from day to day, the flow of work in class becomes predictable for students. This reduces cognitive demand and enables students to focus on the mathematics at hand rather than the mechanics of the lesson.

Concepts developing over time from concrete to abstract:

Mathematical concepts are introduced simply, concretely, and repeatedly, with complexity and abstraction developing over time. Students begin with concrete examples, and transition to diagrams and tables before relying on symbols to represent the mathematics they encounter. Moreover, this CPA approach is repeated throughout lessons, Missions, and across the grade to continually give students access to new ideas.

Co-constructing knowledge with students:

Providing students with time to think through a situation or question independently before engaging with others allows students to carry the weight of their own learning, with support arriving just in time from the community of learners in Collaborative Concept Exploration as well as from the software-based Independent Digital Lessons. This progression allows students to start with what they already know, and continue to build from this base with others.

Opportunities to apply mathematics to real-world contexts:

Giving students opportunities to apply the mathematics they learn through word problems clarifies and deepens their understanding of core math concepts and skills, while also providing motivation and support. Mathematical modeling is a powerful activity for all students, but especially for students with disabilities. Centering instruction on these contextual situations right from the beginning of the lesson during the Warm-Up can provide students with disabilities an anchor upon which to base their mathematical understandings.

SUPPORTING MULTILINGUAL LEARNERS

Zearn believes that language learners of all levels can and should engage with grade-level content that is scaffolded with sufficient linguistic support. Zearn Math provides students opportunities to access grade-level mathematics using existing language skills and to extend their language development in the context of mathematical skill development. Each day with Zearn Math, students learn in a classroom model designed for daily differentiation, experience inclusive environments of social belonging, and build language skills as they learn with the whole class, with peers, and on their own with software-based lessons.

In addition, we provide teachers with a Zearn Math Teacher Edition that provides strategies on how to support students' language development daily within the context of their math class. *(For more, see above section on "Planning for a lesson" under "Implementing Zearn Math.")*

While these features of Zearn Math support all students in building a deep understanding of grade-level mathematics, they are particularly critical for meeting the needs of multilingual learners.

Elements of language

During their daily learning with Zearn Math, students are exposed to many elements of language, such as mathematics vocabulary and spoken language patterns. (In some situations, multilingual learners may benefit from using their first language because processing math in their first language can create a safe space for deeper thinking.) Zearn Math supports students as they develop their mathematical skills by reinforcing:

Essential vocabulary:

Students are not expected to have prior knowledge of essential math vocabulary. Language critical to students' mathematical learning is explicitly introduced, taught, and repeated frequently, which helps all students gain familiarity with new terminology and practice using it as they move through the curriculum.

Patterns of discourse:

As students engage in rich math discussions throughout the lesson, they have opportunities to organize their language in discourse patterns such as "compare and contrast" or "question and answer." Teachers facilitate these structured conversations through instructional routines such as "Think Pair Shares," which allow students to make claims, provide evidence, communicate thinking, and critique others' reasoning. The Zearn Math lessons include specific notes with conversation starters, sentence frames, and modeling guidance that can help enhance discussion quality for all students and ensure multilingual learners are supported in participating.

Math discussion:

Students share their own thinking aloud and discuss classmates' problem-solving strategies throughout daily Warm Up and Collaborative Concept Exploration. Teachers facilitate thoughtful mathematical discussions between students that allow learners to refer to and build on each others' ideas. The Zearn Math Teacher Edition provides guidance on instructional routines that further math discussions for all students, with additional notes on supporting multilingual learners.

Mathematical language routines (MLRs)

For instances where students need even more support than what's in the curriculum, Zearn recommends that teachers read and consider using the mathematical language routines (MLRs) listed below. A mathematical language routine is a structured but adaptable format developed by the SStanford University UL/SCALE team (Zwiers et al 2017) for amplifying, assessing, and developing students' language in order to provide various types

of learners, including multilingual learners, with greater access by supporting them with the language demands of a specific activity without reducing the mathematical demand of the task. These routines emphasize uses of language that are meaningful and purposeful, rather than just getting correct answers. These routines can be adapted and incorporated across lessons in each unit wherever there are productive opportunities to support students in using and improving their English and disciplinary language.

These eight routines were selected for inclusion in this curriculum because they are effective and practical for simultaneously learning mathematical practices, content, and language. They are:

MLR 1 Stronger and Clearer Each Time:

Students think and write individually about a question, use a structured pairing strategy to have multiple opportunities to refine and clarify their response through conversation, and then finally revise their original written response.

MLR 2 Collect and Display:

Teacher listens for, and scribes, the language students use during discussions using written words, diagrams and pictures. This collected output can be organized, re-voiced, or explicitly connected to other language in a display that all students can refer to, build on, or make connections with during future discussion or writing.

MLR 3 Clarify, Critique, Correct:

Teacher provides students with an incorrect, incomplete, or ambiguous written mathematical statement, and students improve upon the written work by correcting errors and clarifying meaning.

MLR 4 Information Gap:

Teachers facilitate meaningful interactions by positioning some students as holders of information that is needed by other students to accomplish a goal, such as solving a problem or winning a game.

MLR 5 Co-Craft Questions:

Students use conversation skills to generate, choose (argue for the best one), and improve questions and situations, as well as develop meta-awareness of the language used in mathematical questions and problems.

MLR 6 Three Reads:

Students read a mathematical text, situation, or word problem three times, each with a particular focus. The intended question or main prompt is intentionally withheld until the third read so that students can concentrate on making sense of what is happening in the text before rushing to a solution or method.

MLR 7 Compare and Connect:

Students make sense of mathematical strategies other than their own by creating visual displays and then relating and connecting other approaches to their own.

MLR 8 Discussion Supports:

This collection of instructional moves can be combined and used together with any of the other routines to help students make sense of complex language, ideas, and classroom communication, and to invite and incentivize more student participation, conversation, and meta-awareness of language.

To learn more about our approach to supporting multilingual learners, including a full description of each MLR, see Appendix II: Access for Multilingual Learners. Zearn is committed to offering comprehensive curriculum resources in Spanish. As of the 2021–22 school year, paper-based teacher and student instructional materials have been fully translated into Spanish.

SUPPORTING STUDENTS WITH DISABILITIES

Students with disabilities can and should engage with Zearn Math. While a student’s Individualized Education Plan should be the first resource teachers use when determining how to differentiate instruction for a student with a disability, Zearn’s curriculum also highlights patterns, critical features, and big math ideas in a way that supports such differentiation. *(For more, see above sections on “Planning for a lesson” as well as “Preparing students for Independent Digital Lessons” under “Implementing Zearn Math.”)*

The following design elements, assistive technologies, and accommodations may help students with disabilities access Zearn Math.

Accessible design features

We developed Zearn Math with a wide range of students in mind, and included accessibility features that ensure students with cognitive, physical, and communication challenges can easily use the self-paced, software-based lessons. These design features include:

Visual clarity:

All content in Independent Digital Lessons is visually clear and understandable. These visual accessibility features help all learners, but are particularly important for students with color blindness or any visual impairments:

- **Use of color:** Throughout Zearn Math Independent Digital Lessons, color is never used as the only visual means of conveying information. When a student receives precise feedback on an answer during Independent Digital Lessons, that feedback is provided in multiple ways—with color but also with clear iconography and specific messages such as “Nice,” “Try again,” or “Check the answer.” Additionally, where color is used to draw attention to a specific piece of information, Zearn Math also uses words to convey the same information.
- **Color contrast:** Zearn Math aims to conform to minimum color contrast requirements. Software-based lessons use larger fonts that meet a minimum contrast ratio of 3:1. Font smaller than 18pt or 14pt bold meet a contrast ratio of 4.5:1. Where specific elements of Independent Digital Lessons do not meet contrast standards today, Zearn is making improvements.
- **Font readability:** Throughout software-based lessons, Zearn Math avoids using fonts smaller than 10pt, with most text using at least 16pt fonts. Font types are simple, clear, and have limited variation in order to ensure all text is readable.

Volume consistency:

In order to provide a consistent and non-disruptive audio experience for students, there are no significant volume changes during Independent Digital Lessons. Outside of Zearn Math’s video content, there is no audio that plays automatically for more than 3 seconds. This accessibility feature is particularly important for students who are sensitive to changes in volume, students who have difficulty focusing on visual content (including text) when audio is playing, students on the autism spectrum, and students with hearing impairments.

Assistive technology

Assistive technology may be helpful to increase and maintain access for students with disabilities. Many assistive technology features are embedded into Zearn's digital materials.

Text to Speech:

All instructional prompts and directions that students see in the Zearn Math digital program can be read aloud by selecting buttons.

Screen Reader and Braille Translation Software:

Zearn Math can be accessed by screen reading software. All student-facing PDFs are screen-reader accessible. Screen readers enable blind students to read the text that is displayed on the computer screen with a speech synthesizer or braille display. However, students who are blind or have limited sight will need teacher, caregiver, or screen reader assistance in understanding Zearn Math's dynamic digital manipulatives within the Guided Practice, given the nature of how they are built.

Keyboard accessibility:

While Zearn Math requires the use of a mouse, trackpad, or touchscreen device today, Zearn has made keyboard accessibility a priority. In the coming years, Zearn will be adding new features to ensure that students can use keyboards for all interactive elements in Independent Digital Lessons. This accessibility feature is particularly important for students with impaired mobility or dexterity or students with low vision.

Instructional accommodations

Teachers can and should provide student specific accommodations for students with disabilities. The following accommodations may increase access for students:

Translated Materials:

All of Zearn's core student-facing paper-based instructional materials will be available during the 2022-2023 school year in various accessible formats, including large print, Braille, and tactile, from APH.org (American Printing House). Educators will be able to search APH's "Louis" catalog, and place orders for the Zearn Math materials they need. These materials will also be on file with the National Instructional Materials Accessibility Center (NIMAC).

Guided Notes and Graphic Organizers:

All Zearn Math lessons include Student Notes to help keep students focused and organized. Zearn also uses graphic organizers in digital content and in paper-based materials to help students organize and internalize information.

Read Aloud:

Students who struggle with word decoding and/or reading comprehension may benefit from having question prompts read aloud. Students who are blind or have limited sight may benefit from hearing oral descriptions of graphs and of other visual representations of problems or math concepts.

Scribe:

Students with scribe accommodation will need support transferring their math thinking, problem solving, and answers into digital form or as a written answer when prompted to write or input an answer.

Separate Location or Quiet Space:

When completing digital lessons, some students may benefit from working in a separate space where they can process out loud, work without headphones, and input text or numerical answers with their voice.

Breaks:

Students may benefit from structured breaks when completing Zearn Math lessons in order to rest or refocus. All Zearn Math digital lessons can be paused, rewound or restarted.

Checklists and Other Self-Monitoring Activities:

Self-monitoring checklists may be helpful for students to use in determining the best approach to solve a problem, guiding problem-solving processes, or evaluating work habits or progress made toward a goal.

Physical Math Manipulatives:

All students benefit from access to physical manipulatives. Zearn Math notes required materials in the Mission Overview and in each Lesson. For some students, more work with physical manipulatives may be beneficial.

Assessments and Reports

Zearn offers a series of formative assessments designed to provide teachers with precise and actionable feedback they can use to inform instruction and respond to the needs of each student, as well as student- and class-level reports that provide teachers with real-time data and insights into student pace, progress, and areas of struggle during Zearn Math digital lessons. To address areas of unfinished learning, Zearn Math contains both embedded supports within each Independent Digital Lesson, as well as foundational lessons that teachers may assign as interventions.

