ZEARN Math

HI THERE,

Welcome to Mission 5!

Arithmetic in Base Ten

NAME ______________________________

GRADE 6

Printed in the U.S.A.

Pilot Edition

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Lesson 1: Using Decimals in a Shopping Context

Let's use what we know about decimals to make shopping decisions.

Warm-Up

1

Clare went to a concession stand that sells pretzels for \$3.25, drinks for \$1.85, and bags of popcorn for \$0.99 each. She bought at least one of each item and spent no more than \$10.

1. Could Clare have purchased 2 pretzels, 2 drinks, and 2 bags of popcorn? Explain your reasoning.

2. Could she have bought 1 pretzel, 1 drink, and 5 bags of popcorn? Explain your reasoning.

Concept Exploration

ACTIVITY 1

2

You are planning a dinner party with a budget of \$50 and a menu that consists of 1 main dish, 2 side dishes, and 1 dessert. There will be 8 guests at your party.

Choose your menu items and decide on the quantities to buy so you stay on budget. If you choose meat, fish, or poultry for your main dish, plan to buy at least 0.5 pound per person.

Use the worksheet to record your choices and estimated costs. Then find the estimated total cost and cost per person. See examples in the first two rows.

1. The budget is \$ ________ per guest.

Item	Quantity Needed	Advertised Price	Estimated Subtotal (in dollars)	Estimated cost per person (in dollars)
Ex. Main Dish: Fish	4 pounds	\$6.69 per pound	$4 \cdot 7 = 28$	
Ex. Dessert: Cupcakes	8 cupcakes	\$2.99 per 6 cupcakes	$2 \cdot 3 = 6$	
Main Dish				
Side Dish 1				
Side Dish 2				
Dessert				
Estimated Total				

2. Is your estimated total close to your budget? If so, continue to the next question. If not, revise your menu choices until your estimated total is close to the budget.
3. Calculate the actual costs of the two most expensive items and add them. Show your reasoning.

4. How will you know if your total cost for all menu items will or will not exceed your budget? Is there a way to predict this without adding all the exact costs? Explain your reasoning.

Digital Lesson

Solve 23.49 + 31.70 using a place value chart. Then, record the sum.

tens	ones	tenths	hundredths

23.49 + 31.70 = ________

Lesson Summary

We often use decimals when dealing with money. In these situations, sometimes we round and make estimates, and other times we calculate the numbers more precisely.

There are many different ways we can add, subtract, multiply, and divide decimals. When we perform these computations, it is helpful to understand the meanings of the digits in a number and the properties of operations. We will investigate how these understandings help us work with decimals in upcoming lessons.

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GRADE 6 / MISSION 5 / LESSON 1

Exit Ticket

Planning your menu involved many calculations with decimals. Reflect on how you made these calculations:

1. How did you compute sums of dollar amounts that were not whole numbers? For example, how did you compute the sum of $5.89 and $1.45? Use this example to explain your strategy.

2. How did you compute products of dollar amounts that were not whole numbers? For example, how did you compute the cost of 4 pounds of beef at $5.89 per pound? Use this example to explain your strategy.

Lesson 2: Using Diagrams to Represent Addition and Subtraction

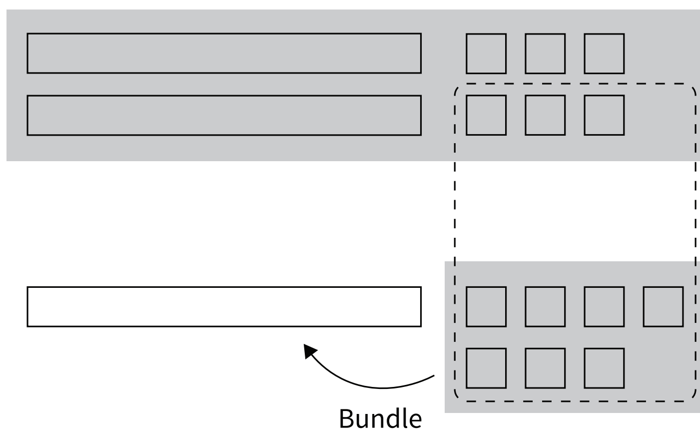
Let's represent addition and subtraction of decimals.

