ZEARN Math

MISSION OVERVIEW

GRADE 8

M1 Rigid Transformations and Congruence

Introduction

In this Mission, students learn to understand and use the terms “reflection,” “rotation,” “translation,” recognizing what determines each type of transformation, e.g., two points determine a translation. They learn to understand and use the terms “transformation” and “rigid transformation.” They identify and describe translations, rotations, and reflections, and sequences of these, using the terms “corresponding sides” and “corresponding angles,” and recognizing that lengths and angle measures are preserved. They draw images of figures under rigid transformations on and off square grids and the coordinate plane. They use rigid transformations to generate shapes and to reason about measurements of figures. They learn to understand congruence of plane figures in terms of rigid transformations. They recognize when one plane figure is congruent or not congruent to another. Students use the definition of “congruent” and properties of congruent figures to justify claims of congruence or non-congruence.

Overview of Topics and Lesson Objectives

Each mission is broken down into topics. A topic is a group of lessons that teach the same concept. There is a balance of Independent Digital Lessons and Concept Explorations in each topic of a mission to ensure every student learns with a mix of modalities, feedback, and support while engaging in grade-level content. Throughout each mission, students work on grade-level content with embedded remediation to address unfinished learnings.

	Objective	INDEPENDENT DIGITAL LESSON	CONCEPT EXPLORATION
Topic A	**Rigid Transformations**		
Lesson 1	Describe a translation or rotation of a shape using informal language, such as “slide” and “turn.”	✓	✓
Lesson 2	Describe the movement of shapes using the terms “clockwise,” “counterclockwise,” “translations,” “rotations,” and “reflections” of figures.	✓	✓
Lesson 3	Draw the image of a figure that results from a translation, rotation, and reflection on a grid.	✓	✓
Lesson 4	Use the terms translation, rotation, and reflection to precisely describe transformations and explain a sequence of transformations that takes one figure to its image.	✓	✓
Lesson 5	Apply transformations to points on a coordinate plane and name the coordinates of points in the image of a transformation.	✓	✓
Lesson 6	Apply a sequence of transformations to points on a coordinate plane. Determine whether the order of a sequence of transformations has an effect on the image.	✓	✓
Topic B	**Properties of Rigid Transformations**		
Lesson 7	Compare measurements of sides and angles on a shape before and after rigid transformations.	✓	✓
Lesson 8	Rotate a line segment 180 degrees around its midpoint, a point on the segment, and a point not on the segment. Generalize the outcomes of rotating a segment 180 degrees around different points.	✓	✓
Lesson 9	Describe the effects of a rigid transformation on a pair of parallel lines and understand that a rotation by 180 degrees about a point of two intersecting lines moves each angle to the angle that is vertical to it.	✓	✓
Lesson 10	Identify side lengths and angle measures using properties of rigid transformations.	X	✓
Mid-Mission Assessment: Topics A-B			

	Objective	INDEPENDENT DIGITAL LESSON	CONCEPT EXPLORATION
Topic C	**Congruence**		
Lesson 11	Determine whether two shapes are congruent by considering their area and perimeter and using rigid transformations.	✓	✓
Lesson 12	Use rigid transformations to decide whether or not two figures are congruent.	✓	✓
Lesson 13	Determine whether shapes are congruent by measuring the distances between corresponding points.	X	✓
Topic D	**Angles in a Triangle**		
Lesson 14	Calculate angle measures using alternate interior, vertical, and supplementary angles to solve problems.	✓	✓
Lesson 15	Use the measure of a straight angle to prove that the sum of the angles in a triangle is 180 degrees.	✓	✓
Lesson 16	Use parallel lines to show that the sum of the angles in any triangle is 180 degrees. Given the measure of two angles in a triangle, find the measure of the third angle.	✓	✓
Topic E	**Let's Put It to Work**		
Lesson 17	Create tessellations and designs with rotational symmetry using rigid transformations.	X	✓
End-of-Mission Assessment: Topics C-E			

Foundational Missions

For each mission, Zearn Math highlights the foundational missions, the earlier content where concepts are introduced and developed. Teachers can access foundational missions directly from the mission page of their Teacher Account to address any unfinished learnings. Zearn recommends that teachers assign foundational missions during Flex Day or during additional non-core instruction time. It is important to use a foundational mission to support students who are struggling, rather than an unaligned mission, because the content students learn in each foundational mission supports their Core Day learning.

