ZEARN Math

MISSION OVERVIEW

GRADE 7

M3 Measuring Circles

Introduction

In this mission, students learn to understand and use the term “circle” to mean the set of points that are equally distant from a point called the “center.” They gain an understanding of why the circumference of a circle is proportional to its diameter, with constant of proportionality π. They see informal derivations of the fact that the area of a circle is equal to π times the square of its radius. Students use the relationships of circumference, radius, diameter, and area of a circle to find lengths and areas, expressing these in terms of π or using appropriate approximations of π to express them numerically.

Overview of Topics and Lesson Objectives

Each mission is broken down into topics. A topic is a group of lessons that teach the same concept. There is a balance of Independent Digital Lessons and Concept Explorations in each topic of a mission to ensure every student learns with a mix of modalities, feedback, and support while engaging in grade-level content. Throughout each mission, students work on grade-level content with embedded remediation to address unfinished learnings.

	Objective	INDEPENDENT DIGITAL LESSON	CONCEPT EXPLORATION
Topic A	**Circumference of a Circle**		
Lesson 1	Examine quotients and use a graph to decide whether two associated quantities are in a proportional relationship.	✓	✓
Lesson 2	Describe characteristics that make a shape a circle and identify the diameter, radius, and center of a circle.	✓	✓
Lesson 3	Explore the relationship between the diameter and circumference of circles.	✓	✓
Lesson 4	Apply an understanding of circumference to find the distance around a shape.	✓	✓
Lesson 5	Use the radius or diameter of a wheel to find the distance the wheel travels in any number of complete rotations.	X	OPTIONAL
Topic B	**Area of a Circle**		
Lesson 6	Calculate and estimate the area of irregular polygons.	✓	✓
Lesson 7	Estimate the area of a circle using the area of a square with same side length as the radius.	✓	✓
Lesson 8	Explain how the area of a circle and its circumference are related to each other, and know and use the formula for area of a circle.	✓	✓
Lesson 9	Calculate the area of more complicated shapes that include fractions of circles.	✓	✓
Topic C	**Let's Put it to Work**		
Lesson 10	Decide whether the circumference or area of a circle is most useful in solving a problem and use formulas for area and circumference to solve problems relating to real-world situations.	✓	✓
Lesson 11	Apply understanding of area and circumference of circles to solve design problems.	X	OPTIONAL

Objective	INDEPENDENT DIGITAL LESSON	CONCEPT EXPLORATION
End-of-Mission Assessment: Topics A-C		

Foundational Missions

For each mission, Zearn Math highlights the foundational missions, the earlier content where concepts are introduced and developed. Teachers can access foundational missions directly from the mission page of their Teacher Account to address any unfinished learnings. Zearn recommends that teachers assign foundational missions during Flex Day or during additional non-core instruction time. It is important to use a foundational mission to support students who are struggling, rather than an unaligned mission, because the content students learn in each foundational mission supports their Core Day learning.

Foundational Mission(s) for G7M3: G6M1 Topic A

Mission Overview

In this mission, students extend their knowledge of circles and geometric measurement, applying their knowledge of proportional relationships to the study of circles. They extend their grade 6 work with perimeters of polygons to circumferences of circles, and recognize that the circumference of a circle is proportional to its diameter, with constant of proportionality π. They encounter informal derivations of the relationship between area, circumference, and radius.

The mission begins with activities designed to help students come to a more precise understanding of the characteristics of a circle (MP6): a "circle" is the set of points that are equally distant from a point called the "center"; the diameter of a circle is a line segment that passes through its center with endpoints on the circle; the radius is a line segment with one endpoint on the circle and one endpoint at the center. Students identify these characteristics in a variety of contexts (MP2). They use compasses to draw circles with given diameters or radii, and to copy designs that involve circles. Using their newly gained familiarity with circumference and diameter, students measure circular objects, investigating the relationship between measurements of circumference and diameter by making tables and graphs.

The second section involves area. Students encounter two informal derivations of the fact that the area of a circle is equal to π times the square of its radius. The first involves dissecting a disk into sectors and rearranging them to form a shape that approximates a parallelogram of height r and width πr. A second argument involves considering a disk as formed of concentric rings, "cutting" the rings with a radius, and "opening" the rings to form a shape that approximates an isosceles triangle of height r and base $2\pi \cdot r$.

In the third and last section, students select and use formulas for the area and circumference of a circle to solve abstract and real-world problems that involve calculating lengths and areas. They express measurements in terms of π or using appropriate approximations of π to express them numerically. In grade 8, they will use and extend their knowledge of circles and radii at the beginning of a mission on dilations and similarity.

On using the term circle. Strictly speaking, a circle is one-dimensional—the boundary of a two-dimensional region rather than the region itself. Because students are not yet expected to make this distinction, these materials refer to both circular regions (i.e., disks) and boundaries of disks as "circles," using illustrations to eliminate ambiguity.