Concept Exploration

ACTIVITY 1

1 Here are two ways to calculate the value of 0.26 + 0.07. In the diagram, each rectangle represents 0.1 and each square represents 0.01.

$$\begin{array}{r} {\scriptstyle 1} \\ 0.26 \\ +\ 0.07 \\ \hline 0.33 \end{array}$$



Use what you know about base-ten units and addition of base-ten numbers to explain:

a) Why ten squares can be "bundled" into a rectangle.

b) How this "bundling" is reflected in the computation.

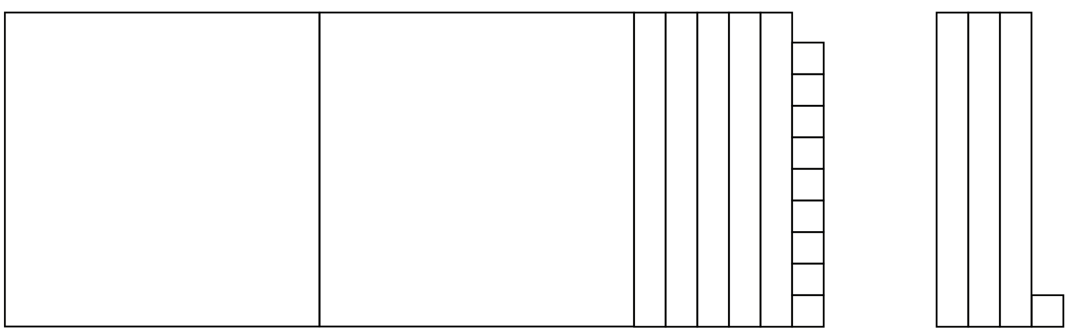
2 Use place value diagrams to help you solve the following problems.

1. Find the value of 0.38 + 0.69 by drawing a diagram. Can you find the sum without bundling? Would it be useful to bundle some pieces? Explain your reasoning.

2. Calculate 0.38 + 0.69. Check your calculation against your diagram in the previous question.

3. Find each sum. The larger square represents 1, the rectangle represents 0.1, and the smaller square represents 0.01.

a)



b)

$$\begin{array}{r} 6.03 \\ +\ 0.098 \\ \hline \end{array}$$

Digital Lesson

Solve each problem using the place value chart or the algorithm. Choose whichever strategy works best for you.

1.

ones	tenths	hundredths	thousandths

$$\begin{array}{r} 4.032 \\ +\ 2.079 \\ \hline \end{array}$$

2.

