ZEARN Math

MISSION OVERVIEW

GRADE 8

M2 Dilations, Similarity, and Introducing Slope

Introduction

In this Mission, students learn to understand and use the term “dilation,” and to recognize that a dilation is determined by a point called the “center” and a number called the “scale factor.” They learn that under a dilation, the image of a circle is a circle and the image of a line is a line parallel to the original. They draw images of figures under dilations on and off the coordinate plane. They use the terms “corresponding sides” and “corresponding angles” to describe correspondences between a figure and its dilated image, and recognizing that angle measures are preserved, but lengths are multiplied by the scale factor. They learn to understand similarity of plane figures in terms of rigid transformations and dilations. They learn to recognize when one plane figure is similar or not similar to another. They use the definition of “similar” and properties of similar figures to justify claims of similarity or non-similarity. Students learn the terms “slope” and “slope triangle,” and use the similarity of slope triangles on the same line to understand that any two distinct points on a line determine the same slope.

Overview of Topics and Lesson Objectives

Each mission is broken down into topics. A topic is a group of lessons that teach the same concept. There is a balance of Independent Digital Lessons and Concept Explorations in each topic of a mission to ensure every student learns with a mix of modalities, feedback, and support while engaging in grade-level content. Throughout each mission, students work on grade-level content with embedded remediation to address unfinished learnings.

	Objective	INDEPENDENT DIGITAL LESSON	CONCEPT EXPLORATION
Topic A	**Dilations**		
Lesson 1	Understand that "dilation" produces scaled copies, and identify whether one rectangle is a scaled copy of another rectangle.	✓	✓
Lesson 2	Dilate polygons using a circular grid given a scale factor and center of dilation.	✓	✓
Lesson 3	Perform dilations without a circular grid given a scale factor and center of dilation.	✓	✓
Lesson 4	Dilate polygons on a square grid using the origin as the center of dilation and a given scale factor. Use coordinates to describe points on the original polygon and the image.	✓	✓
Lesson 5	Dilate polygons on a square grid using any point as the center of dilation and a given scale factor. Use coordinates to describe points on the original polygon and the image.	✓	✓
Topic B	**Similarity**		
Lesson 6	Apply a sequence of transformations to one figure to get a similar figure, and use a sequence of transformations to explain why two figures are similar.	✓	✓
Lesson 7	Understand that similar polygons have congruent corresponding angles and proportional side lengths.	✓	✓
Lesson 8	Determine that two triangles are similar by finding a sequence of transformations that takes one triangle to the other or checking that two pairs of corresponding angles are congruent.	✓	✓
Lesson 9	"Calculate unknown side lengths in similar triangles using the scale factor between similar triangles and understand that the quotients of pairs of side lengths in similar triangles are equal."	✓	✓

	Objective	INDEPENDENT DIGITAL LESSON	CONCEPT EXPLORATION
Topic C	**Slope**		
Lesson 10	Find the slope of a line on a grid using properties of slope triangles.	✓	✓
Lesson 11	Write an expression for the slope of a line using the quotient of the vertical and horizontal side lengths of a slope triangle.	✓	✓
Lesson 12	Write an equation for a line and use line equations to determine if a point is on a line.	✓	✓
Topic D	**Let's Put It to Work**		
Lesson 13	Use the properties of similar triangles in a real-word context to find the height of unknown object	X	✓
End-of-Mission Assessment: Topics A-D			

Foundational Missions

For each mission, Zearn Math highlights the foundational missions, the earlier content where concepts are introduced and developed. Teachers can access foundational missions directly from the mission page of their Teacher Account to address any unfinished learnings. Zearn recommends that teachers assign foundational missions during Flex Day or during additional non-core instruction time. It is important to use a foundational mission to support students who are struggling, rather than an unaligned mission, because the content students learn in each foundational mission supports their Core Day learning.

Foundational Mission(s) for G8M2: G7M1 and G7M2

Mission Overview

Work with transformations of plane figures in grade 8 builds on earlier work with geometry and geometric measurement, using students' familiarity with geometric figures, their knowledge of formulas for the areas of rectangles, parallelograms, and triangles, and their abilities to use rulers and protractors. Grade 7 work with scaled copies is especially relevant. This work was limited to pairs of figures with the same rotation and mirror orientations (i.e. that are not rotations or reflections of each other). In grade 8, students study pairs of scaled copies that have different rotations or mirror orientations, examining how one member of the pair can be transformed into the other, and describing these transformations. Initially, they view transformations as moving one figure in the plane onto another figure in the plane. As the mission progresses, they come to view transformations as moving the entire plane.