ONGOING FORMATIVE ASSESSMENTS

Assessments focus on the big ideas of mathematics and allow students to demonstrate their understanding across multiple modalities through a thoughtful balance of software- and paper-based experiences. All assessments are designed to fit into the classroom model and allocated time and to enhance — rather than distract from — instruction.

Daily lesson-level assessments: The Tower of Power (digital) and Exit Tickets (paper)

Lesson-level assessments are embedded into the curriculum and occur as part of recommended daily core instructional time, not in addition to it.

Tower of Power (digital):

This scaffolded assessment focuses on the content of a single lesson, and is administered automatically at the end of each Independent Digital Lesson. If students make a mistake they receive real-time support at the point of misconception, allowing them to correct their understanding and continue through the assessment. Each Tower of Power contains two to four stages of problems that increase in complexity and decrease in scaffolding as students progress. The problems in each stage are carefully designed to focus on the big ideas of each lesson, mirroring the progression of learning students have just completed. Students are not permitted to move on to their next Independent Digital lesson without successful completion of the Tower of Power. Since the Tower of Power is software-based, teachers can access a report to determine how well students are progressing through the Tower of Power assessments, enabling them to adjust instruction to support students' progress. *(See next section on Reports.)*

Exit Ticket (paper):

This assessment also focuses on the content of a single lesson, and it is administered at the culmination of each Lesson to help teachers monitor daily learning. As the companion to the Tower of Power, the Exit Ticket uses a single problem (or multiple problems where appropriate) to determine if the student can transfer their thinking and work from the Concept Exploration to an open-response item that requires students to show their thinking and work, including drawing models and/or writing explanations. Exit Ticket problems are designed to highlight the big mathematical idea of each lesson, or a piece thereof, and, as such, should not be edited.

Mission-level assessments (paper)

Mission-level assessments take an average of 30 minutes to complete and should be administered at the end of a Mission during built-in weekly Flex time; longer Missions will sometimes be broken into a Mid-Mission Assessment, roughly halfway through the Mission and an end-of-Mission assessment at the end, both of similar length. All questions assess student understanding of content within the specific Mission and do not include questions related to other Missions. Each assessment includes a carefully selected number of problems that give teachers rich feedback on student learning, while limiting the time students spend on any given assessment and teachers spend analyzing assessments.

These paper assessments consist of open-response items that require students to show their work or explain their thinking in a variety of ways, including drawing models and writing explanations, similar to an Exit Ticket. Some assessment items highlight a student's understanding of a big mathematical idea, while others focus on students' procedural fluency. Each part of any multi-step problem has a clear objective, is aligned with the explicit expectations of the target standards, and allows teachers to identify whether students are struggling with the foundational math concept or the multi-step aspect of the problem.

Zearn provides teachers with an answer key for each Mission-level assessment that contains an exemplar student response for each item, as well as specific standards-alignment information. Exemplar student responses can be used to inform teacher feedback, but are not the only correct answer or solution method; many of the problems on a Mission-level assessment allow students multiple entry points and acceptable solution paths or strategies.

Zearn also offers assessment rubrics to provide teachers with actionable feedback they can use to respond to the learning and misconceptions students demonstrate on the Mission-level assessment. Each rubric models a progression towards understanding, offering detailed examples of where students might go wrong, along with guidance on what incorrect answers may indicate about a student's unfinished learning.

The rubrics also include guidance on scoring each item. Scoring guidance helps teachers assess the depth of students' learning and provide students with the precise feedback they need to continue to develop their understanding. Each rubric is designed to produce a score out of 100 so that teachers can more easily input that score into their gradebooks. To help create inclusive classroom math communities, in which all students feel they belong and can deeply learn the math content of their grade, Zearn's scoring guidance raises the floor for grades, ensuring that any student who is at least initiating understanding on all items is guaranteed a minimum score of 60 or higher. When connecting possible scores to the Progression Towards Understanding, scores now have meaning: a student's score communicates where they are in the progression from initial understanding to full understanding.

- A student scoring above 90 points on an assessment should be considered as having **full understanding** of the content of the Mission.
- A student scoring between 80 and 90 points on an assessment should be considered as **nearing [full] understanding** of the content of the Mission.

- A student scoring between 70 and 80 points on an assessment should be considered as **developing understanding** of the content of the Mission.
- A student scoring between 60 and 70 points on an assessment should be considered as **initiating understanding** of the content of the Mission.

The points possible for each item vary based on the amount of understanding a student can demonstrate in any single item, in addition to the mathematical focus of the item and the extent to which it connects to the big ideas of the Mission.

*Note: Given the coherent structure of Zearn Math, if unfinished learning is evident on Mission-level assessments, teachers should move forward with additional supports and address misconceptions during collaborative Concept Exploration and on Flex Days, understanding that the unfinished learning may best be completed by connecting it to new ideas presented in the latter half of a Mission or a subsequent Mission. Students with unfinished learnings should also be supported during flexible math time or other specific intervention time with work on foundational lessons. For more information on Zearn Math’s approach to unfinished learning, see the Approach to Unfinished Learning section.

CLASS AND STUDENT REPORTS

Zearn provides teachers with reports that provide real-time visibility into student pace, progress, and areas of struggle during software-based learning, which they can use to differentiate instruction and ensure all students receive the support and enrichment they need. Go online to Zearn’s Help Center to find information about all of these reports, including video overviews.³ Zearn encourages teachers to check reports at least twice a week to stay up-to-date on students’ learning and to use the insights to inform instruction.

Class reports: Progress, Pace, and Tower Alerts

Progress Report:

This report shows teachers where each student is in the digital sequence of all grade-level content. Teachers can view the percentage of Independent Digital Lessons students have completed for each Mission. By checking the Progress Report, teachers can understand how far along students are in exploring grade-level math content.

Pace Report:

This report helps teachers keep students on track each week to complete the recommended four Independent Digital Lessons. Teachers can access a real-time view of how many lessons students have completed, the time it took to complete each lesson, and whether students have completed any bookmarked foundational lessons, if applicable (to learn more about foundational lessons, see the section on “Addressing unfinished learning” below). By checking the Pace Report, teachers can identify groups of students who need more time to meet weekly learning goals with Independent Digital Lessons and students who have already met their goal and can begin working on Bonuses for an extra challenge. Teachers can also use the Pace Report to track student progress on any foundation lessons the teacher has bookmarked, and can filter by lesson grade level to monitor how many still remain.

³ To learn about the Zearn Class Reports and Student Reports you may access, visit <https://help.zearn.org/hc/en-us/articles/4403432402071-Teacher-Reports>

Tower Alerts Report:

This report allows teachers to identify the part of the lesson where the student struggled, and whether or not the student was able to complete the independent practice portion of an Independent Digital Lesson and move on to the next Independent Digital Lesson. Mistakes are magic and not all struggle is bad:

- If a student struggles in the Tower of Power, the student receives scaffolded support at the precise moment of misconception called a Boost.
- If the Boost allows the student to move past their initial point of struggle, this struggle was productive.
- If a student struggles multiple times, it's a sign that they have moved past the point of productive struggle to unproductive struggle, and their teacher receives a Tower Alert. Teachers can then use the Tower Alerts Report to identify which students need additional support on specific concepts and can bookmark recommended foundational lessons for each of these students to complete. (For more, see the next section on unfinished learning.)

By checking the report at least twice each week, teachers can ensure no students are “stuck” with unproductive struggle in a Tower for long periods of time before they can move on to the next lesson and continue learning.

Student Reports

Zearn also offers Student Reports that contain real-time data and insights into student pace, progress, and areas of struggle during Independent Digital Lessons. Teachers can use these reports, along with other formative assessment data, to gain insight into individual student learning, including topics where that student excels and topics where they may still struggle.

Within a Student Report, teachers can see the breakdown of Pace, Progress, and Tower Alerts, as well as all the activities that the student has completed and when they did so. Combined with Zearn's automated recommendations on foundational lessons that support students with unfinished learning (see next section), this report gives teachers the information they need to choose deeper interventions when necessary.

APPROACH TO UNFINISHED LEARNING

Zearn Math helps teachers address students' unfinished learning in the context of new learning, setting students up for success with just-in-time supports built into daily digital lessons, as well as targeted foundational lessons that are coherently aligned to their grade's core content:

Embedded supports:

Each Zearn Math grade-level digital lesson includes built-in support on concepts from previous topics and grades, so that students can strengthen foundational understanding while learning grade-level concepts.

- **In-lesson adaptive support:** All Independent Digital Lessons contain built-in supportive pathways that teach new concepts through concrete and pictorial representations that help students make sense of new concepts by anchoring to ideas they already know or intuitively make sense to them. This approach emphasizes the big ideas in mathematics and strengthens conceptual and procedural knowledge to address unfinished learning so that students can move smoothly to and make connections with other mathematics.
- **Boosts help during struggle:** In addition, the Tower of Power, Zearn's embedded daily diagnostic, assesses each student's understanding and automatically launches a Boost exactly when kids need it, with support

and scaffolding they need from prior grades or prior units. Thus, Zearn continually assesses, diagnoses, and gives kids the Boost they need, built into their grade-level learning.

Foundational lessons to address significant unfinished learning:

If a student continues to struggle, teachers receive a notification in their Tower Alerts Report, which they should monitor regularly. Teachers can then check the Student Report to see precisely which topics a student may be excelling in, which topics a student may be struggling with, and how deep the struggle is. The information in the Student Reports empowers teachers to assess struggle side-by-side with other information—such as productivity of the struggle, where in the scope and sequence struggle is occurring, and other formative assessment data—so that teachers are empowered with the full information they need to choose deeper interventions when necessary. In addition to alerts and reports, Zearn helps teachers address misconceptions and unfinished learning through a recommendation engine that suggests precise targeted foundational content that will be most supportive based on an individual student’s area of struggle:

- **Bookmark foundational lessons:** Teachers can bookmark foundational lessons recommended by Zearn as an additional assignment to be completed outside of the core math block. The Zearn team has identified foundational lessons based on an analysis of data on student struggle from all problems completed in our digital lessons. Each foundational lesson focuses on the big math idea that connects with and promotes the same grade-level content students are learning during their math block.
- **Direct to Math Library:** Students can then access their bookmarked foundational lessons alongside their grade-level assignments in their Math Library, and spend flexible math time or other specific intervention time working on these foundational lessons in a way that is directly tied to core grade-level learning. Students can access both their foundational and grade-level assignments on Zearn’s online math platform with the same login. Zearn’s student experience is designed to feel safe and supportive, so students do not see the word “intervention” or the grade level of the bookmarked lessons. Zearn Math’s database provides the essential foundational lessons for understanding specific grade-level math concepts, as well as an additional layer of support lessons that may be helpful to students.
- **Monitor student progress:** Teachers and administrators can log in to their Zearn Accounts to track student progress on unfinished learning. In their digital reports, teachers and administrators can see progress on a student’s lesson assignments (including grade-level and bookmarked foundational lessons) and areas of struggle. Administrators will be able to see this same information for the school, not just the class and student.

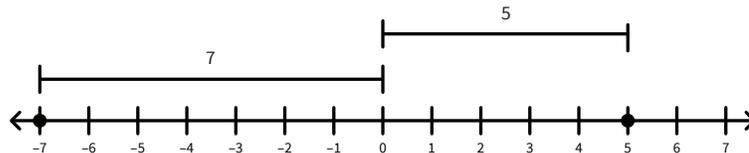
This targeted and coherent approach maximizes effectiveness by allowing students to move fluidly between grade-level and intervention content as needed in order to fill conceptual gaps and get back to grade-level learning as quickly as possible.