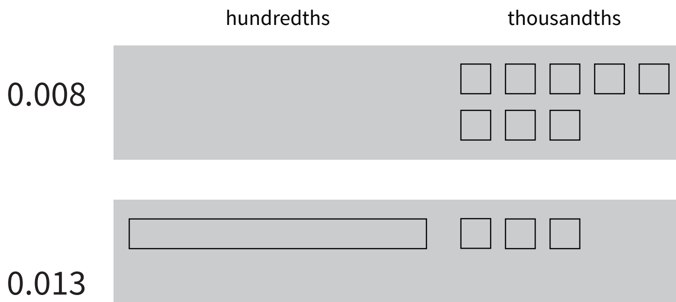
ones	tenths	hundredths	thousandths

$$\begin{array}{r} 2.463 \\ -\ 1.749 \\ \hline \end{array}$$

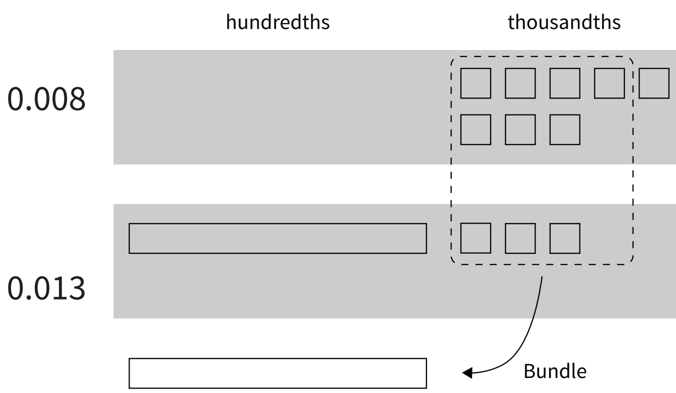
Lesson Summary

Base-ten diagrams represent collections of base-ten units—tens, ones, tenths, hundredths, etc. We can use them to help us understand sums of decimals.

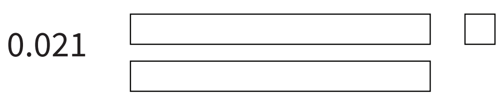
Here is a diagram of 0.008 and 0.013, where a square represents 0.001 and a rectangle (made up of ten squares) represents 0.01.



To find the sum, we can “bundle” (or compose) 10 thousandths as 1 hundredth.



Here is a diagram of the sum, which shows 2 hundredths and 1 thousandth.



We can use vertical calculation to find . Notice that here 10 thousandths are also bundled (or composed) as 1 hundredth.

$$\begin{array}{r} {}^{1} \\ 0.008 \\ +\ 0.013 \\ \hline 0.021 \end{array}$$

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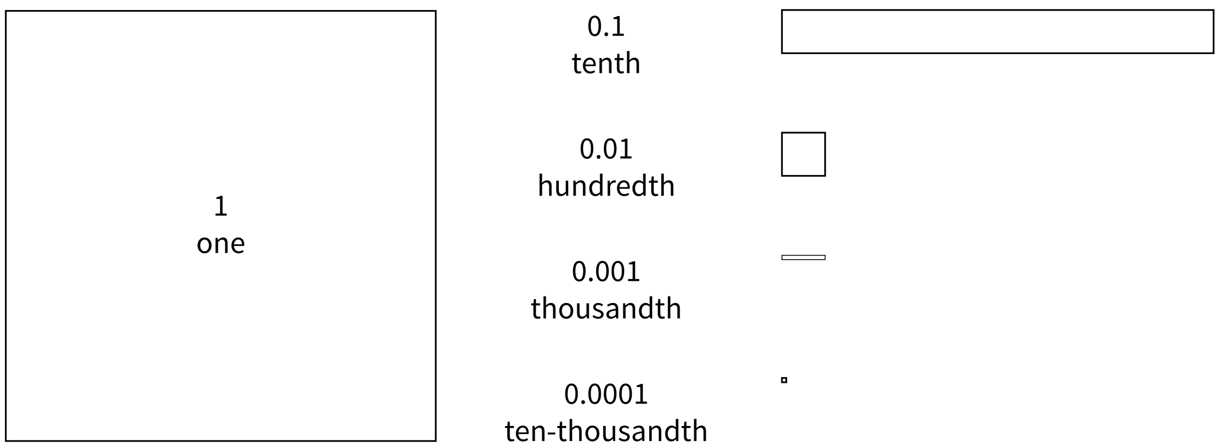
GRADE 6 / MISSION 5 / LESSON 2

Exit Ticket

Is this equation true?

$$0.025 + 0.17 = 0.042$$

Use a diagram or numerical calculation to explain or show your reasoning. Here are diagrams that you could use to represent base-ten units.



Lesson 3: Adding and Subtracting Decimals with Few Non-Zero Digits

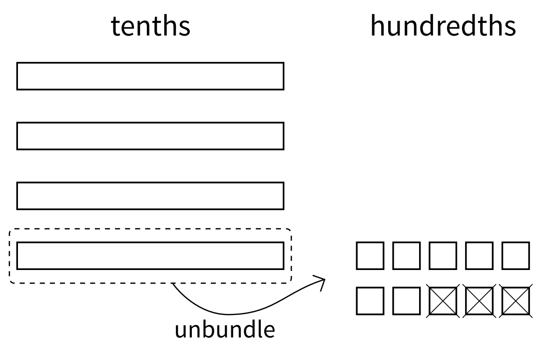
Let's add and subtract decimals.