Through activities designed and sequenced to allow students to make sense of problems and persevere in solving them (MP1), students use and extend their knowledge of geometry and geometric measurement. Students begin the first lesson of the mission by looking at cut-out figures, first comparing them visually to determine if they are scaled copies of each other, then representing the figures in a diagram, and finally representing them on a circular grid with radial lines. They encounter the term "scale factor" (familiar from grade 7) and the new terms "dilation" and "center of dilation." In the next lesson, students again use a circular grid with radial lines to understand that under a dilation the image of a circle is a circle and the image of a line is a line parallel to the original. During the rest of the mission, students draw images of figures under dilations on and off square grids and the coordinate plane. In describing correspondences between a figure and its dilation, they use the terms "corresponding points," "corresponding sides," and "image." Students learn that angle measures are preserved under a dilation, but lengths in the image are multiplied by the scale factor. They learn the definition of "similar": two figures are said to be similar if there is a sequence of translations, rotations, reflections, and dilations that takes one figure to the other. They use the definition of "similar" and properties of similar figures to justify claims of similarity or non-similarity and to reason about similar figures (MP3). Using these properties, students conclude that if two triangles have two angles in common, then the triangles must be similar. Students also conclude that the quotient of a pair of side lengths in a triangle is equal to the quotient of the corresponding side lengths in a similar triangle. This conclusion is used in the lesson that follows: students learn the terms "slope" and "slope triangle," and use the similarity of slope triangles on the same line to understand that any two distinct points on a line determine the same slope (MP7). In the following lesson, students use their knowledge of slope to find an equation for a line. They will build on this initial work with slope in a subsequent grade 8 mission on linear relationships. Throughout the mission, students discuss their mathematical ideas and respond to the ideas of others (MP3, MP6).

Many of the lessons in this mission ask students to work on geometric figures that are not set in a real-world context. This design choice respects the significant intellectual work of reasoning about area. Tasks set in real-world contexts are sometimes contrived and hinder rather than help understanding. Moreover, mathematical contexts are legitimate contexts that are worthy of study. Students do have opportunities in the mission to tackle real-world applications. In the culminating activity of the mission, students examine shadows cast by objects in the sun. This is an opportunity for them to apply what they have learned about similar triangles (MP4).

In this mission, several lesson plans suggest that each student have access to a geometry toolkit. Each toolkit contains tracing paper, graph paper, colored pencils, scissors, ruler, protractor, and an index card to use as a straightedge or to mark right angles, giving students opportunities to develop their abilities to select appropriate tools and use them strategically to solve problems (MP5). To support students in their developing understanding, Zearn Math Independent Digital Lessons utilize a variety of digital manipulatives that mirror what students will see in their geometry toolkit. The combination of physical and digital tools will build a deep understanding of important geometric and measurement concepts.

Progression of Disciplinary Language

In this mission, teachers can anticipate students using language for mathematical purposes such as describing, explaining, representing, and justifying. Throughout the mission, students will benefit from routines designed to grow robust disciplinary language, both for their own sense-making and for building shared understanding with peers.

Teachers can formatively assess how students are using language in these ways, particularly when students are using language to:

Describe

- observations about scaled rectangles (Lesson 1)
- observations about dilated points, circles, and polygons (Lesson 2)
- sequences of transformations (Lesson 6)
- observations about side lengths in similar triangles (Lesson 9)

Explain

- how to apply dilations to find specific images (Lesson 5)
- how to determine whether triangles are congruent, similar, or neither (Lesson 8)
- strategies for finding missing side lengths (Lesson 9)
- how to apply dilations to find specific images of points (Lesson 12)
- reasoning for a conjecture (Lesson 13)

Represent

- dilations using given scale factors and coordinates (Lesson 4)
- figures using specific transformations (Lesson 6)
- graphs of lines using equations (Lesson 12)

In addition, students are expected to use language to interpret directions for dilating figures and for creating triangles; compare dilated polygons and methods for determining similarity; critique reasoning about angles, sides, and similarity; justify whether polygons are similar; and generalize about points on a line and similar triangles.

The table shows lessons where new terminology is first introduced, including when students are expected to understand the word or phrase receptively and when students are expected to produce the word or phrase in their own speaking or writing. Terms from the glossary appear bolded. Teachers should continue to support students' use of a new term in the lessons that follow the lesson in which it was first introduced.