Terminology

- Absolute value**

The absolute value of a number is its distance from 0 on the number line.

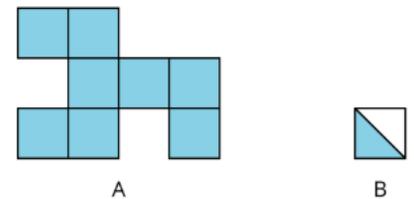
The absolute value of -7 is 7, because it is 7 units away from 0. The absolute value of 5 is 5, because it is 5 units away from 0.



- Area**

Area is the number of square units that covers a two-dimensional region, without any gaps or overlaps.

For example, the area of region A is 8 square units. The area of the shaded region of B is $\frac{1}{2}$ square unit.



- Average**

The average is another name for the mean of a data set.

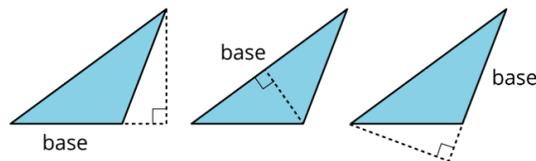
For the data set 3, 5, 6, 8, 11, 12, the average is 7.5.

$$3 + 5 + 6 + 8 + 11 + 12 = 45$$

$$45 \div 6 = 7.5$$

- Base (of a parallelogram or triangle)**

We can choose any side of a parallelogram or triangle to be the shape's base. Sometimes we use the word base to refer to the length of this side.

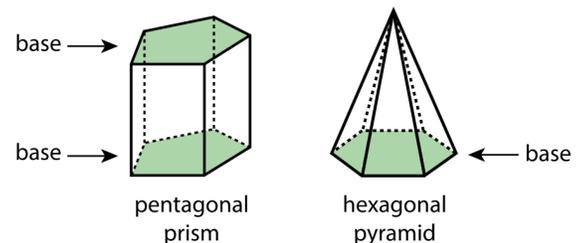


- Base (of a prism or pyramid)**

The word *base* can also refer to a face of a polyhedron.

A prism has two identical bases that are parallel. A pyramid has one base.

A prism or pyramid is named for the shape of its base.



- **Box plot**

A box plot is a way to represent data on a number line. The data is divided into four sections. The sides of the box represent the first and third quartiles. A line inside the box represents the median. Lines outside the box connect to the minimum and maximum values.

For example, this box plot shows a data set with a minimum of 2 and a maximum of 15. The median is 6, the first quartile is 5, and the third quartile is 10.



number of books

- **Categorical data**

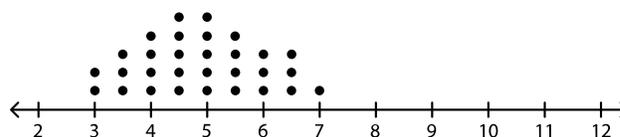
A set of categorical data has values that are words instead of numbers.

For example, Han asks 5 friends to name their favorite color. Their answers are: blue, blue, green, blue, orange.

- **Center**

The center of a set of numerical data is a value in the middle of the distribution. It represents a typical value for the data set.

For example, the center of this distribution of cat weights is between 4.5 and 5 kilograms.



cat weights in kilograms

- **Coefficient**

A coefficient is a number that is multiplied by a variable.

For example, in the expression $3x + 5$, the coefficient of x is 3. In the expression $y + 5$, the coefficient of y is 1, because $y = 1 \cdot y$.

- **Common factor**

A common factor of two numbers is a number that divides evenly into both numbers. For example, 5 is a common factor of 15 and 20, because $15 \div 5 = 3$ and $20 \div 5 = 4$. Both of the quotients, 3 and 4, are whole numbers.

The factors of 15 are 1, 3, 5, and 15.

The factors of 20 are 1, 2, 4, 5, 10, and 20.

- **Common multiple**

A common multiple of two numbers is a product you can get by multiplying each of the two numbers by some whole number. For example, 30 is a common multiple of 3 and 5, because $3 \cdot 10 = 30$ and $5 \cdot 6 = 30$. Both of the factors, 10 and 6, are whole numbers.

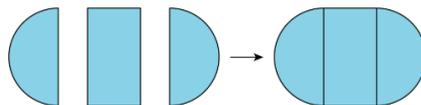
The multiples of 3 are 3, 6, 9, 12, 15, 18, 21, 24, 27, 30, 33 . . .

The multiples of 5 are 5, 10, 15, 20, 25, 30, 35, 40 . . .

The common multiples of 3 and 5 are 15, 30, 45, 60 . . .

- Compose**

Compose means “put together.” We use the word compose to describe putting more than one figure together to make a new shape.

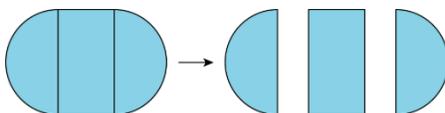


- Cubed**

We use the word *cubed* to mean “to the third power.” This is because a cube with side length s has a volume of $s \cdot s \cdot s$, or s^3 .

- Decompose**

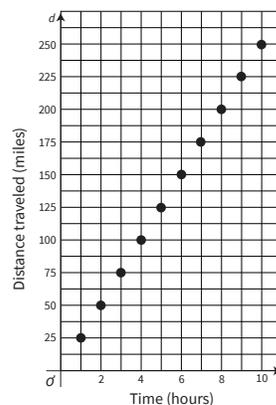
Decompose means “take apart.” We use the word decompose to describe taking a figure apart to make more than one new shape.



- Dependent variable**

The dependent variable is the result of a calculation.

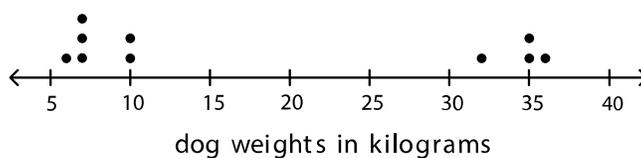
For example, a boat travels at a constant speed of 25 miles per hour. The equation $d = 25t$ describes the relationship between the boat’s distance and time. The dependent variable is the distance traveled, because d is the result of multiplying 25 by t .



- Distribution**

The distribution tells how many times each value occurs in a data set. For example, in the data set blue, blue, green, blue, orange, the distribution is 3 blues, 1 green, and 1 orange.

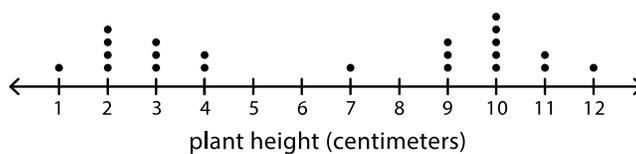
Here is a dot plot that shows the distribution for the data set 6, 10, 7, 35, 7, 36, 32, 10, 7, 35.



- Dot plot**

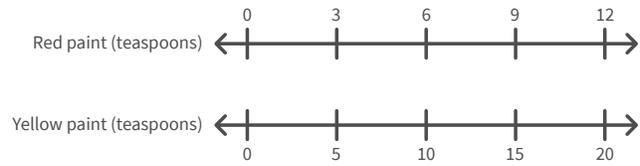
A dot plot is a way to represent data on a number line. Each time a value appears in the data set, we put another dot above that number on the number line.

For example, in this dot plot there are three dots above the 9. This means that three different plants had a height of 9 cm.



- **Double number line diagram**

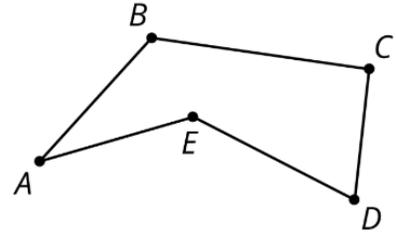
A double number line diagram uses a pair of parallel number lines to represent equivalent ratios. The locations of the tick marks match on both number lines. The tick marks labeled 0 line up, but the other numbers are usually different.



- **Edge**

Each straight side of a polygon is called an edge.

For example, the edges of this polygon are segments AB, BC, CD, DE, and EA.



- **Equivalent expressions**

Equivalent expressions are always equal to each other. If the expressions have variables, they are equal whenever the same value is used for the variable in each expression.

For example, $3x + 4x$ is equivalent to $5x + 2x$. No matter what value we use for x , these expressions are always equal. When $x = 3$, both expressions equal 21. When $x = 10$, both expressions equal 70.

- **Equivalent ratios**

Two ratios are equivalent if you can multiply each of the numbers in the first ratio by the same factor to get the numbers in the second ratio. For example, $6 : 8$ is equivalent to $3 : 4$, because $8 \cdot \frac{1}{2} = 4$ and $6 \cdot \frac{1}{2} = 3$.

Cups of water	Number of lemons
8	6
4	3

A recipe for lemonade says to use 8 cups of water and 6 lemons. If we use 4 cups of water and 3 lemons, it will make half as much lemonade. Both recipes taste the same, because $8 : 6$ and $4 : 3$ are equivalent ratios.

- **Exponent**

In expressions like 5^3 and 8^2 , the 3 and the 2 are called exponents. They tell you how many factors to multiply. For example, $5^3 = 5 \cdot 5 \cdot 5$, and $8^2 = 8 \cdot 8$.

- **Face**

Each flat side of a polyhedron is called a face. For example, a cube has 6 faces, and they are all squares.

- **Frequency**

The frequency of a data value is how many times it occurs in the data set.

For example, there were 20 dogs in a park. The table shows the frequency of each color.

Color	Frequency
white	4
black	7
brown	3
multi-color	6

- **Greatest common factor**

The greatest common factor of two numbers is the largest number that divides evenly into both numbers. Sometimes we call this the GCF. For example, 15 is the greatest common factor of 45 and 60.

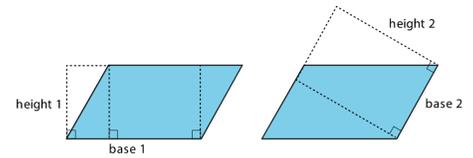
The factors of 45 are 1, 3, 5, 9, 15, and 45.

The factors of 60 are 1, 2, 3, 4, 5, 6, 10, 12, 15, 20, 30, and 60.

- **Height (of a parallelogram or triangle)**

The height is the shortest distance from the base of the shape to the opposite side (for a parallelogram) or opposite vertex (for a triangle).

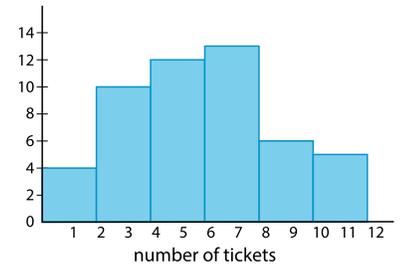
We can show the height in more than one place, but it will always be perpendicular to the chosen base.



- **Histogram**

A histogram is a way to represent data on a number line. Data values are grouped by ranges. The height of the bar shows how many data values are in that group.

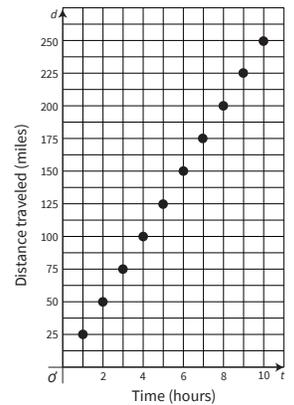
For example, this histogram shows there were 10 people who earned 2 or 3 tickets. We can't tell how many of them earned 2 tickets or how many earned 3.