Concept Exploration

ACTIVITY 1 TASK 1

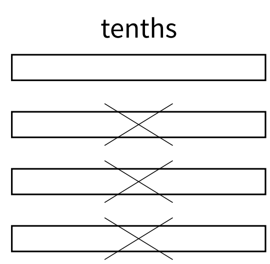
To represent 0.4 - 0.03, Diego and Noah drew different diagrams. Each rectangle shown here represents 0.1. Each square represents 0.01.

Diego started by drawing 4 rectangles for 0.4. He then replaced 1 rectangle with 10 squares and crossed out 3 squares for the subtraction of 0.03, leaving 3 rectangles and 7 squares in his drawing.



Diego's method

Noah started by drawing 4 rectangles for 0.4. He then crossed out 3 of them to represent the subtraction, leaving 1 rectangle in his drawing.



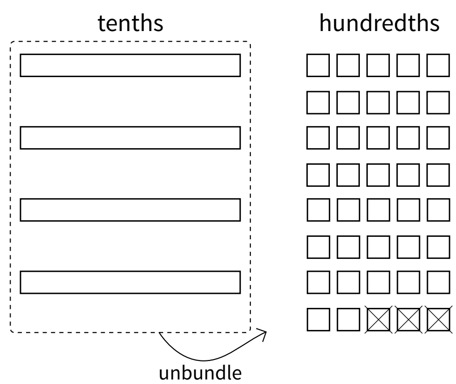
Noah's method

Do you agree that either diagram correctly represents 0.4 - 0.03? Discuss your reasoning with a partner.

ACTIVITY 1 TASK 2

2 To represent 0.4 - 0.03, Elena drew another diagram.

She also started by drawing 4 rectangles. She then replaced all 4 rectangles with 40 squares and crossed out 3 squares for the subtraction of 0.03, leaving 37 squares in her drawing. Is her diagram correct? Discuss your reasoning with a partner.



Elena's method

ACTIVITY 1 TASK 3

3 Find each difference. Explain or show your reasoning.

a) 0.3 - 0.05

b) 2.1 - 0.4

c) 1.03 - 0.06

d) 0.02 - 0.007

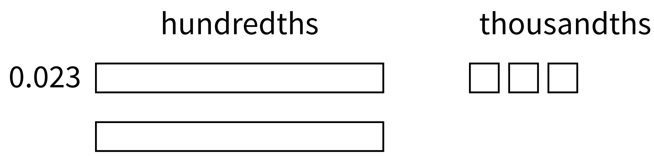
Digital Lesson

The work below shows how a student solved 5.408 - 1.07.
What error did the student make? Explain why this student's work is incorrect and show the correct solution using the subtraction algorithm or place value chart.

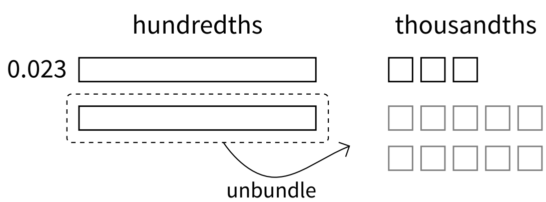
$$\begin{array}{r} 5.408 \\ -\quad 1.07 \\ \hline 1.301 \end{array}$$

Lesson Summary

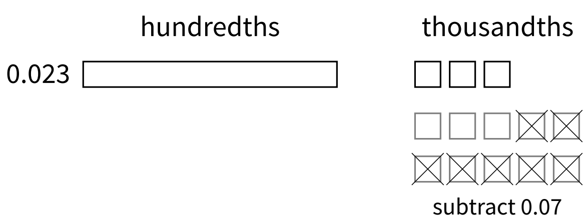
Base-ten diagrams can help us understand subtraction as well as addition. Suppose we are finding 0.023 - 0.007. Here is a diagram showing 0.023, or 2 hundredths and 3 thousandths.



