ZEARN Math

MISSION OVERVIEW

GRADE 7

# M2 Introducing Proportional Relationships

## Introduction

In this Mission, students learn to understand and use the terms “proportional,” “constant of proportionality,” and “proportional relationship,” and recognize when a relationship is or is not proportional. They represent proportional relationships with tables, equations, and graphs. Students use these terms and representations in reasoning about situations that involve constant speed, unit pricing, and measurement conversions.

## Overview of Topics and Lesson Objectives

Each mission is broken down into topics. A topic is a group of lessons that teach the same concept. There is a balance of Independent Digital Lessons and Concept Explorations in each topic of a mission to ensure every student learns with a mix of modalities, feedback, and support while engaging in grade-level content. Throughout each mission, students work on grade-level content with embedded remediation to address unfinished learnings.

| | Objective | INDEPENDENT DIGITAL LESSON | CONCEPT EXPLORATION |
|---|---|---|---|
| **Topic A** | **Representing Proportional Relationships with Tables** | | |
| Lesson 1 | Compare and create representations to compare ratios in the context of recipes or scaled copies. | ✓ | ✓ |
| Lesson 2 | Use a table to describe a proportional relationship, calculate the constant of proportionality, and find missing values. | ✓ | ✓ |
| Lesson 3 | Find the constant of proportionality from information given on a table and use the constant of proportionality to fill information on a table. | ✓ | ✓ |
| **Topic B** | **Representing Proportional Relationships with Equations** | | |
| Lesson 4 | Write equations to represent a proportional relationship described in a table. | ✓ | ✓ |
| Lesson 5 | Write two equations that represent the same proportional relationship. | ✓ | ✓ |
| Lesson 6 | Use tables and equations to solve problems involving proportional relationships. | ✓ | ✓ |
| **Topic C** | **Comparing Proportional and Nonproportional Relationships** | | |
| Lesson 7 | Use a table of values to determine if a relationship is proportional. | ✓ | ✓ |
| Lesson 8 | Recognize that proportional relationships are characterized by equations in the form $y = kx$. | ✓ | ✓ |
| Lesson 9 | Write an equation to represent a proportional relationship and solve problems about proportional relationships. | ✓ | ✓ |
| **Mid-Mission Assessment: Topics A-C** | | | |

| | Objective | INDEPENDENT DIGITAL LESSON | CONCEPT EXPLORATION |
|---|---|---|---|
| **Topic D** | **Representing Proportional Relationships with Graphs** | | |
| Lesson 10 | Generalize that the graph of a proportional relationship lies on a line through the origin. | ✓ | ✓ |
| Lesson 11 | Interpret points on the graph of a proportional relationship, and identify the constant of proportionality from the graph of a proportional relationship. | ✓ | ✓ |
| Lesson 12 | Interpret and compare two related proportional relationships on the same graph. | ✓ | ✓ |
| Lesson 13 | Interpret and compare the same proportional relationship using two different sets of tables, graphs, and equations. | ✓ | ✓ |
| **Topic E** | **Let's Put It To Work** | | |
| Lesson 14 | Represent a proportional relationship in four different ways. | ✓ | ✓ |
| Lesson 15 | Use proportional relationships to analyze a problem about water usage. | X | ✓ |
| **End-of-Mission Assessment: Topics D-E** | | | |

## Foundational Missions

For each mission, Zearn Math highlights the foundational missions, the earlier content where concepts are introduced and developed. Teachers can access foundational missions directly from the mission page of their Teacher Account to address any unfinished learnings. Zearn recommends that teachers assign foundational missions during Flex Day or during additional non-core instruction time. It is important to use a foundational mission to support students who are struggling, rather than an unaligned mission, because the content students learn in each foundational mission supports their Core Day learning.

**Foundational Missions for G7M2: G6M2 Topics C-D and G6M3 Topic C**

## Mission Overview

In this mission, students develop the idea of a proportional relationship out of the grade 6 idea of equivalent ratios. Proportional relationships prepare the way for the study of linear functions in grade 8.

In grade 6, students learned two ways of looking at equivalent ratios. First, if you multiply both values in a ratio $a : b$ by the same positive number $s$ (called the scale factor) you get an equivalent ratio $sa : sb$. Second, two ratios are equivalent if they have the same unit rate. A unit rate is the "amount per 1" in a ratio; the ratio $a : b$ is equivalent to $\frac{a}{b}$ : 1, and $\frac{a}{b}$ is a unit rate giving the amount of the first quantity per unit of the second quantity. You could also talk about the amount of the second quantity per unit of the first quantity, which is the unit rate $\frac{b}{a}$, coming from the equivalent ratio $1 : \frac{b}{a}$.