- **Independent variable**

The independent variable is used to calculate the value of another variable.

For example, a boat travels at a constant speed of 25 miles per hour. The equation $d = 25t$ describes the relationship between the boat's distance and time. The independent variable is time, because t is multiplied by 25 to get d .



- **Interquartile range (IQR)**

The interquartile range is one way to measure how spread out a data set is. We sometimes call this the IQR. To find the interquartile range we subtract the first quartile from the third quartile.

22	29	30	31	32	43	44	45	50	50	59
		Q1			Q2			Q3		

For example, the IQR of this data set is 20 because $50 - 30 = 20$.

- **Least common multiple**

The least common multiple of two numbers is the smallest product you can get by multiplying each of the two numbers by some whole number. Sometimes we call this the LCM. For example, 30 is the least common multiple of 6 and 10.

The multiples of 6 are 6, 12, 18, 24, 30, 36, 42, 48, 54, 60 . . .

The multiples of 10 are 10, 20, 30, 40, 50, 60, 70, 80 . . .

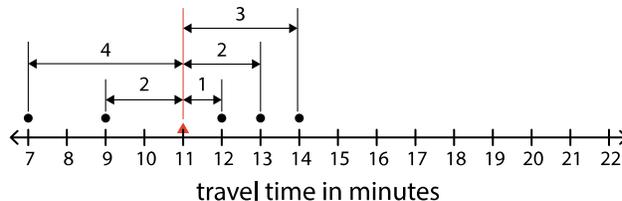
- **Long division**

Long division is a way to show the steps for dividing numbers in decimal form. It finds the quotient one digit at a time, from left to right. For example, here is the long division for $57 \div 4$.

$$\begin{array}{r} 14.25 \\ 4 \overline{)57.00} \\ \underline{-4} \\ 17 \\ \underline{-16} \\ 10 \\ \underline{-8} \\ 20 \\ \underline{-20} \\ 0 \end{array}$$

- **Mean**

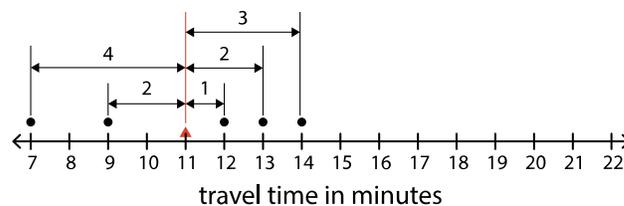
The mean is one way to measure the center of a data set. We can think of it as a balance point. For example, for the data set 7, 9, 12, 13, 14, the mean is 11.



To find the mean, add up all the numbers in the data set. Then, divide by how many numbers there are. $7 + 9 + 12 + 13 + 14 = 55$ and $55 \div 5 = 11$.

- **Mean absolute deviation (MAD)**

The mean absolute deviation is one way to measure how spread out a data set is. Sometimes we call this the MAD. For example, for the data set 7, 9, 12, 13, 14, the MAD is 2.4. This tells us that these travel times are typically 2.4 minutes away from the mean, which is 11.



To find the MAD, add up the distance between each data point and the mean. Then, divide by how many numbers there are. $4 + 2 + 1 + 2 + 3 = 12$ and $12 \div 5 = 2.4$.

- **Measure of center**

A measure of center is a value that seems typical for a data distribution.

Mean and median are both measures of center.

Median

The median is one way to measure the center of a data set. It is the middle number when the data set is listed in order.

For the data set 7, 9, 12, 13, 14, the median is 12.

For the data set 3, 5, 6, 8, 11, 12, there are two numbers in the middle. The median is the average of these two numbers. $6 + 8 = 14$ and $14 \div 2 = 7$.

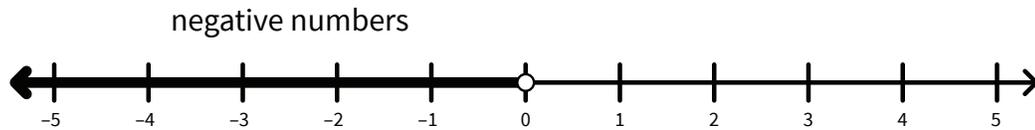
- **Meters per second**

Meters per second is a unit for measuring speed. It tells how many meters an object goes in one second.

For example, a person walking 3 meters per second is going faster than another person walking 2 meters per second.

- **Negative number**

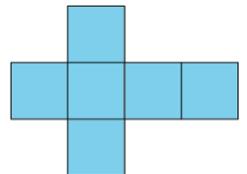
A negative number is a number that is less than zero. On a horizontal number line, negative numbers are usually shown to the left of 0.



- **Net**

A net is a two-dimensional figure that can be folded to make a polyhedron.

Here is a net for a cube.



- **Numerical data**

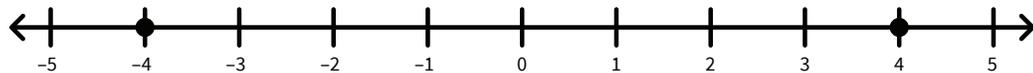
A set of numerical data has values that are numbers.

For example, Han lists the ages of people in his family: 7, 10, 12, 36, 40, 67.

- **Opposite**

Two numbers are opposites if they are the same distance from 0 and on different sides of the number line.

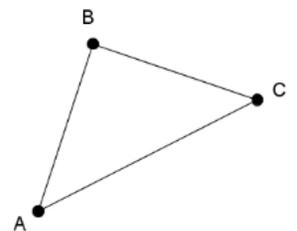
For example, 4 is the opposite of -4, and -4 is the opposite of 4. They are both the same distance from 0. One is negative, and the other is positive.



- **Opposite Vertex**

For each side of a triangle, there is one vertex that is not on that side. This is the opposite vertex.

For example, point *A* is the opposite vertex to side *BC*.



- **Pace**

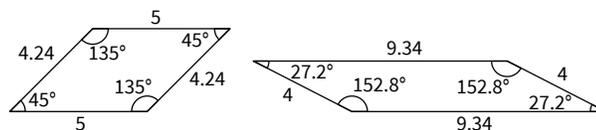
Pace is one way to describe how fast something is moving. Pace tells how much time it takes the object to travel a certain distance.

For example, Diego walks at a pace of 10 minutes per mile. Elena walks at a pace of 11 minutes per mile. Elena walks slower than Diego, because it takes her more time to travel the same distance.

- **Parallelogram**

A parallelogram is a four-sided polygon with two pairs of parallel sides.

Here are two examples of parallelograms.



- Per**

The word *per* means “for each.” For example, if the price is \$5 per ticket, that means you will pay \$5 for *each* ticket. Buying 4 tickets would cost \$20, because $4 \cdot 5 = 20$.

- Percent**

The word percent means “for each 100.” The symbol for percent is %.

For example, a quarter is worth 25 cents, and a dollar is worth 100 cents. We can say that a quarter is worth 25% of a dollar.



1 Quarter

25¢

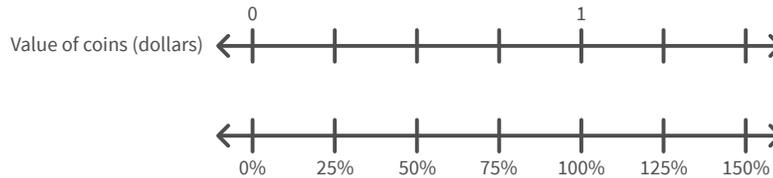
1 Dollar

100¢

- Percentage**

A percentage is a rate per 100.

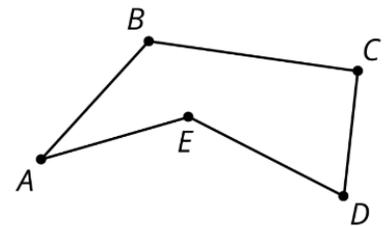
For example, a fish tank can hold 36 liters. Right now there is 27 liters of water in the tank. The percentage of the tank that is full is 75%.



- Polygon**

A polygon is a closed, two-dimensional shape with straight sides that do not cross each other.

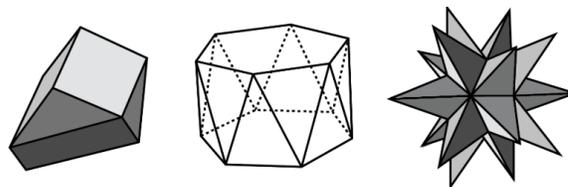
Figure ABCDE is an example of a polygon.



- Polyhedron**

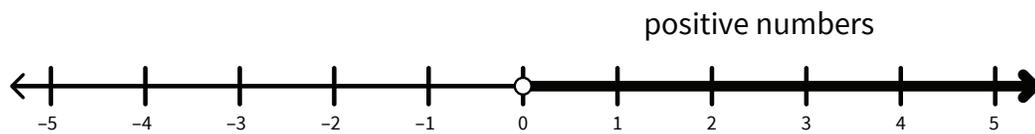
A polyhedron is a closed, three-dimensional shape with flat sides. When we have more than one polyhedron, we call them polyhedra.

Here are some drawings of polyhedra.



- **Positive number**

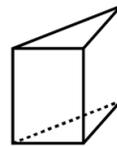
A positive number is a number that is greater than zero. On a horizontal number line, positive numbers are usually shown to the right of 0.



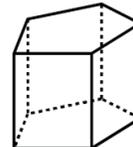
- **Prism**

A prism is a type of polyhedron that has two bases that are identical copies of each other. The bases are connected by rectangles or parallelograms.

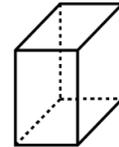
Here are some drawings of prisms.



triangular
prism



pentagonal
prism



rectangular
prism

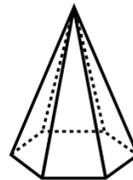
- **Pyramid**

A pyramid is a type of polyhedron that has one base. All the other faces are triangles, and they all meet at a single vertex.

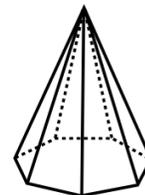
Here are some drawings of pyramids.



rectangular
pyramid



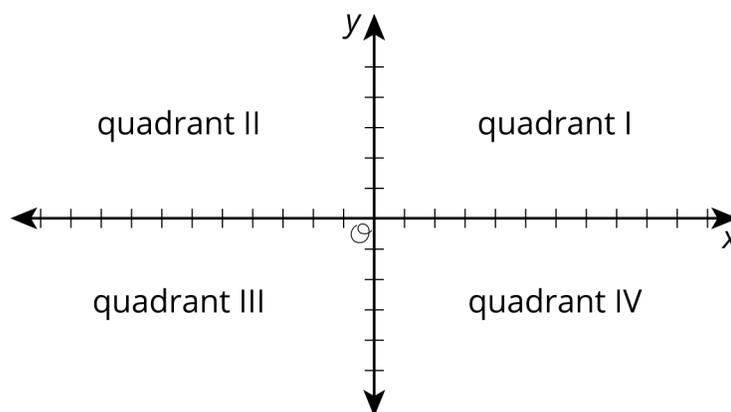
hexagonal
pyramid



heptagonal
pyramid

- **Quadrant**

The coordinate plane is divided into 4 regions called quadrants. The quadrants are numbered using Roman numerals, starting in the top right corner.



- **Quadrilateral**

A quadrilateral is a type of polygon that has 4 sides. A rectangle is an example of a quadrilateral. A pentagon is not a quadrilateral, because it has 5 sides.

- **Quartile**

Quartiles are the numbers that divide a data set into four sections that each have the same number of values.