Foundational Mission(s) for G8M1: G7M7 Topics A-B and G6M7 Topic C

Mission Overview

Work with transformations of plane figures in grade 8 draws on earlier work with geometry and geometric measurement. Students began to learn about two- and three-dimensional shapes in kindergarten, and continued this work in grades 1 and 2, composing, decomposing, and identifying shapes. Students' work with geometric measurement began with length and continued with area. Students learned to "structure two-dimensional space," that is, to see a rectangle with whole-number side lengths as composed of an array of unit squares or composed of iterated rows or iterated columns of unit squares. In grade 3, students distinguished between perimeter and area. They connected rectangle area with multiplication, understanding why (for whole-number side lengths) multiplying the side lengths of a rectangle yields the number of unit squares that tile the rectangle. They used area diagrams to represent instances of the distributive property. In grade 4, students applied area and perimeter formulas for rectangles to solve real-world and mathematical problems, and learned to use protractors. In grade 5, students extended the formula for the area of rectangles to rectangles with fractional side lengths. In grade 6, students combined their knowledge of geometry and geometric measurement to produce formulas for the areas of parallelograms and triangles, using these formulas to find surface areas of polyhedra. In grade 7, students worked with scaled copies and scale drawings, learning that angle measures are preserved in scaled copies, but areas increase or decrease proportionally to the square of the scale factor. Their study of scaled copies was limited to pairs of figures with the same rotation and mirror orientation. Viewed from the perspective of grade 8, a scaled copy is a dilation and translation, not a rotation or reflection, of another figure.

In grade 8, students extend their reasoning to plane figures with different rotation and mirror orientations.

Through activities designed and sequenced to allow students to make sense of problems and persevere in solving them (MP1), students use and extend their knowledge of geometry and geometric measurement. They begin the mission by looking at pairs of cartoons, each of which illustrates a translation, rotation, or reflection. Students describe in their own words how to move one cartoon figure onto another. As the mission progresses, they solidify their understanding of these transformations, increase the precision of their descriptions (MP6), and begin to use associated terminology, recognizing what determines each type of transformation, e.g., two points determine a translation. They identify and describe translations, rotations, and reflections, and sequences of these. In describing images of figures under rigid transformations on and off square grids and the coordinate plane, students use the terms "corresponding points," "corresponding sides," and "image." Students learn that angles and distances are preserved by any sequence of translations, rotations, and reflections, and that such a sequence is called a "rigid transformation." They learn the definition of "congruent": two figures are said to be congruent if there is a rigid transformation that takes one figure to the other. Students experimentally verify the properties of translations, rotations, and reflections, and use these properties to reason about plane figures, understanding informal arguments showing that the alternate interior angles cut by a transversal have the same measure and that the sum of the angles in a triangle is 180°. The latter will be used in a subsequent grade 8 mission on similarity and dilations. Throughout the mission, students discuss their mathematical ideas and respond to the ideas of others (MP3, MP6).

Many of the lessons in this mission ask students to work on geometric figures that are not set in a real-world context. This design choice respects the significant intellectual work of reasoning about area. Tasks set in real-world contexts are sometimes contrived and hinder rather than help understanding. Moreover, mathematical contexts are legitimate contexts that are worthy of study. Students do have opportunities in the mission to tackle real-world applications. In the culminating activity of the mission, students examine and create different patterns formed by plane figures. This is an opportunity for them to apply what they have learned in the mission (MP4).