In a table of equivalent ratios, a multiplicative relationship between the pair of rows is given by a scale factor. By contrast, the multiplicative relationship between the columns is given by a unit rate. Every number in the second column is obtained by multiplying the corresponding number in the first column by one of the unit rates, and every number in the first column is obtained by multiplying the number in the second column by the other unit rate. The relationship between pairs of values in the two columns is called a proportional relationship, the unit rate that describes this relationship is called a constant of proportionality, and the quantity represented by the right column is said to be proportional to the quantity represented by the left. (Although a proportional relationship between two quantities represented by $a$ and $b$ is associated with *two* constants of proportionality, $\frac{a}{b}$ and $\frac{b}{a}$, throughout the mission, the convention is if $a$ and $b$ are, respectively, in the left and right columns of a table, then $\frac{b}{a}$ is the constant of proportionality for the relationship represented by the table.)

For example, if a person runs at a constant speed and travels 12 miles in 2 hours, then the distance traveled is proportional to the time elapsed, with constant of proportionality 6, because

$$\text{distance} = 6 \cdot \text{time}.$$

The time elapsed is proportional to distance traveled with constant of proportionality $\frac{1}{6}$, because

$$\text{time} = \frac{1}{6} \cdot \text{distance}.$$

Students learn that any proportional relationship can be represented by an equation of the form $y = kx$ where $k$ is the constant of proportionality, that its graph lies on a line through the origin that passes through Quadrant I, and that the constant of proportionality indicates the steepness of the line. By the end of the mission, students should be able to easily work with common contexts associated with proportional relationships (such as constant speed, unit pricing, and measurement conversions) and be able to determine whether a relationship is proportional or not.

Because this mission focuses on understanding what a proportional relationship is, how it is represented, and what types of contexts give rise to proportional relationships, the contexts have been carefully chosen. The first tasks in the mission employ contexts such as servings of food, recipes, constant speed, and measurement conversion, that should be familiar to students from the grade 6 course. These contexts are revisited throughout the mission as new aspects of proportional relationships are introduced.

Associated with the contexts from the grade 6 course are derived units: miles per hour; meters per second; dollars per pound; or cents per minute. In this mission, students build on their grade 6 experiences in working with a wider variety of derived units, such as cups of flour per tablespoon of honey, hot dogs eaten per minute, and centimeters per millimeter. The tasks in this mission avoid discussion of measurement error and statistical variability, which will be addressed in later missions.

*On using the terms quantity, ratio, proportional relationship, unit rate, and fraction*. In these materials, a quantity is a measurement that is or can be specified by a number and a unit, e.g., 4 oranges, 4 centimeters, "my height in feet," or "my height" (with the understanding that a unit of measurement will need to be chosen, MP6). The term ratio is used to mean a type of association between two or more quantities. A *proportional relationship* is a collection of equivalent ratios.

A *unit rate* is the numerical part of a rate per 1 unit, e.g., the 6 in 6 miles per hour. The fractions $\frac{a}{b}$ and $\frac{b}{a}$ are never called ratios. The fractions $\frac{a}{b}$ and $\frac{b}{a}$ are identified as "unit rates" for the ratio $a : b$. In high school—after their study of ratios, rates, and proportional relationships—students discard the term "unit rate," referring to $a$ to $b$, $a : b$, and $\frac{a}{b}$ as "ratios."

In grades 6–8, students write rates without abbreviated units, for example as "3 miles per hour" or "3 miles in every 1 hour." Use of notation for derived units such as $\frac{\text{mi}}{\text{hr}}$ waits for high school—except for the special cases of area and volume. Students have worked with area since grade 3 and volume since grade 5. Before grade 6, they have learned the meanings of such things as sq cm and cu cm. After students learn exponent notation in grade 6, they also use $\text{cm}^2$ and $\text{cm}^3$.

A *fraction* is a point on the number line that can be located by partitioning the segment between 0 and 1 into equal parts, then finding a point that is a whole number of those parts away from 0. A fraction can be written in the form $\frac{a}{b}$ or as a decimal.

## Progression of Disciplinary Language

In this mission, teachers can anticipate students using language for mathematical purposes such as comparing, interpreting, and generalizing. Throughout the mission, students will benefit from routines designed to grow robust disciplinary language, both for their own sense-making and for building shared understanding with peers. Teachers can formatively assess how students are using language in these ways, particularly when students are using language to:

Compare:

- drink mixtures and figures (Lesson 1)
- approaches to solving problems involving proportional relationships (Lesson 6)
- proportional relationships with nonproportional relationships (Lesson 8)
- tables, descriptions, and graphs representing the same situations (Lesson 10)
- graphs of proportional relationships (Lesson 12)

Interpret

- representations showing equivalent ratios (Lesson 1)
- tables showing equivalent ratios (Lesson 2)
- situations involving proportional relationships (Lesson 6 and 9)
- how a graph represents features of a situation (Lesson 11)

Generalize

- about proportional relationships (Lesson 4)
- about equations that represent proportional relationships (Lesson 5)
- about how a constant of proportionality is represented by graphs and tables (Lesson 13)

In addition, students are expected to describe proportional relationships and constants of proportionality, explain how to determine whether or not a relationship is proportional and how to compare and represent situations with different constants of proportionality, justify whether or not a relationship is proportional, and represent proportional and nonproportional relationships in multiple ways.