For example, in this data set the first quartile is 20. The second quartile is the same thing as the median, which is 33. The third quartile is 40.

12	19	20	21	22	33	34	35	40	40	49
		Q1			Q2			Q3		

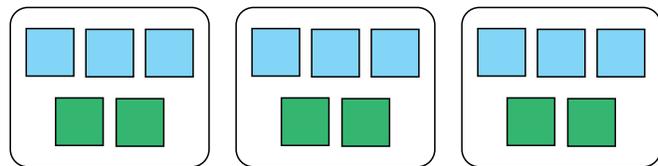
- **Range**

The range is the distance between the smallest and largest values in a data set. For example, for the data set 3, 5, 6, 8, 11, 12, the range is 9, because $12 - 3 = 9$.

- **Ratio**

A ratio is an association between two or more quantities.

For example, the ratio 3 : 2 could describe a recipe that uses 3 cups of flour for every 2 eggs, or a boat that moves 3 meters every 2 seconds. One way to represent the ratio 3: 2 is with a diagram that has 3 blue squares for every 2 green squares.



- **Rational number**

A rational number is a fraction or the opposite of a fraction.

For example, 8 and -8 are rational numbers because they can be written as $\frac{8}{1}$ and $-\frac{8}{1}$.

Also, 0.75 and -0.75 are rational numbers because they can be written as $\frac{75}{100}$ and $-\frac{75}{100}$.

- **Region**

A region is the space inside of a shape. Some examples of two-dimensional regions are inside a circle or inside a polygon. Some examples of three-dimensional regions are the inside of a cube or the inside of a sphere.

- **Same rate**

We use the words *same rate* to describe two situations that have equivalent ratios.

For example, a sink is filling with water at a rate of 2 gallons per minute. If a tub is also filling with water at a rate of 2 gallons per minute, then the sink and the tub are filling at the same rate.

- **Sign**

The sign of any number other than 0 is either positive or negative.

For example, the sign of 6 is positive. The sign of -6 is negative. Zero does not have a sign, because it is not positive or negative.

- **Solution to an equation**

A solution to an equation is a number that can be used in place of the variable to make the equation true.

For example, 7 is the solution to the equation $m + 1 = 8$, because it is true that $7 + 1 = 8$. The solution to $m + 1 = 8$ is not 9, because $9 + 1 \neq 8$.

- **Solution to an inequality**

A solution to an inequality is a number that can be used in place of the variable to make the inequality true.

For example, 5 is a solution to the inequality $c < 10$, because it is true that $5 < 10$. Some other solutions to this inequality are 9.9, 0, and -4.

- **Speed**

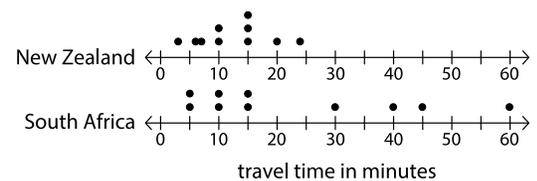
Speed is one way to describe how fast something is moving. Speed tells how much distance the object travels in a certain amount of time.

For example, Tyler walks at a speed of 4 miles per hour. Priya walks at a speed of 5 miles per hour. Priya walks faster than Tyler, because she travels more distance in the same amount of time.

- **Spread**

The spread of a set of numerical data tells how far apart the values are.

For example, the dot plots show that the travel times for students in South Africa are more spread out than for New Zealand.



- **Squared**

We use the word *squared* to mean “to the second power.” This is because a square with side length s has an area of $s \cdot s$, or s^2 .

- **Statistical question**

A statistical question can be answered by collecting data that has variability. Here are some examples of statistical questions:

- How many minutes do sixth grade students spend on homework each week?
- What is the typical bedtime of a seventh grade student?
- How many pets does an eighth grade student have?

- **Surface Area**

The surface area of a polyhedron is the number of square units that covers all the faces of the polyhedron, without any gaps or overlaps.

For example, if the faces of a cube each have an area of 9 cm^2 , then the surface area of the cube is $6 \cdot 9$, or 54 cm^2 .

- **Table**

A table organizes information into horizontal rows and vertical columns. The first row or column usually tells what the numbers represent.

For example, here is a table showing the tail lengths of three different pets. This table has four rows and two columns.

Pet	Tail length (inches)
Dog	225
Cat	-150
Mouse	-475

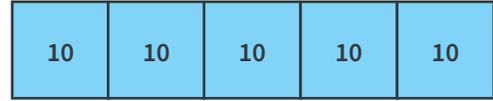
- **Tape diagram**

A tape diagram is a group of rectangles put together to represent a relationship between quantities.

For example, this tape diagram shows a ratio of 30 gallons of yellow paint to 50 gallons of blue paint.



If each rectangle were labeled 5, instead of 10, then the same picture could represent the equivalent ratio of 15 gallons of yellow paint to 25 gallons of blue paint.



- **Unit price**

The unit price is the cost for one item or for one unit of measure.

For example, if 10 feet of chain link fencing cost \$150, then the unit price is $150 \div 10$, or \$15 per foot.

- **Unit rate**

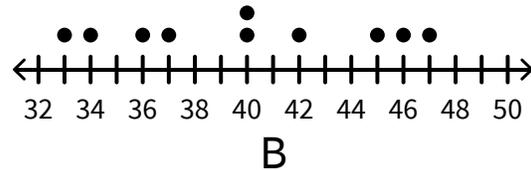
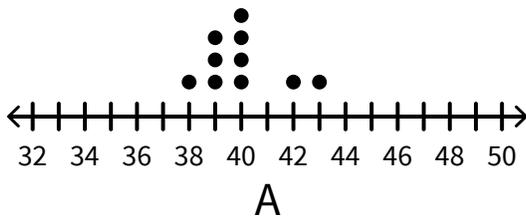
A unit rate is a rate per 1.

For example, 12 people share 2 pies equally. One unit rate is 6 people per pie, because $12 \div 2 = 6$. The other unit rate is $\frac{1}{6}$ of a pie per person, because $2 \div 12 = \frac{1}{6}$.

- **Variability**

Variability means having different values.

For example, data set B has more variability than data set A. Data set B has many different values, while data set A has more of the same values.



- **Variable**

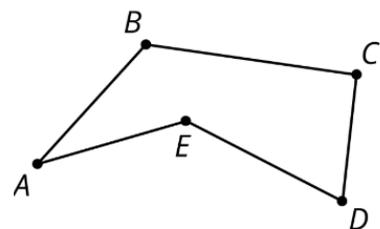
A variable is a letter that represents a number. You can choose different numbers for the value of the variable.

For example, in the expression $10 - x$, the variable is x . If the value of x is 3, then $10 - x = 7$, because $10 - 3 = 7$. If the value of x is 6, then $10 - x = 4$, because $10 - 6 = 4$.

- **Vertex**

A vertex is a point where two or more edges meet. When we have more than one vertex, we call them vertices.

The vertices in this polygon are labeled A, B, C, D, and E.



Required Materials

- $\frac{1}{2}$ -inch cubes
- $\frac{1}{2}$ -inch graph paper
- base-ten blocks
- beakers
- bingo chips
- blank paper
- colored pencils
- computers
- copies of template
- cuisenaire rods
- decks of playing cards
- demonstration nets with and without flaps
- dot stickers
- drink mix
- empty containers
- food coloring
- four-function calculators
- gallon-sized jug
- geometry toolkits
- glue or gluesticks
- graduated cylinders
- graph paper
- grocery store circulars
- headphones, and connection requirements
- household items
- inch cubes
- index cards
- internet-enabled device
- liter-sized bottle
- markers
- masking tape
- materials assembled from the template
- measuring tapes
- metal paper fasteners
- meter sticks
- nets of polyhedra
- origami paper
- paper cups
- pattern blocks
- pre-assembled or commercially produced polyhedra
- pre-assembled or commercially produced tangrams
- pre-assembled polyhedra
- pre-printed cards, cut from copies of the template
- pre-printed slips, cut from copies of the template
- quart-sized bottle
- rulers
- rulers marked with centimeters
- rulers marked with inches
- salt
- scale
- scissors
- snap cubes
- sticky notes
- stopwatches
- straightedges
- string
- students' collections of objects
- tape
- teacher's collection of objects
- teaspoon
- tools for creating a visual display
- tracing paper
- tray
- water
- yardsticks

Appendix I: Instructional Routines

The kind of instruction appropriate in any particular lesson depends on the learning goals of that lesson. Some lessons may be devoted to developing a concept, others to mastering a procedural skill, yet others to applying mathematics to a real-world problem. These aspects of mathematical proficiency are interwoven into Zearn Math. The Zearn Math Teacher Edition includes a small set of activity structures and references a small, high-leverage set of teacher moves that become more and more familiar to teachers and students as the year progresses.

ALGEBRA TALK

What:

One expression is displayed at a time. Students are given a few minutes to quietly think and give a signal when they have an answer and a strategy. The teacher selects students to share different strategies for each one (“Who thought about it a different way?”). Their explanations are recorded for all to see. Students might be pressed to provide more details about why they decided to approach a problem a certain way. It may not be possible to share every possible strategy for the given limited time; the teacher may only gather two or three distinctive strategies per problem. Problems are purposefully chosen to elicit different approaches.

Where:

Warm Up

Why:

Algebra Talks build algebraic thinking by encouraging students to think about the numbers and variables in an expression and rely on what they know about structure, patterns, and properties of operations to mentally solve a problem. Algebra Talks promote seeing structure in expressions and thinking about how changing one number affects others in an equation. While participating in these activities, students need to be precise in their word choice and use of language.

ANTICIPATE, MONITOR, SELECT, SEQUENCE, CONNECT

What:

Fans of 5 Practices for Orchestrating Productive Mathematical Discussions (Smith and Stein, 2011) will recognize these as the 5 Practices. In this curriculum, much of the work of anticipating, sequencing, and connecting is handled by the materials in the activity narrative, launch, and synthesis sections. Teachers will need to develop their capacity to prepare for and conduct whole-class discussions.

Where:

Warm Up, Concept Exploration, Wrap Up

Why:

In Zearn Math, many activities can be described as “do math and talk about it,” but the 5 Practices lend more structure to these activities so that they more reliably result in students making connections and learning new mathematics.

NOTICE AND WONDER

What:

Students are shown some media or a mathematical representation. The prompt to students is “What do you notice? What do you wonder?” Students are given a few minutes to write down things they notice and things they wonder. After students have had a chance to write down their responses, the teacher asks several students to share things they noticed and things they wondered; these are recorded by the teacher for all to see. Usually, the teacher steers the conversation to wondering about something mathematical that the class is about to focus on.

Where:

Warm Up, Concept Exploration

Why:

The purpose is to make a mathematical task accessible for all students with these two low-stakes questions; by thinking about them and responding, students gain entry into the context and might get their curiosity piqued. Taking steps to become familiar with a context and the mathematics that might be involved is making sense of problems. Note: Notice and Wonder and I Notice/I Wonder are trademarks of NCTM and the Math Forum and used in these materials with permission.

NUMBER TALK

What:

One problem is displayed at a time. Students are given a few minutes to quietly think and give a signal when they have an answer and a strategy. The teacher selects students to share different strategies for each problem (“Who thought about it a different way?”). Their explanations are recorded for all to see. Students might be pressed to provide more details about why they decided to approach a problem a certain way. It may not be possible to share every possible strategy for the given limited time; the teacher may only gather two or three distinctive strategies per problem. Problems are purposefully chosen to elicit different approaches, often in a way that builds from one problem to the next.