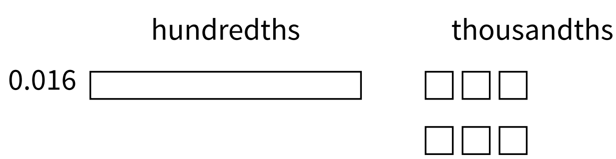
Subtracting 7 thousandths means removing 7 small squares, but we do not have enough to remove. Because 1 hundredth is equal to 10 thousandths, we can “unbundle” (or decompose) one of the hundredths (1 rectangle) into 10 thousandths (10 small squares).



We now have 1 hundredth and 13 thousandths, from which we can remove 7 thousandths.



We have 1 hundredth and 6 thousandths remaining, so 0.023 - 0.007 = 0.016.



Here is a vertical calculation of 0.023 - 0.007.

$$\begin{array}{r} \scriptstyle 1\ 13 \\ 0.0\not{2}\not{3} \\ -\ 0.007 \\ \hline 0.016 \end{array}$$

In both calculations, notice that a hundredth is unbundled (or decomposed) into 13 thousandths in order to subtract 7 thousandths.

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GRADE 6 / MISSION 5 / LESSON 3

Exit Ticket

1. Find the sum 1.56 + 0.083. Show your reasoning.

2. Find the difference 0.2 - 0.05. Show your reasoning.

3. You need to be at least 39.37 inches tall (about a meter) to ride on a bumper car. Diego's cousin is 35.75 inches tall. How many more inches will he need to grow before Diego can take him on the bumper car ride? Explain or show your reasoning.

Lesson 4: Adding and Subtracting Decimals with Many Non-Zero Digits

Let's practice adding and subtracting decimals.

Warm-Up

1 Clare bought a photo for 17 cents and paid with a $5 bill. Answer the following questions about this situation.

```
    5               5               5
 – 0.17          – 0.17          – 0.17
 ------          ------          ------
```

1. Which way of writing the numbers could Clare use to find the change she should receive? Be prepared to explain how you know.

2. Find the amount of change that Clare should receive. Show your reasoning, and be prepared to explain how you calculate the difference of 0.17 and 5.

Concept Exploration

ACTIVITY 1

2 Solve the following subtraction problems.

1. Find the value of each expression. Show your reasoning.
   a) 11.3 - 9.5

   b) 318.8 - 94.63

   c) 0.02 - 0.0116

2. Discuss with a partner:
   - Which method or methods did you use in the previous question? Why?
   - In what ways were your methods effective? Was there an expression for which your methods did not work as well as expected?

3 For each situation, write an equation and use it to solve.

1. Lin's grandmother ordered needles that were 0.3125 inch long to administer her medication, but the pharmacist sent her needles that were 0.6875 inch long. How much longer were these needles than the ones she ordered? Show your reasoning.

2. There is 0.162 liter of water in a 1-liter bottle. How much more water should be put in the bottle so it contains exactly 1 liter? Show your reasoning.

3. One micrometer is 1 millionth of a meter. A red blood cell is about 7.5 micrometers in diameter. A coarse grain of sand is about 70 micrometers in diameter. Find the difference between the two diameters in meters. Show your reasoning.

ACTIVITY 2

4 Write the missing digits in each calculation so that the value of each sum or difference is correct. Be prepared to explain your reasoning.

1.
```
  0.404
+ □.□□□
-------
  1
```

2.
```
  9.8765
+ □.□□□□
--------
 10
```

3.
```
  0.7
− □.□□□
-------
  0.012
```

4.
```
  7
− □.□□□□
--------
  3.4567
```

5.
```
  70
− □□.□□□□
---------
   0.0089
```

Digital Lesson

How could you solve this using subtraction?

0.391 + ________ = 0.4017

Lesson Summary

Base-ten diagrams work best for representing subtraction of numbers with few non-zero digits, such as 0.16 - 0.09. For numbers with many non-zero digits, such as 0.25103 - 0.04671, it would take a long time to draw the base-ten diagram. With vertical calculations, we can find this difference efficiently.