The table shows lessons where new terminology is first introduced, including when students are expected to understand the word or phrase receptively and when students are expected to produce the word or phrase in their own speaking or writing. Terms from the glossary appear bolded. Teachers should continue to support students' use of a new term in the lessons that follow where it was first introduced.

| New Terminology | | |
|---|---|---|
| **Lesson** | **Receptive** | **Productive** |
| 1 | **equivalent ratios** | |
| 2 | **constant of proportionality**<br>**proportional relationship**<br>value | **equivalent ratios**<br>row<br>column |
| 3 | ________ is proportional to ________<br>relate<br>constant | per<br>reciprocal |
| 4 | equation<br>quotient | ________ is proportional to ________ |
| 5 | steady<br>situation | |
| 6 | | equation<br>quotient |
| 7 | | **constant of proportionality**<br>**proportional relationship** |
| 8 | volume<br>surface area | constant |
| 10 | **origin**<br>plot<br>coordinate plane | |
| 11 | quantity<br>axes<br>coordinates | |
| 13 | $x$-coordinate<br>$y$-coordinate | **origin** |

| New Terminology | | |
|---|---|---|
| **Lesson** | **Receptive** | **Productive** |
| 14 | | axes |
| 15 | reasonable | |

## Terminology

### Constant of proportionality

In a proportional relationship, the values for one quantity are each multiplied by the same number to get the values for the other quantity. This number is called the constant of proportionality.

In this example, the constant of proportionality is 3, because $2 \cdot 3 = 6$, $3 \cdot 3 = 9$, and $5 \cdot 3 = 15$. This means that there are 3 apples for every 1 orange in the fruit salad.

| Number of oranges | Number of apples |
|---|---|
| 2 | 6 |
| 3 | 9 |
| 5 | 15 |

### Equivalent ratios

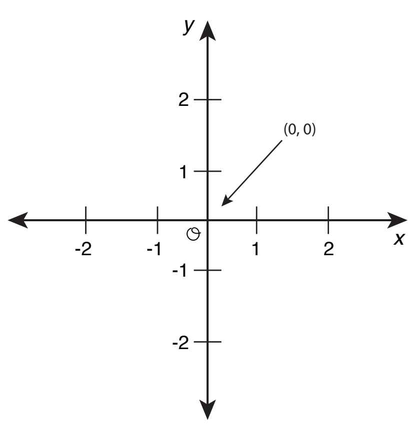
Two ratios are equivalent if you can multiply each of the numbers in the first ratio by the same factor to get the numbers in the second ratio. For example, 8 : 6 is equivalent to 4 : 3, because $8 \cdot \frac{1}{2} = 4$ and $6 \cdot \frac{1}{2} = 3$.

A recipe for lemonade says to use 8 cups of water and 6 lemons. If we use 4 cups of water and 3 lemons, it will make half as much lemonade. Both recipes taste the same, because 8 : 6 and 4 : 3 are equivalent ratios.

| Cups of Water | Number of Lemons |
|---|---|
| 8 | 6 |
| 4 | 3 |

### Origin

The origin is the point (0, 0) in the coordinate plane. This is where the horizontal axis and the vertical axis cross.



**Proportional relationship**

In a proportional relationship, the values for one quantity are each multiplied by the same number to get the values for the other quantity.

For example, in this table every value of $p$ is equal to 4 times the value of $s$ on the same row.

We can write this relationship as $p = 4s$. This equation shows that $p$ is proportional to $s$.

| $s$ | $p$ |
|---|---|
| 2 | 8 |
| 3 | 12 |
| 5 | 20 |
| 10 | 40 |

## Required Materials

**Colored pencils**

**Drink mix**

**Four-function calculators**

**Graph paper**

**Internet-enabled device**

**Measuring cup**

**Measuring spoons**

**Mixing containers**

**Rulers**

**Small disposable cups**

**Snap cubes**

**Templates**

Copies of template

Pre-printed slips, cut from copies of the template

Lesson 9 Activity 1

Lesson 10 Activity 2

Lesson 14 Activity 1

**Tools for creating a visual display**

Any way for students to create work that can be easily displayed to the class. Examples: chart paper and markers, whiteboard space and markers, shared online drawing tool, access to a document camera.

**Water**