ZEARN Math

MISSION OVERVIEW

GRADE 8

M3 Linear Relationships

Introduction

In this Mission, students learn to understand and use the terms “rate of change,” “linear relationship,” and “vertical intercept.” They deepen their understanding of slope, and they learn to recognize connections among rate of change, slope, and constant of proportionality, and between linear and proportional relationships. They learn to understand that lines with the same slope are translations of each other. They represent linear relationships with tables, equations, and graphs that include lines with negative slopes or vertical intercepts, and horizontal and vertical lines. They learn to use the term “solution of an equation” when working with one or two linear equations in two variables, and learn to understand the graph of a linear equation as the set of its solutions. Students use these terms and representations in reasoning about situations involving one or two constant rates.

Overview of Topics and Lesson Objectives

Each mission is broken down into topics. A topic is a group of lessons that teach the same concept. There is a balance of Independent Digital Lessons and Concept Explorations in each topic of a mission to ensure every student learns with a mix of modalities, feedback, and support while engaging in grade-level content. Throughout each mission, students work on grade-level content with embedded remediation to address unfinished learnings.

	Objective	INDEPENDENT DIGITAL LESSON	CONCEPT EXPLORATION
Topic A	**Proportional Relationships**		
Lesson 1	Create appropriate graphs and equations for proportional relationships by finding a constant of proportionality.	✓	✓
Lesson 2	Compare graphs that represent proportional relationships using differently scaled axes. Use the graph of a proportional relationship to find the constant of proportionality and an equation that represents the relationship.	✓	✓
Lesson 3	Find the rate of change of a proportional relationship given the graph, equation, table, or situation.	✓	✓
Lesson 4	Compare the rates of change of proportional relationships presented in multiple ways.	✓	✓
Topic B	**Representing Linear Relationships**		
Lesson 5	Find the rate of change of a linear relationship by figuring out the slope of the line representing the relationship. Determine if a linear relationship is proportional or non-proportional by looking at its graph.	✓	✓
Lesson 6	Interpret the slope and vertical intercept of a graph of a real-world situation.	✓	✓
Lesson 7	Interpret the slope and vertical intercept in context, and write equations that represent linear relationships.	✓	✓
Lesson 8	Create and compare equations that represent linear relationships with the same rate of change but different initial values.	✓	✓
Mid-Mission Assessment: Topics A-B			
Topic C	**Finding Slopes**		
Lesson 9	Create and interpret a graph of a line representing a linear relationship with a non-positive rate of change.	✓	✓

	Objective	INDEPENDENT DIGITAL LESSON	CONCEPT EXPLORATION
Lesson 10	Calculate the slope of any line given the coordinates of two points on the line.	✓	✓
Lesson 11	Write equations for vertical and horizontal lines, given the graph or a situation in which one variable does not vary and the other can take any value.	✓	✓
Topic D	**Linear Equations**		
Lesson 12	Determine pairs of values that satisfy or do not satisfy a linear relationship using an equation or graph.	✓	✓
Lesson 13	Calculate the solution to a linear equation given one variable, and determine whether a point is a solution to an equation of a line.	✓	✓
Topic E	**Let's Put It to Work**		
Lesson 14	Write and interpret linear equations representing real-world situations.	X	✓
End-of-Mission Assessment: Topics C-E			

Foundational Missions

For each mission, Zearn Math highlights the foundational missions, the earlier content where concepts are introduced and developed. Teachers can access foundational missions directly from the mission page of their Teacher Account to address any unfinished learnings. Zearn recommends that teachers assign foundational missions during Flex Day or during additional non-core instruction time. It is important to use a foundational mission to support students who are struggling, rather than an unaligned mission, because the content students learn in each foundational mission supports their Core Day learning.

Foundational Mission for G8M3: G7M4 Topic A

Mission Overview

Work with linear relationships in grade 8 builds on earlier work with rates and proportional relationships in grade 7, and grade 8 work with geometry. At the end of the previous mission on dilations, students learned the terms "slope" and "slope triangle," used the similarity of slope triangles on the same line to understand that any two distinct points on a line determine the same slope, and found an equation for a line with a positive slope and vertical intercept. In this mission, students gain experience with linear relationships and their representations as graphs, tables, and equations through activities designed and sequenced to allow them to make sense of problems and persevere in solving them (MP1). Because of this dependency, this mission and the previous one should be done in order.

The mission begins by revisiting different representations of proportional relationships (graphs, tables, and equations), and the role of the constant of proportionality in each representation and how it may be interpreted in context (MP2).