Where:

Warm Up

Why:

Number talks build computational fluency by encouraging students to think about the numbers in a computation problem and rely on what they know about structure, patterns, and properties of operations to mentally solve a problem. Dot images are similar to number talks, except the image used is an arrangement of dots that students might count in different ways. While participating in these activities, students need to be precise in their word choice and use of language.

POLL THE CLASS

What:

Used to register an initial response or an estimate, most often in activity launches or to kick off a discussion - every student in class reports a response to a prompt. This can also be used when data needs to be collected from

each student in class, for example, “What is the length of your ear in centimeters?” Teachers need to develop a mechanism by which poll results are collected and displayed so that this frequent form of classroom interaction is seamless. Smaller classes might be able to conduct a roll call by voice. For larger classes, students might be given mini-whiteboards or a set of colored index cards to hold up. Free and paid commercial tools are also readily available.

Where:

Concept Exploration

Why:

Going on record with an estimate or a gut reaction makes people want to know if they were right and increases investment in the outcome. If coming up with an estimate is too daunting, ask students for a guess that they are sure is too low or too high. Putting some boundaries on possible outcomes of a problem is an important skill for mathematical modeling. Collecting data from the class to use in an activity makes the outcome of the activity more interesting.

TAKE TURNS

What:

Students work with a partner or small group. They take turns in the work of the activity, whether it be spotting matches, explaining, justifying, agreeing or disagreeing, or asking clarifying questions. If they disagree, they are expected to support their case and listen to their partner’s arguments. The first few times students engage in these activities, the teacher should demonstrate, with a partner, how the discussion is expected to go. Once students are familiar with these structures, less setup will be necessary. While students are working, the teacher can ask students to restate their question more clearly or paraphrase what their partner said.

Where:

Concept Exploration

Why:

Building in an expectation, through the routine, that students explain the rationale for their choices and listen to another’s rationale deepens the understanding that can be achieved through these activities. Specifying that students take turns deciding, explaining, and listening limits the phenomenon where one student takes over and the other does not participate. Taking turns can also give students more opportunities to construct logical arguments and critique others’ reasoning.

THINK PAIR SHARE

What:

Students have quiet time to think about a problem and work on it individually, and then time to share their response or their progress with a partner. Once these partner conversations have taken place, some partnerships are selected to share their thoughts with the class.

Where:

Concept Exploration

Why:

This is a teaching routine useful in many contexts whose purpose is to give all students enough time to think about a prompt and form a response before they are expected to try to verbalize their thinking. First they have an opportunity to share their thinking in a low-stakes way with one partner, so that when they share with the class they can feel calm and confident, as well as say something meaningful that might advance everyone's understanding. Additionally, the teacher has an opportunity to eavesdrop on the partner conversations so that she can purposefully select students to share with the class.

TRUE OR FALSE

What:

One statement is displayed at a time. Students are given a few minutes to quietly think and give a signal when they have an answer. The teacher selects students to share different ways of reasoning for each statement (“Who thought about it a different way?”). While students may evaluate each side of the equation to determine if it is true or false, encourage students to think about ways to reason that do not require complete computations. It may not be possible to share every possible reasoning approach for the given limited time; the teacher may only gather two or three distinctive strategies per problem. Statements are purposefully chosen to elicit different approaches, often in a way that builds from one statement to the next.

Where:

Warm Up

Why:

Depending on the purpose of the task, the true or false structure encourages students to reason about numeric and algebraic expressions using base-ten structure, the meaning of fractions, meaning and properties of operations, and the meaning of comparison symbols. While the structure of a true or false is similar to that of a number talk, number talks are often focused on computational strategies, while true or false tasks are more likely to focus on more structural aspects of the expressions. Often students can determine whether an equation, an inequality, or a statement is true or false without doing any direct computation. While participating in these activities, students need to be precise in their word choice and use of language.

WHICH ONE DOESN'T BELONG?

What:

Students are presented with four figures, diagrams, graphs, or expressions with the prompt “Which one doesn't belong?” Typically, each of the four options “doesn't belong” for a different reason, and the similarities and differences are mathematically significant. Students are prompted to explain their rationale for deciding that one option doesn't belong and given opportunities to make their rationale more precise.

Where:

Warm Up

Why:

Which One Doesn't Belong fosters a need to define terms carefully and use words precisely in order to compare and contrast a group of geometric figures or other mathematical representations.

GROUP PRESENTATIONS

Some activities instruct students to work in small groups to solve a problem with mathematical modeling, invent a new problem, design something, or organize and display data, and then create a visual display of their work. Teachers need to help groups organize their work so that others can follow it, and then facilitate different groups' presentation of work to the class. Teachers can develop specific questioning skills to help more students make connections and walk away from these experiences with desired mathematical learning. For example, instead of asking if anyone has any questions for the group, it is often more productive to ask a member of the class to restate their understanding of the group's findings in their own words.

Appendix II: Access for Multilingual Learners

INTRODUCTION

Zearn Math for Sixth Grade builds on foundational principles for supporting language development for all students. This appendix aims to provide guidance to help teachers recognize and support students' language development in the context of mathematical sense-making. Embedded within the Zearn Math Teacher Edition are instructional supports and practices to help teachers address the specialized academic language demands in math when planning and delivering lessons, including the demands of reading, writing, speaking, listening, conversing, and representing in math (Aguirre & Bunch, 2012). Therefore, while these instructional supports and practices can and should be used to support all students learning mathematics, they are particularly well-suited to meet the needs of linguistically and culturally diverse students who are learning mathematics while simultaneously acquiring English.

This table reflects the attention and support for language development at each level of the Zearn Math curriculum:

Course	<ul style="list-style-type: none"> • foundation of curriculum: theory of action and design principles that drive a continuous focus on language development • student terminology
Mission	<ul style="list-style-type: none"> • Mission-specific progression of language development included in each Mission Overview
Lesson	<ul style="list-style-type: none"> • language goals embedded in learning goals describe the language demands of the lesson • definitions of new terminology • additional supports for multilingual learners based on language demands of the activity
Activity	<ul style="list-style-type: none"> • additional supports for multilingual learners based on language demands of the activity • mathematical language routines

THEORY OF ACTION

Zearn believes that language development can be built into teachers' instructional practice and students' classroom experience through intentional design of materials, teacher commitments, administrative support, and professional development. Our theory of action is grounded in the interdependence of language learning and content learning, the importance of scaffolding routines that foster students' independent participation, the value of instructional responsiveness in the teaching process, and the central role of student agency in the learning process.

Mathematical understandings and language competence develop interdependently.

Deep conceptual learning is gained through language. Ideas take shape through words, texts, illustrations, conversations, debates, examples, etc. Teachers, peers, and texts serve as language resources for learning. Instructional attention to academic language development, historically limited to vocabulary instruction, has now shifted to also include instruction around the demands of argumentation, explanation, generalization, analyzing the purpose and structure of text, and other mathematical discourse.

Scaffolding provides temporary supports that foster student autonomy.

Learners with emerging language—at any level—can engage deeply with central mathematical ideas under specific instructional conditions. Mathematical language development occurs when students use their developing language to make meaning and engage with challenging problems that are beyond students' mathematical ability to solve independently and therefore require interaction with peers. However, these interactions should be structured with temporary supports that students can use to make sense of what is being asked of them, to help organize their own thinking, and to give and receive feedback.

Instruction supports learning when teachers respond to students' verbal and written work.

Eliciting student thinking through language allows teachers and students to respond formatively to the language students generate. Formative peer and teacher feedback creates opportunities for revision and refinement of both content understanding and language.

Students are agents in their own mathematical and linguistic sense-making.

Mathematical language proficiency is developed through the process of actively exploring and learning mathematics. Language is action: in the very doing of math, students have naturally occurring opportunities to need, learn, and notice mathematical ways of making sense and talking about ideas and the world. These experiences support learners in using as well as expanding their existing language toolkits.

Additional supports for multilingual learners are embedded within lessons in the Zearn Math Teacher Edition; these offer instructional strategies for teachers to meet the individual needs of a diverse group of learners when support beyond existing strategies embedded in Zearn Math is required. Lesson- and activity-level supports for multilingual learners stem from the design principles (below) and are aligned to the language domains of reading, writing, speaking, listening, conversing, and representing in math (Aguirre & Bunch, 2012). These lesson-specific supports (examples found below) provide students with access to the mathematics by supporting them with the language demands of a specific activity, without reducing the mathematical demand of the task. Using these supports will help maintain student engagement in mathematical discourse and ensure that the struggle remains productive. All of the supports are designed to be used as needed, and use should be faded out as students develop understanding and fluency with the English language. Teachers should use their professional judgment about which supports to use and when based on their knowledge of the individual needs of students in their classroom.

Based on their observations of student language, teachers can make adjustments to their teaching and provide additional language support where necessary. Teachers can select from the “Supports for multilingual learners” provided in the Zearn Math Teacher Edition as appropriate. When selecting from these supports, teachers should take into account the language demands of the specific activity and the language needed to engage the content more broadly, in relation to their students’ current ways of using language to communicate ideas as well as their students’ English language proficiency.

DESIGN PRINCIPLES FOR PROMOTING MATHEMATICAL LANGUAGE USE AND DEVELOPMENT

The framework for supporting multilingual learners in Zearn Math includes four design principles for promoting mathematical language use and development in curriculum and instruction. The design principles and related routines work to make language development an integral part of planning and delivering instruction while guiding teachers to amplify the most important language that students are expected to bring to bear on the central mathematical ideas of each Mission.

Principle 1: Support Sense-Making

Scaffold tasks and amplify language so students can make their own meaning. Students do not need to understand a language completely before they can engage with academic content in that language. Language learners of all levels can and should engage with grade-level content that is appropriately scaffolded. Students need multiple opportunities to talk about their mathematical thinking, negotiate meaning with others, and collaboratively solve problems with targeted guidance from the teacher.

Teachers can make language more accessible for students by amplifying rather than simplifying speech or text. Simplifying includes avoiding the use of challenging words or phrases. Amplifying means anticipating where students might need support in understanding concepts or mathematical terms, and providing multiple ways to access them. Providing visuals or manipulatives, demonstrating problem-solving, engaging in think-alouds, and creating analogies, synonyms, or context are all ways to amplify language so that students are supported in taking an active role in their own sense-making of mathematical relationships, processes, concepts, and terms.

Principle 2: Optimize Output

Strengthen opportunities and supports for students to describe their mathematical thinking to others orally, visually, and in writing. Linguistic output is the language that students use to communicate their ideas to others (oral, written, visual, etc.), and refers to all forms of student linguistic expressions except those that include significant back-and-forth negotiation of ideas. (That type of conversational language is addressed in the third principle.) All students benefit from repeated, strategically optimized, and supported opportunities to articulate mathematical ideas into linguistic expression.

Opportunities for students to produce output should be strategically optimized for both (a) important concepts of the Mission or grade level, and (b) important disciplinary language functions (for example, making conjectures and claims, justifying claims with evidence, explaining reasoning, critiquing the reasoning of others, making generalizations, and comparing approaches and representations). The focus for optimization must be determined, in part, by how students are currently using language to engage with important disciplinary concepts. When opportunities to produce output are optimized in these ways, students will get spiraled practice in making their thinking about important mathematical concepts stronger, with more robust reasoning and examples, and making their thinking clearer, with more precise language and visuals.