New Terminology		
Lesson	**Receptive**	**Productive**
1	scale factor scaled copy scaling	

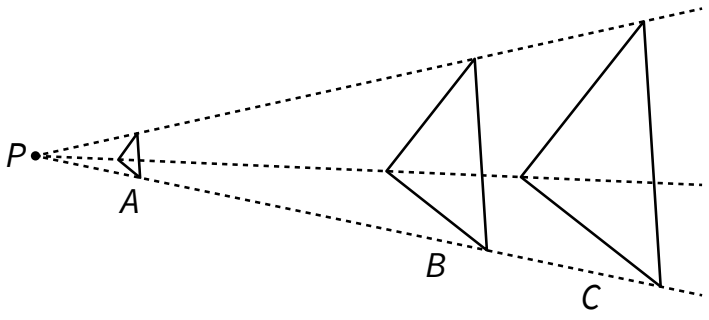
New Terminology		
Lesson	**Receptive**	**Productive**
2	**dilation** **center of a dilation** dilate radius	
4		**center of dilation** scale factor
6	**similar**	dilate
7		**dilation**
9	quotient	
10	**similar**	**slope** slope triangle
11	similarity *x*-coordinate *y*-coordinate equation of a line	quotient
13	estimate approximate/approximately	

Terminology

Center of a dilation

The center of a dilation is a fixed point on a plane. It is the starting point from which we measure distances in a dilation.

In this diagram, point *P* is the center of the dilation.

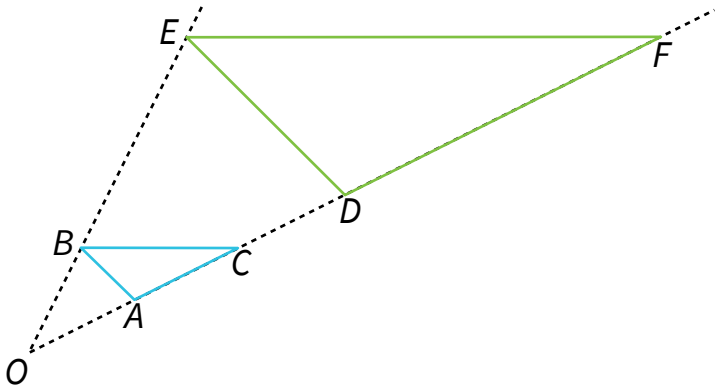


Dilation

A dilation is a transformation in which each point on a figure moves along a line and changes its distance from a fixed point. The fixed point is the center of the dilation. All of the original distances are multiplied by the same scale factor.

For example, triangle *DEF* is a dilation of triangle *ABC*. The fixed center point is *O* and the scale factor is 3.

This means that every point of triangle *DEF* is 3 times as far from *O* as every corresponding point of triangle *ABC*.

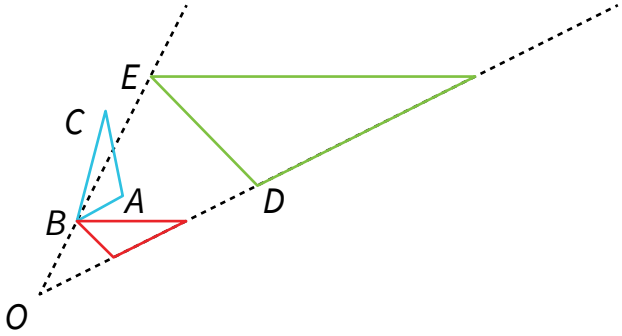


Similar

Two figures are similar if one can fit exactly over the other after rigid transformations and dilations.

In this figure, triangle *ABC* is similar to triangle *DEF*.

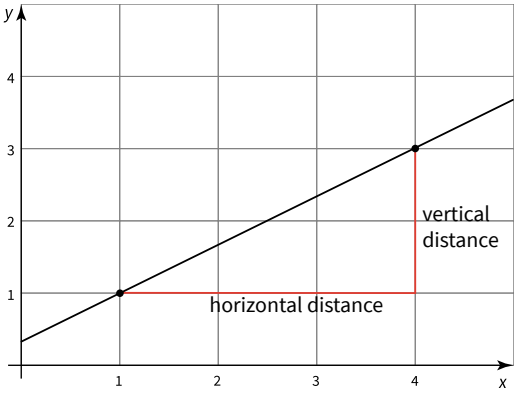
If *ABC* is rotated around point *B* and then dilated with center point *O*, then it will fit exactly over *DEF*. This means that they are similar.



Slope

The slope of a line is a number we can calculate using any two points on the line. To find the slope, divide the vertical distance between the points by the horizontal distance.

The slope of this line is 2 divided by 3 or $\frac{2}{3}$.



Required Materials

Blank paper

Dried linguine pasta

Four-function calculators

Geometry toolkits

Tracing paper, graph paper, colored pencils, scissors, ruler, protractor, and an index card to use as a straightedge or to mark right angles

Long straightedge

Measuring tapes

Rulers marked with inches

Scissors

Straightedges

Tape

Templates

Pre-printed slips, cut from copies of the templates

L4 Activity 2

L5 Activity 1

L6 Activity 3

L7 Activity 2

L8 Activity 1