In this mission, several lesson plans suggest that each student have access to a *geometry toolkit*. These contain tracing paper, graph paper, colored pencils, scissors, ruler, protractor, and an index card to use as a straightedge or to mark right angles, giving students opportunities to develop their abilities to select appropriate tools and use them strategically to solve problems (MP5). To support students in their developing understanding, Zearn Math Independent Digital Lessons

utilize a variety of digital manipulatives that mirror what students will see in their geometry toolkit. The combination of physical and digital tools will build a deep understanding of important geometric and measurement concepts.

Progression of Disciplinary Language

In this mission, teachers can anticipate students using language for mathematical purposes such as describing, generalizing, and justifying. Throughout the mission, students will benefit from routines designed to grow robust disciplinary language, both for their own sense-making and for building shared understanding with peers. Teachers can formatively assess how students are using language in these ways, particularly when students are using language to:

Describe

- movements of figures (Lessons 1 and 2)
- observations about transforming parallel lines (Lesson 9)
- transformations using corresponding points, line segments, and angles (Lesson 10)
- observations about angle measurements (Lesson 16)
- transformations found in tessellations and in designs with rotational symmetry (Lesson 17)

Generalize

- about categories for movement (Lesson 2)
- about rotating line segments (Lesson 8)
- about the relationship between vertical angles (Lesson 9)
- about transformations and congruence (Lesson 12)
- about corresponding segments and length (Lesson 13)
- about alternate interior angles (Lesson 14)
- about the sum of angles in a triangle (Lesson 16)

Justify

- whether or not rigid transformations could produce an image (Lesson 7)
- whether or not shapes are congruent (Lesson 11)
- whether or not polygons are congruent (Lesson 12)
- whether or not ovals are congruent (Lesson 13)
- whether or not triangles can be created from given angle measurements (Lesson 15)

In addition, students are expected to explain and interpret directions for transforming figures and how to apply transformations to find specific images. Students are also asked to use language to compare rotations of a line segment and compare perimeters and areas of rectangles. Over the course of the mission, teachers can support students'

mathematical understandings by amplifying (not simplifying) language used for all of these purposes as students demonstrate and develop ideas.

The table shows lessons where new terminology is first introduced, including when students are expected to understand the word or phrase receptively and when students are expected to produce the word or phrase in their own speaking or writing. Terms from the glossary appear bolded. Teachers should continue to support students' use of a new term in the lessons that follow the one in which it was first introduced.

New Terminology		
Lesson	**Receptive**	**Productive**
1	plane measure vertex/vertices direction	slide turn
2	**clockwise** **corresponding** **counterclockwise** **image** **reflection** **rotation** **translation**	opposite
3	angle of rotation center (of rotation) line of reflection	vertex / vertices
4	**sequence of transformations** distance	**clockwise** **counterclockwise** reflect rotate translate
5	point segment coordinates *x*-axis *y*-axis	
6	polygon	angle of rotation center (of rotation) line of reflection

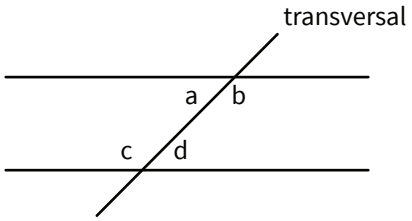
New Terminology		
Lesson	**Receptive**	**Productive**
7	**rigid transformation** measurements preserve	**reflection** **rotation** **translation** measure point
8	midpoint	segment
9	**vertical angles** parallel intersect	distance
10		**image** **rigid transformation** midpoint parallel
11	**congruent** perimeter area	
12		*x*-axis *y*-axis area right angle
13		**corresponding**
14	**alternate interior angles** **transversal**	**vertical angles** **congruent** supplementary angles
15	**straight angle**	
16		**alternate interior angles** **transversal** **straight angle**
17	tessellation symmetry	

Terminology

Alternate interior angles

Alternate interior angles are created when two parallel lines are crossed by another line called a transversal. Alternate interior angles are inside the parallel lines and on opposite sides of the transversal.