Next, students analyze the relationship between number of cups in a given stack of cups and the height of the stack—a relationship that is linear but not proportional—in order to answer the question "How many cups are needed to get to a height of 50 cm?" They are not asked to solve this problem in a specific way, giving them an opportunity to choose and use strategically (MP5) representations that appeared earlier in this mission (table, equation, graph) or in the previous mission (equation, graph). Students are introduced to "rate of change" as a way to describe the rate per 1 in a linear relationship and note that its numerical value is the same as that of the slope of the line that represents the relationship. Students analyze another linear relationship (height of water in a cylinder vs number of cubes in the cylinder) and establish a way to compute the slope of a line from any two distinct points on the line via repeated reasoning (MP8). They learn a third way to obtain an equation for a linear relationship by viewing the graph of a line in the coordinate plane as the vertical translation of a proportional relationship (MP7).

So far, the mission has involved only lines with positive slopes and y-intercepts. Students next consider the graph of a line with a negative y-intercept and equations that might represent it. They consider situations represented by linear relationships with negative rates of change, graph these (MP4), and interpret coordinates of points on the graphs in context (MP2).

The mission concludes with two lessons that involve graphing two equations in two unknowns and finding and interpreting their solutions (MP2). Doing this involves considering correspondences among different representations (MP1), in particular, what it means for a pair of values to be a solution for an equation and the correspondence between coordinates of points on a graph and solutions of an equation.

In this mission, several lesson plans suggest that each student have access to a *geometry toolkit*. Each toolkit contains tracing paper, graph paper, colored pencils, scissors, ruler, protractor, and an index card to use as a straightedge or to mark right angles, giving students opportunities to select appropriate tools and use them strategically to solve problems (MP5). To support students in their developing understanding, Zearn Math Independent Digital Lessons utilize a variety of digital manipulatives that mirror what students will see in their geometry toolkit. The combination of physical and digital tools will build a deep understanding of important geometric and measurement concepts.

On using the terms ratio, rate, and proportion. In these materials, a *quantity* is a measurement that is or can be specified by a number and a unit, e.g., 4 oranges, 4 centimeters, "my height in feet," or "my height" (with the understanding that a unit of measurement will need to be chosen). The term *ratio* is used to mean an association between two or more quantities and the fractions $\frac{a}{b}$ and $\frac{b}{a}$ are never called ratios. The fractions $\frac{a}{b}$ and $\frac{b}{a}$ are identified as "unit rates" for the ratio $a : b$. The word "per" is used with students in interpreting a unit rate in context, as in "$3 per ounce," and "at the same rate" is used to signify a situation characterized by equivalent ratios.

In grades 6–8, students write rates without abbreviated units, for example as "3 miles per hour" or "3 miles in every 1 hour." Use of notation for derived units such as $\frac{\text{mi}}{\text{hr}}$ waits for high school—except for the special cases of area and volume. Students have worked with area since grade 3 and volume since grade 5. Before grade 6, they have learned the meanings of such things as sq cm and cu cm. After students learn exponent notation in grade 6, they also use cm^2 and cm^3.

A proportional relationship is a collection of equivalent ratios. In high school—after their study of ratios, rates, and proportional relationships—students discard the term "unit rate," referring to a to b, $a : b$, and $\frac{a}{b}$ as "ratios."

A proportional relationship between two quantities represented by a and b is associated with two constants of proportionality: $\frac{a}{b}$ and $\frac{b}{a}$. Throughout the mission, the convention is if a and b are represented by columns in a table and the column for a is to the left of the column for b, then $\frac{b}{a}$ is the constant of proportionality for the relationship represented by the table.

Progression of Disciplinary Language

In this mission, teachers can anticipate students using language for mathematical purposes such as representing, generalizing, and explaining. Throughout the mission, students will benefit from routines designed to grow robust disciplinary language, both for their own sense-making and for building shared understanding with peers. Teachers can formatively assess how students are using language in these ways, particularly when students are using language to:

Represent

- situations involving proportional relationships (Lesson 1)
- constants of proportionality in different ways (Lesson 3)
- slope using expressions (Lesson 7)
- linear relationships using graphs, tables, equations, and verbal descriptions (Lesson 8)
- situations using negative slopes and slopes of zero (Lesson 9)
- situations by graphing lines and writing equations (Lesson 12)
- situations involving linear relationships (Lesson 14)

Generalize

- categories for graphs (Lesson 2)
- about equations and linear relationships (Lesson 7)
- in order to make predictions about the slope of lines (Lesson 10)

Explain

- how to graph proportional relationships (Lesson 3)
- how to use a graph to determine information about a linear situation (Lessons 5 and 6)
- how to graph linear relationships (Lesson 10)
- how slope relates to changes in a situation (Lesson 11)

In addition, students are expected to describe observations about the equation of a translated line and describe features of an equation that could make one variable easier or harder to solve for than the other. Students will also have opportunities to use language to interpret situations involving proportional relationships, interpret graphs using different scales, interpret slopes and intercepts of linear graphs, justify reasoning about linear relationships, justify

correspondences between different representations, and justify which equations correspond to graphs of horizontal and vertical lines.