Principle 3: Cultivate Conversation

Strengthen opportunities and supports for constructive mathematical conversations (pairs, groups, and whole-class). Conversations are back-and-forth interactions with multiple turns that build up ideas about math. Conversations act as scaffolds for students developing mathematical language because they provide opportunities to simultaneously make meaning, communicate that meaning, and refine the way content understandings are communicated.

When students have a purpose for talking and listening to each other, communication is more authentic. During effective discussions, students pose and answer questions, clarify what is being asked and what is happening in a problem, build common understandings, and share experiences relevant to the topic. As mentioned in Principle 2, learners must be supported in their use of language, including when having conversations, making claims, justifying claims with evidence, making conjectures, communicating reasoning, critiquing the reasoning of others, engaging in other mathematical practices, and above all when making mistakes. Meaningful conversations depend on the teacher using lessons and activities as opportunities to build a classroom culture that motivates and values efforts to communicate.

Principle 4: Maximize Meta-awareness

Strengthen the meta-connections and distinctions between mathematical ideas, reasoning, and language. Language is a tool that not only allows students to communicate their math understanding to others, but also to organize their own experiences, ideas, and learning for themselves. Meta-awareness is consciously thinking about one's own thought processes or language use. Meta-awareness develops when students and teachers engage in classroom activities or discussions that bring explicit attention to what students need to do to improve communication and reasoning about mathematical concepts. When students are using language in ways that are purposeful and meaningful for themselves, in their efforts to understand—and be understood by—each other, they are motivated to think of ways in which language can be both clarified and clarifying.

Meta-awareness in students can be strengthened when, for example, teachers ask students to explain to each other the strategies they brought to bear to solve a challenging problem. They might be asked, “How does yesterday’s method connect with the method we are learning today?” or, “What ideas are still confusing to you?” These questions are metacognitive because they help students reflect on their own and others’ learning. Students can also reflect on their expanding use of language—for example, by comparing the language they used to clarify a mathematical concept with the language used by their peers in a similar situation. This is called metalinguistic awareness because students reflect on English as a language, their own growing use of that language, and the particular ways ideas are communicated in mathematics. Students learning English benefit from being aware of how language choices are related to the purpose of the task and the intended audience, especially if oral or written work is required. Both metacognitive and metalinguistic awareness are powerful tools to help students self-regulate their academic learning and language acquisition.

These four principles are guides for curriculum development, as well as for the planning and execution of instruction, including the structure and organization of interactive opportunities for students. They also serve as guides for observation, analysis, and reflection on student language and learning.

MATHEMATICAL LANGUAGE ROUTINES

For instances where students need even more support than what's in the curriculum, Zearn recommends that teachers read and consider using the mathematical language routines (MLRs) listed below:

- **MLR 1: Stronger and Clearer Each Time**
- **MLR 2: Collect and Display**
- **MLR 3: Clarify, Critique, Correct**
- **MLR 4: Information Gap**
- **MLR 5: Co-Craft Questions**
- **MLR 6: Three Reads**
- **MLR 7: Compare and Connect**
- **MLR 8: Discussion Supports**

The mathematical language routines (MLRs) were selected because they are effective and practical for simultaneously learning mathematical practices, content, and language. A mathematical language routine is a structured but adaptable format for amplifying, assessing, and developing students' language. The routines emphasize uses of language that is meaningful and purposeful, rather than just getting answers. These routines can be adapted and incorporated across lessons in each Mission to fit the mathematical work wherever there are productive opportunities to support students in using and improving their English and disciplinary language use.

Each MLR facilitates attention to student language in ways that support in-the-moment teacher-, peer-, and self-assessment for all learners. The feedback enabled by these routines will help students revise and refine not only the way they organize and communicate their own ideas, but also how they ask questions to clarify their understanding of others' ideas.

Mathematical Language Routine 1: Stronger and Clearer Each Time

Purpose: To provide a structured and interactive opportunity for students to revise and refine both their ideas and their verbal and written output (Zwiers, 2013). This routine also provides a purpose for student conversation through the use of a discussion-worthy and iteration-worthy prompt. The main idea is to have students think and write individually about a question, use a structured pairing strategy to have multiple opportunities to refine and clarify their response through conversation, and then finally revise their original written response. Subsequent conversations and second drafts should naturally show evidence of incorporating or addressing new ideas and language. They should also show evidence of refinement in precision, communication, expression, examples, and reasoning about mathematical concepts.

How it happens:

- **PROMPT:** This routine begins by providing a thought-provoking question or prompt. The prompt should guide students to think about a concept or big idea connected to the content goal of the lesson, and should be answerable in a format that is connected with the activity's primary disciplinary language function.
- **RESPONSE - FIRST DRAFT:** Students draft an initial response to the prompt by writing or drawing their initial thoughts in a first draft. Responses should attempt to align with the activity's primary language function. It is not necessary that students finish this draft before moving to the structured pair meetings step. However, students should be encouraged to write or draw something before meeting with a partner. This

encouragement can come over time as class culture is developed, strategies and supports for getting started are shared, and students become more comfortable with the low stakes of this routine. (2–3 min)

- **STRUCTURED PAIR MEETINGS:** Next, use a structured pairing strategy to facilitate students having 2–3 meetings with different partners. Each meeting gives each partner an opportunity to be the speaker and an opportunity to be the listener. As the speaker, each student shares their ideas (without looking at their first draft, when possible). As a listener, each student should (a) ask questions for clarity and reasoning, (b) press for details and examples, and (c) give feedback that is relevant for the language goal. (1–2 min each meeting)
- **RESPONSE - SECOND DRAFT:** Finally, after meeting with 2–3 different partners, students write a second draft. This draft should naturally reflect borrowed ideas from partners, as well as refinement of initial ideas through repeated communication with partners. This second draft will be stronger (with more or better evidence of mathematical content understanding) and clearer (more precision, organization, and features of disciplinary language function). After students are finished, their first and second drafts can be compared. (2–3 min)

Mathematical Language Routine 2: Collect and Display

Purpose:

To capture a variety of students' oral words and phrases into a stable, collective reference. The intent of this routine is to stabilize the varied and fleeting language in use during mathematical work, in order for students' own output to become a reference in developing mathematical language. The teacher listens for and scribes the language students use during partner, small group, or whole class discussions using written words, diagrams and pictures. This collected output can be organized, revoiced, or explicitly connected to other language in a display that all students can refer to, build on, or make connections with during future discussion or writing. Throughout the course of a Mission (and beyond), teachers can reference the displayed language as a model, update and revise the display as student language changes, and make bridges between prior student language and new disciplinary language (Zwiers et al, 2017). This routine provides feedback for students in a way that supports sense-making while simultaneously increasing meta-awareness of language.

How it happens:

- **COLLECT:** During this routine, circulate and listen to student talk during paired, group, or as a whole-class discussion. Jot down the words, phrases, drawings, or writing students use. Capture a variety of uses of language that can be connected to the lesson content goals, as well as the relevant disciplinary language function(s). Collection can happen digitally, with a clipboard, or directly onto poster paper; capturing on a whiteboard is not recommended due to risk of erasure.
- **DISPLAY:** Display the language collected visually for the whole class to use as a reference during further discussions throughout the lesson and Mission. Encourage students to suggest revisions, updates, and connections be added to the display as they develop—over time—both new mathematical ideas and new ways of communicating ideas. The display provides an opportunity to showcase connections between student ideas and new vocabulary and also highlights examples of students using disciplinary language functions, beyond just vocabulary words.

Mathematical Language Routine 3: Clarify, Critique, Correct

Purpose:

To give students a piece of mathematical writing that is not their own to analyze, reflect on, and develop. The intent is to prompt student reflection with an incorrect, incomplete, or ambiguous written mathematical statement, and for students to improve upon the written work by correcting errors and clarifying meaning.

Teachers can demonstrate how to effectively and respectfully critique the work of others with meta-think-alouds and pressing for details when necessary. This routine fortifies output and engages students in meta-awareness. More than just error analysis, this routine purposefully engages students in considering both the author’s mathematical thinking as well as the features of their communication.

How it happens:

- **ORIGINAL STATEMENT:** Create or curate a written mathematical statement that intentionally includes conceptual (or common) errors in mathematical thinking as well as ambiguities in language. The mathematical errors should be driven by the content goals of the lesson and the language ambiguities should be driven by common or typical challenges with the relevant disciplinary language function. This mathematical text is read by the students and used as the draft, or “original statement,” that students improve. (1–2 min)
- **DISCUSSION WITH PARTNER:** Next, students discuss the original statement in pairs. The teacher provides guiding questions for this discussion such as, “What do you think the author means?,” “Is anything unclear?,” or “Are there any reasoning errors?” In addition to these general guiding questions, 1–2 questions can be added that specifically address the content goals and disciplinary language function relevant to the activity. (2–3 min)
- **IMPROVED STATEMENT:** Students individually revise the original statement, drawing on the conversations with their partners, to create an “improved statement.” In addition to resolving any mathematical errors or misconceptions, clarifying ambiguous language, other requirements can be added as parameters for the improved response. These specific requirements should be aligned with the content goals and disciplinary language function of the activity. (3–5 min)

Mathematical Language Routine 4: Information Gap**Purpose:**

To create a need for students to communicate (Gibbons, 2002). This routine allows teachers to facilitate meaningful interactions by positioning some students as holders of information that is needed by other students. The information is needed to accomplish a goal, such as solving a problem or winning a game. An information gap creates a need for students to orally (or visually) share ideas and information in order to bridge a gap and accomplish something that they could not have done alone. Teachers should demonstrate how to ask for and share information, how to justify a request for information, and how to clarify and elaborate on information. This routine cultivates conversation.

How it happens:

- **PROBLEM/DATA CARDS:** Students are paired into Partner A and Partner B. Partner A is given a card with a problem that must be solved, and Partner B has the information needed to solve it on a “data card.” Data cards can also contain diagrams, tables, graphs, etc. Neither partner should read nor show their cards to their partners. Partner A determines what information they need and prepares to ask Partner B for that specific information. Partner B should not share the information unless Partner A specifically asks for it and justifies the need for the information. Because partners don’t have the same information, Partner A must work to produce clear and specific requests, and Partner B must work to understand more about the problem through Partner A’s requests and justifications.
- **BRIDGING THE GAP:** First, Partner B asks, “What specific information do you need?” Then, Partner A asks for specific information from Partner B. Before sharing the requested information, Partner B asks Partner A for a

justification: “Why do you need that information?” Partner A explains how they plan to use the information. Finally, Partner B asks clarifying questions as needed, and then provides the information. These four steps are repeated until Partner A is satisfied that they have information they need to solve the problem.

- **SOLVING THE PROBLEM:** First, Partner A shares the problem card with Partner B. Partner B does not share the data card. Then, both students solve the problem independently, then discuss their strategies. Finally, Partner B can share the data card after discussing their independent strategies.

Mathematical Language Routine 5: Co-Craft Questions

Purpose:

To allow students to get inside of a context before feeling pressure to produce answers, to create space for students to produce the language of mathematical questions themselves, and to provide opportunities for students to analyze how different mathematical forms and symbols can represent different situations. Through this routine, students are able to use conversation skills to generate, choose (argue for the best one), and improve questions and situations as well as develop meta-awareness of the language used in mathematical questions and problems.