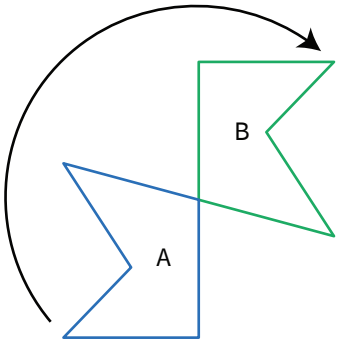
This diagram shows two pairs of alternate interior angles. Angles *a* and *d* are one pair and angles *b* and *c* are another pair.



Clockwise

Clockwise means to turn in the same direction as the hands of a clock. It is a turn to the right.

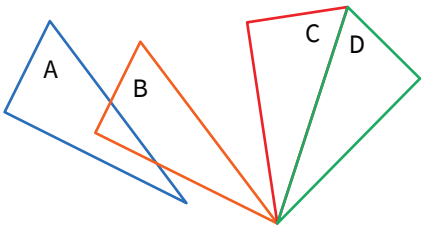
This diagram shows Figure A turned clockwise to make Figure B.



Congruent

One figure is congruent to another if it can be moved with translations, rotations, and reflections to fit exactly over the other.

In the figure, Triangle A is congruent to Triangles B, C, and D. A translation takes Triangle A to Triangle B, a rotation takes Triangle B to Triangle C, and a reflection takes Triangle C to Triangle D.

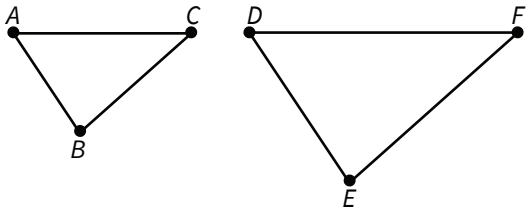


Corresponding

When part of an original figure matches up with part of a copy, we call them corresponding parts. These could be points, segments, angles, or distances.

For example, point *B* in the first triangle corresponds to point *E* in the second triangle.

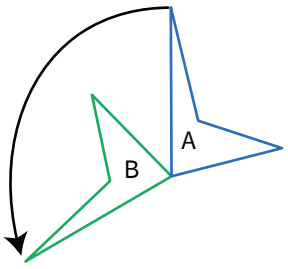
Segment *AC* corresponds to segment *DF.*



Counterclockwise

Counterclockwise means to turn opposite of the way the hands of a clock turn. It is a turn to the left.

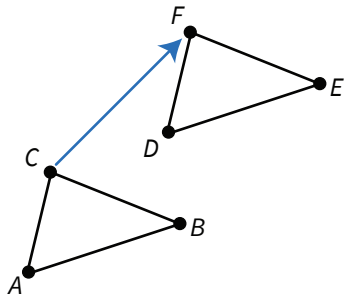
This diagram shows Figure A turned counterclockwise to make Figure B



Image

An image is the result of translations, rotations, and reflections on an object. Every part of the original object moves in the same way to match up with a part of the image.

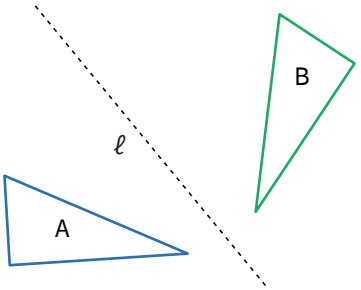
In this diagram, triangle ***ABC*** has been translated up and to the right to make triangle ***DEF***. Triangle ***DEF*** is the image of the original triangle ***ABC***.



Reflection

A reflection across a line moves every point on a figure to a point directly on the opposite side of the line. The new point is the same distance from the line as it was in the original figure.

This diagram shows a reflection of A over line ℓ that makes the mirror image B.