Progression of Disciplinary Language

In this mission, teachers can anticipate students using language for mathematical purposes such as generalizing, justifying, and interpreting. Throughout the mission, students will benefit from routines designed to grow robust disciplinary language, both for their own sense-making and for building shared understanding with peers. Teachers can formatively assess how students are using language in these ways, particularly when students are using language to:

Generalize:

- about categories for sorting circles (Lesson 2)
- about the relationship between circumference and diameter (Lesson 3)
- about circumference and rotation (Lesson 5)
- about the relationship between radius and area of a circle (Lesson 8)

Justify:

- reasoning about circumference and perimeter (Lesson 4)
- estimates for the areas of circles (Lesson 7)

- reasoning about areas of curved figures (Lesson 9)
- reasoning about the cost of stained glass windows (Lesson 11)

Interpret:

- situations involving circles (Lessons 5 and 8)
- floor plans and maps (Lesson 6)
- situations involving circumference and area (Lesson 10)

In addition, students are expected to critique reasoning about circles and circle measurements, explain reasoning, including about different approximations of pi, and describe features of graphs and of deconstructed circles.

The table shows lessons where new terminology is first introduced, including when students are expected to understand the word or phrase receptively and when students are expected to produce the word or phrase in their own speaking or writing. Terms from the glossary appear bolded. Teachers should continue to support students' use of a new term in the lessons that follow where it was first introduced.

New Terminology		
Lesson	**Receptive**	**Productive**
1	relationship perimeter	
2	**radius** **diameter** **circumference** center (of a circle)	**circle**
3	**pi**	
4	half-circle rotation approximation	
5		**diameter** **circumference** **pi** travel
6		approximate estimate
7	**area of a circle**	
8	**radius** **area of a circle**	formula squared

New Terminology		
Lesson	**Receptive**	**Productive**
10		center (of a circle) formula squared
11	design	

Terminology

Area of a circle

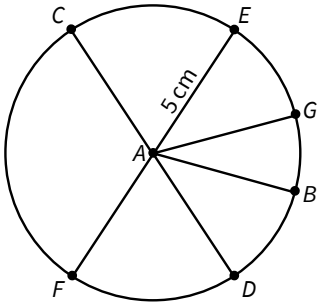
The area of a circle whose radius is r units is πr^2 square units.

A circle has radius 3 inches. Its area is $\pi 3^2 = 9\pi$ square inches, which is approximately 28.3 square inches.

Circle

A circle is made out of all the points that are the same distance from a given point.

For example, every point on this circle is 5 cm away from point A, which is the center of the circle.



Circumference

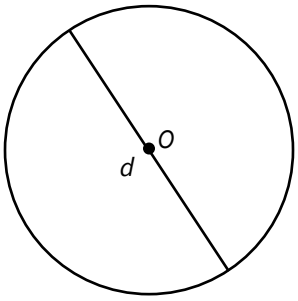
The circumference of a circle is the distance around the circle. If you imagine the circle as a piece of string, it is the length of the string. If the circle has radius r then the circumference is $2\pi r$.

The circumference of a circle of radius 3 is $2 \cdot \pi \cdot 3 = 6\pi$, which is about 18.85.

Diameter

A diameter is a line segment that goes from one edge of a circle to the other and passes through the center. A diameter can go in any direction. Every diameter of the circle is the same length. We also use the word *diameter* to mean the length of this segment.

For example, d is the diameter of this circle with center O.

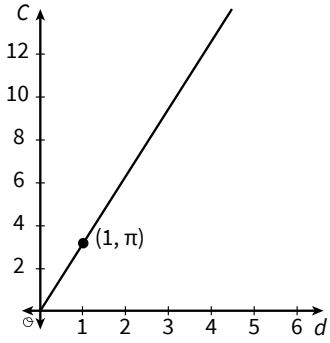


Pi (π)

There is a proportional relationship between the diameter and circumference of any circle. The constant of proportionality is pi. The symbol for pi is π.

We can represent this relationship with the equation $C = \pi d$, where C represents the circumference and d represents the diameter.

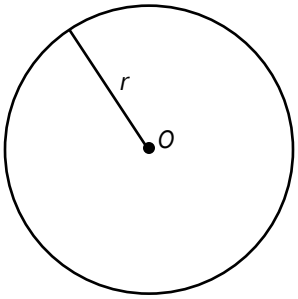
Some approximations for π are $\frac{22}{7}$, 3.14, and 3.14159.



Radius

A radius is a line segment that goes from the center to the edge of a circle. A radius can go in any direction. Every radius of the circle is the same length. We also use the word *radius* to mean the length of this segment.

For example, r is the radius of this circle with center O.



Required Materials

Blank paper

Compasses

Cylindrical household items

Empty toilet paper roll

Four-function calculators

Geometry toolkits

Tracing paper, graph paper, colored pencils, scissors, and an index card to use as a straightedge or to mark right angles.

Glue or gluesticks

Markers

Measuring tapes

Receipt tape

Rulers marked with centimeters

Scissors

Templates

Copies of template

Pre-printed slips, cut from copies of the template

Lesson 1 Activity 1

Lesson 2 Activity 1

Lesson 7 Activity 1

Lesson 10 Activity 1