How it happens:

- **HOOK:** Begin by presenting students with a hook—a context or a stem for a problem, with or without values included. The hook can also be a picture, video, or list of interesting facts.
- **STUDENTS WRITE QUESTIONS:** Next, students write down possible mathematical questions that might be asked about the situation. These should be questions that they think are answerable by doing math and could be questions about the situation, information that might be missing, and even about assumptions that they think are important. (1–2 minutes)
- **STUDENTS COMPARE QUESTIONS:** Students compare the questions they generated with a partner (1–2 minutes) before sharing questions with the whole class. Demonstrate (or ask students to demonstrate) identifying specific questions that are aligned to the content goals of the lesson as well as the disciplinary language function. If there are no clear examples, teachers can demonstrate adapting a question or ask students to adapt questions to align with specific content or function goals. (2–3 minutes)
- **ACTUAL QUESTION(S) REVEALED/IDENTIFIED:** Finally, the actual questions students are expected to work on are revealed or selected from the list that students generated.

Mathematical Language Routine 6: Three Reads

Purpose:

To ensure that students know what they are being asked to do, create opportunities for students to reflect on the ways mathematical questions are presented, and equip students with tools used to actively make sense of mathematical situations and information (Kelemanik, Lucenta, & Creighton, 2016). This routine supports reading comprehension, sense-making, and meta-awareness of mathematical language. It also supports negotiating information in a text with a partner through mathematical conversation.

How it happens:

In this routine, students are supported in reading a mathematical text, situation, or word problem three times, each with a particular focus. The intended question or main prompt is intentionally withheld until the third read so that students can concentrate on making sense of what is happening in the text before rushing to a solution or method.

- **READ #1: SHARED READING (ONE PERSON READS ALOUD WHILE EVERYONE ELSE READS WITH THEM):** The first read focuses on the situation, context, or main idea of the text. After a shared reading, ask students, "What is this situation about?" This is the time to identify and resolve any challenges with any non-mathematical vocabulary. (1 minute)
- **READ #2: INDIVIDUAL, PAIRS, OR SHARED READING:** After the second read, students list any quantities that can be counted or measured. Students are encouraged not to focus on specific values. Instead they focus on naming what is countable or measurable in the situation. It is not necessary to discuss the relevance of the quantities, just to be specific about them (examples: "number of people in her family" rather than "people," "number of markers after" instead of "markers"). Some of the quantities will be explicit (example: 32 apples) while others are implicit (example: the time it takes to brush one tooth). Record the quantities as a reference to use when solving the problem after the third read. (3–5 minutes)
- **READ #3: INDIVIDUAL, PAIRS, OR SHARED READING:** During the third read, the final question or prompt is revealed. Students discuss possible solution strategies, referencing the relevant quantities recorded after the second read. It may be helpful for students to create diagrams to represent the relationships among quantities identified in the second read, or to represent the situation with a picture (Asturias, 2015). (1–2 minutes)

Mathematical Language Routine 7: Compare and Connect

Purpose:

To foster students' meta-awareness as they identify, compare, and contrast different mathematical approaches and representations. This routine leverages the powerful mix of disciplinary representations available in mathematics as a resource for language development. In this routine, students make sense of mathematical strategies other than their own by relating and connecting other approaches to their own. Students should be prompted to reflect on, and linguistically respond to, these comparisons (for example, exploring why or when one might do or say something a certain way, identifying and explaining correspondences between different mathematical representations or methods, or wondering how a certain concept compares or connects to other concepts). Be sure to demonstrate asking questions that students can ask each other, rather than asking questions to "test" understanding. Use think alouds to demonstrate the trial and error, or fits and starts of sense-making (similar to the way teachers think aloud to demonstrate reading comprehension). This routine supports metacognition and metalinguistic awareness and also supports constructive conversations.

How it happens:

- **STUDENTS PREPARE DISPLAYS OF THEIR WORK:** Students are given a problem that can be approached and solved using multiple strategies, or a situation that can be modeled using multiple representations. Students are assigned the job of preparing a visual display of how they made sense of the problem and why their solution makes sense. Variation is encouraged and supported among the representations that different students use to show what makes sense.
- **COMPARE:** Students investigate each others' work by taking a tour of the visual displays. Tours can be self-guided, a "travellers and tellers" format, or the teacher can act as "docent" by providing questions for students to ask of each other, pointing out important mathematical features, and facilitating comparisons. Comparisons focus on the typical structures, purposes, and affordances of the different approaches or representations: what worked well in this or that approach, or what is especially clear in this or that representation. During this discussion, listen for and amplify any comments about what might make this or that approach or representation more complete or easy to understand.

- **CONNECT:** The discussion then turns to identifying correspondences between different representations. Students are prompted to find correspondences in how specific mathematical relationships, operations, quantities, or values appear in each approach or representation. Guide students to refer to each other's thinking by asking them to make connections between specific features of expressions, tables, graphs, diagrams, words, and other representations of the same mathematical situation. During the discussion, amplify language students use to communicate about mathematical features that are important for solving the problem or modeling the situation. Call attention to the similarities and differences between the ways those features appear.

Mathematical Language Routine 8: Discussion Supports

Purpose:

To support rich and inclusive discussions about mathematical ideas, representations, contexts, and strategies (Chapin, O'Connor, & Anderson, 2009). Rather than another structured format, the examples provided in this routine are instructional moves that can be combined and used together with any of the other routines. They include multimodal strategies for helping students make sense of complex language, ideas, and classroom communication. The examples can be used to invite and incentivize more student participation, conversation, and meta-awareness of language. Eventually, as teachers continue to demonstrate, students should begin using these strategies themselves to prompt each other to engage more deeply in discussions.

How it happens:

Unlike the other routines, this support is a collection of strategies and moves that can be combined and used to support discussion during almost any activity. Examples of possible strategies:

- Revoice student ideas to demonstrate mathematical language use by restating a statement as a question in order to clarify, apply appropriate language, and involve more students.
- Press for details in students' explanations by requesting for students to challenge an idea, elaborate on an idea, or give an example.
- Show central concepts multi-modally by using different types of sensory inputs: acting out scenarios or inviting students to do so, showing videos or images, using gesture, and talking about the context of what is happening.
- Practice phrases or words through choral response.
- Think aloud by talking through thinking about a mathematical concept while solving a related problem or doing a task.
- Demonstrate uses of disciplinary language functions such as detailing steps, describing and justifying reasoning, and questioning strategies.
- Give students time to make sure that everyone in the group can explain or justify each step or part of the problem. Then make sure to vary who is called on to represent the work of the group so students get accustomed to preparing each other to fill that role.
- Prompt students to think about different possible audiences for the statement, and about the level of specificity or formality needed for a classmate vs. a mathematician, for example. [Convince Yourself, Convince a Friend, Convince a Skeptic (Mason, Burton, & Stacey, 2010)]

SENTENCE FRAMES

Sentence frames can support student language production by providing a structure to communicate about a topic. Helpful sentence frames are open-ended so as to amplify language production, not constrain it. The table shows examples of generic sentence frames that can support common disciplinary language functions across a variety of content topics. Some of the lessons in these materials include suggestions of additional sentence frames that could support the specific content and language functions of that lesson.

Language function	Sample sentence frames
describe	<ul style="list-style-type: none"> • It looks like... • I notice that... • I wonder if... • Let's try... • A quantity that varies is _____. • What do you notice? • What other details are important?
explain	<ul style="list-style-type: none"> • First, I _____ because... • Then/Next, I... • I noticed _____ so I... • I tried _____ and what happened was... • How did you get...? • What else could we do?
justify	<ul style="list-style-type: none"> • I know _____ because... • I predict _____ because... • If _____ then _____ because... • Why did you...? • How do you know...? • Can you give an example?
generalize	<ul style="list-style-type: none"> • _____ reminds me of _____ because... • _____ will always _____ because... • _____ will never _____ because... • Is it always true that...? • Is _____ a special case
critique	<ul style="list-style-type: none"> • That could/couldn't be true because... • This method works/doesn't work because... • We can agree that... • _____'s idea reminds me of... • Another strategy would be _____ because... • Is there another way to say/do...?
compare and contrast	<ul style="list-style-type: none"> • Both _____ and _____ are alike because... • _____ and _____ are different because... • One thing that is the same is... • One thing that is different is... • How are _____ and _____ different? • What do _____ and _____ have in common?

Language function	Sample sentence frames
represent	<ul style="list-style-type: none"> • _____ represents _____. • _____ stands for _____. • _____ corresponds to _____. • Another way to show _____ is... • How else could we show this?
interpret	<ul style="list-style-type: none"> • We are trying to... • We will need to know... • We already know... • It looks like _____ represents... • Another way to look at it is... • What does this part of _____ mean? • Where does _____ show...?

Appendix III: Access for Students with Disabilities

INTRODUCTION

All students are individuals who can know, use, and enjoy mathematics. Zearn Math for Sixth Grade empowers students with activities that capitalize on their existing strengths and abilities to ensure that all learners can participate meaningfully in rigorous mathematical content. The Zearn Math lessons support a flexible approach to instruction and provide teachers with options for additional support to address the needs of a diverse group of students.

Zearn Math for Sixth Grade includes in each lesson supplemental instructional strategies, labeled “Support for Students with Disabilities” that are designed to increase access and eliminate barriers. These lesson-specific supports can be used as needed to help students succeed with a specific activity, without reducing the mathematical demand of the task, and can be faded out as students gain understanding and fluency.

DESIGN PRINCIPLES

These materials reflect three key design principles that support and engage all students in today’s diverse mathematics classrooms. The design principles and related supports work together to make each activity in each lesson accessible to all students.

Principle 1: Access for All.

This foundational design principle draws from the Universal Design for Learning (UDL) framework, and shapes the instructional goals, recommended practices, lesson plans, and assessments to support a flexible approach to instruction, ensuring all students have an equitable opportunity to learn.

Principle 2: Presume Competence.

All students are individuals who can learn, apply, and enjoy mathematics. The activities in these materials position students to capitalize on their existing abilities, and provide supports that eliminate potential barriers to learning when they arise. Each lesson is designed for a wide range of abilities, and all students are given access to grade-level problems. Student competence to engage with mathematical tasks should be assumed, with additional supports provided only when needed.

Principle 3: Strengths-based approach.

All students, including students with disabilities, are resourceful and resilient members of the mathematics community. When the unique strengths and interests of students with disabilities are highlighted during class discussions, their contributions enhance the learning of all students in the classroom.

AREAS OF COGNITIVE FUNCTIONING

The lesson and activity-level supports for students with disabilities are aligned to an area of cognitive functioning and are paired with a suggested strategy aimed to increase access and eliminate barriers. All of the supports can be used discreetly and are designed to be used as needed. Many of these supports can be implemented throughout the academic year; for example, peer tutors can help build classroom culture, provide opportunities for teamwork, and build collaboration skills while also supporting those who struggle. Other supports should be faded out as students gain understanding and fluency with key ideas and procedures. Additional supports for students with disabilities are designed to address students' strengths and needs in the following areas of cognitive functioning, which are integral to learning mathematics (Brodesky et al., 2002):

- **Conceptual processing**, which includes perceptual reasoning, problem solving, and metacognition.
- **Expressive & receptive language**, which includes auditory and visual language processing and expression.
- **Visual-spatial processing**, which includes processing visual information and understanding relation in space (e.g., visual mathematical representations and geometric concepts).
- **Executive functioning**, which includes organizational skills, attention, and focus.
- **Memory**, which includes working memory and short-term memory.
- **Social-emotional functioning**, which includes interpersonal skills and the cognitive comfort and safety required in order to take risks and make mistakes.
- **Fine-motor skills**, which includes tasks that require small muscle movement and coordination such as manipulating objects (graphing, cutting, writing).

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