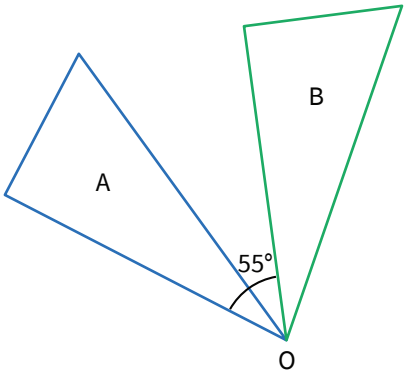
Rigid transformation

A rigid transformation is a move that does not change any measurements of a figure. Translations, rotations, and reflections are rigid transformations, as is any sequence of these.

Rotation

A rotation moves every point on a figure around a center by a given angle in a specific direction.

This diagram shows Triangle A rotated around center ***O*** by 55 degrees clockwise to get Triangle B.

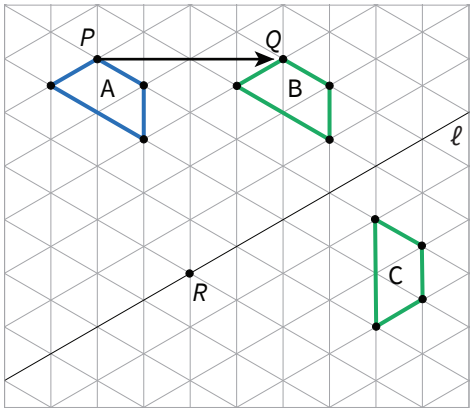


Sequence of transformations

A sequence of transformations is a set of translations, rotations, reflections, and dilations on a figure. The transformations are performed in a given order.

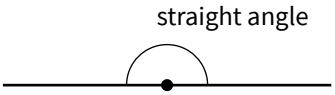
This diagram shows a sequence of transformations to move Figure A to Figure C.

First, A is translated to the right to make B. Next, B is reflected across line ℓ to make C.



Straight angle

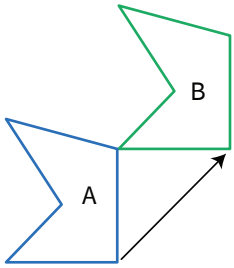
A straight angle is an angle that forms a straight line. It measures 180 degrees.



Translation

A translation moves every point in a figure a given distance in a given direction.

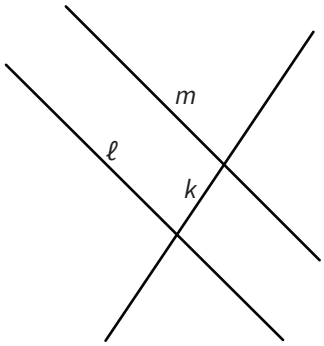
This diagram shows a translation of Figure A to Figure B using the direction and distance given by the arrow.



Transversal

A transversal to two parallel lines is a line that cuts across them, intersecting each one.

This diagram shows a transversal line k intersecting parallel lines m and ℓ.

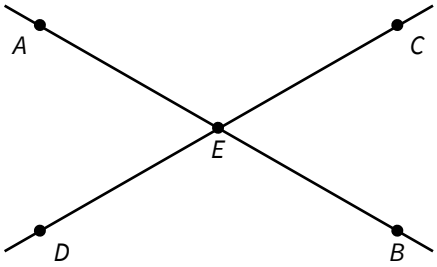


Vertical angles

Vertical angles are opposite angles that share the same vertex. They are formed by a pair of intersecting lines. Their angle measures are equal.

For example, angles *AEC* and *DEB* are vertical angles. If angle *AEC* measures 120°, then angle *DEB* must also measure 120°.

Angles *AED* and *BEC* are another pair of vertical angles.



Required Materials

Blank paper

Geometry toolkits

Tracing paper, graph paper, colored pencils, scissors, ruler, protractor, and an index card to use as a straightedge or to mark right angles

Graph paper

Isometric graph paper

Templates

Copies of templates

Pre-printed cards, cut from copies of the templates

L1 Activity 1

L2 Activity 2

L4 Activity 1

L6 Activity 1

L15 Activity 1

L17 Warm-up

Toothpicks, pencils, straws, or other objects