The table shows lessons where new terminology is first introduced, including when students are expected to understand the word or phrase receptively and when students are expected to produce the word or phrase in their own speaking or writing. Terms from the glossary appear bolded. Teachers should continue to support students' use of a new term in the lessons that follow the one in which it was first introduced.

New Terminology		
Lesson	**Receptive**	**Productive**
1	represent scale label constant of proportionality	productive
2	equation	
3	**rate of change**	constant of proportionality equation
5	**linear relationship** constant rate	**slope**
6	**vertical intercept** *y*-intercept	
7	initial (value or amount)	constant rate
8	relate	
9	horizontal intercept *x*-intercept	
10	intersection point	**rate of change** **vertical intercept** *y*-intercept
11	constraint	horizontal line vertical line
12	**solution to an equation with two variables** variable combination set of solutions	

Terminology

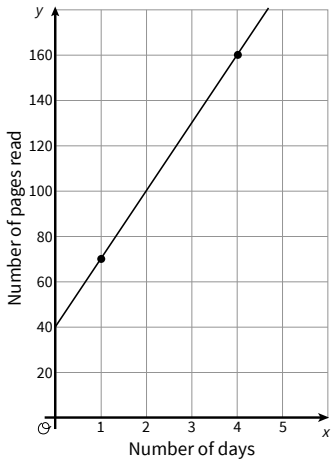
Linear relationship

A linear relationship between two quantities means they are related like this: When one quantity changes by a certain amount, the other quantity always changes by a set amount. In a linear relationship, one quantity has a constant rate of change with respect to the other.

The relationship is called linear because its graph is a line.

The graph shows a relationship between number of days and number of pages read.

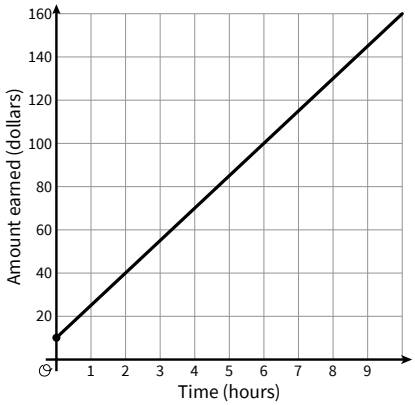
When the number of days increases by 2, the number of pages read always increases by 60. The rate of change is constant, 30 pages per day, so the relationship is linear.



Rate of change

The rate of change in a linear relationship is the amount y changes when x increases by 1. The rate of change in a linear relationship is also the slope of its graph.

In this graph, y increases by 15 dollars when x increases by 1 hour. The rate of change is 15 dollars per hour.



Solution to an equation with two variables

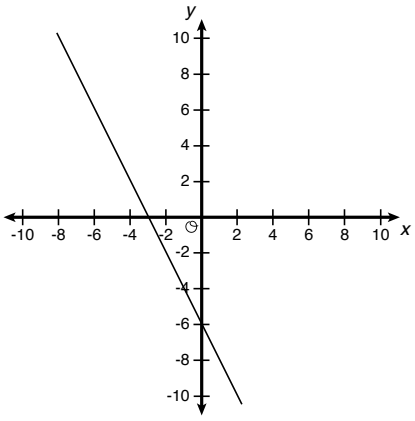
A solution to an equation with two variables is a pair of values of the variables that make the equation true.

For example, one possible solution to the equation $4x + 3y = 24$ is (6, 0). Substituting 6 for x and 0 for y makes this equation true because $4(6) + 3(0) = 24$.

Vertical intercept

The vertical intercept is the point where the graph of a line crosses the vertical axis.

The vertical intercept of this line is (0, -6) or just -6.



Required Materials

Geometry toolkits

Tracing paper, graph paper, colored pencils, scissors, and an index card to use as a straightedge or to mark right angles.

Graph paper

Rulers

Straightedges

String

Templates

Pre-printed cards, cut from copies of the templates

Pre-printed slips, cut from copies of the templates

Tools for creating a visual display