Thinking about base-ten diagrams can help us make sense of this calculation.

$$\begin{array}{rl} & \quad\ \ \ 10 \\ & \quad\ 4\ \ \not{0}\ 10 \\ & 0.2\not{5}\not{1}\not{0}3 \\ - & 0.04671 \\ \hline & 0.20432 \end{array}$$

The thousandth in 0.25103 is unbundled (or decomposed) to make 10 ten-thousandths so that we can subtract 7 ten-thousandths. Similarly, one of the hundredths in 0.25103 is unbundled (or decomposed) to make 10 thousandths.

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GRADE 6 / MISSION 5 / LESSON 4

Exit Ticket

1. Diego is 59.5 inches tall. His brother is 40.125 inches tall. How much taller than his brother is Diego? Show your reasoning.

2. A runner has run 1.192 kilometers of a 10-kilometer race. How much farther does he need to run to finish the race? Show your reasoning.

Lesson 5: Decimal Points in Products

Let's look at products that are decimals.

Concept Exploration

ACTIVITY 1 TASK 1

Work with a partner to answer the following questions. One person should answer the questions labeled "Partner A," and the other should answer those labeled "Partner B." Then compare the results.

1. Find each product or quotient. Be prepared to explain your reasoning.

Partner A	Partner B
a) $250 \cdot \frac{1}{10}$	a) $250 \div 10$
b) $250 \cdot \frac{1}{100}$	b) $250 \div 100$
c) $48 \div 10$	c) $48 \cdot \frac{1}{10}$
d) $48 \div 100$	d) $48 \cdot \frac{1}{100}$

2. Use your work in the previous problems to find $720 \cdot (0.1)$ and $720 \cdot (0.01)$. Explain your reasoning.

ACTIVITY 1 TASK 2

2 Find each product. Show your reasoning.

a) $36 \cdot (0.1)$

d) $54 \cdot (0.01)$

b) $(24.5) \cdot (0.1)$

e) $(9.2) \cdot (0.01)$

c) $(1.8) \cdot (0.1)$

ACTIVITY 1 TASK 3

3 Jada says: "If you multiply a number by 0.001, the decimal point of the number moves three places to the left." Do you agree with her statement? Explain your reasoning.

ACTIVITY 2

Answer the following questions about multiplying decimals and fractions.

1. Select **all** expressions that are equivalent to $(0.6) \cdot (0.5)$. Be prepared to explain your reasoning.

   a) $6 \cdot (0.1) \cdot 5 \cdot (0.1)$

   b) $6 \cdot (0.01) \cdot 5\,(0.1)$

   c) $6 \cdot \frac{1}{10} \cdot 5 \cdot \frac{1}{10}$

   d) $6 \cdot \frac{1}{1000} \cdot 5 \cdot \frac{1}{100}$

   e) $6 \cdot (0.001) \cdot 5 \cdot (0.01)$

   f) $6 \cdot 5 \cdot \frac{1}{10} \cdot \frac{1}{10}$

   g) $\frac{6}{10} \cdot \frac{5}{10}$

2. Find the value of $(0.6) \cdot (0.5)$. Show your reasoning.

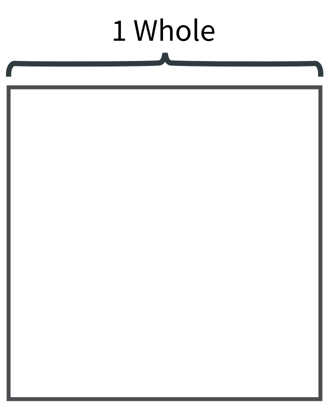
3. Find the value of each product by writing and reasoning with an equivalent expression with fractions.

   a) $(0.3) \cdot (0.02)$

   b) $(0.7) \cdot (0.05)$

Digital Lesson

Solve $\frac{2}{10} \cdot \frac{3}{10}$ You might think of this as $\frac{2}{10}$ of $\frac{3}{10}$. Consider using an area model to explain your answer.



Lesson Summary

We can use fractions like $\frac{1}{10}$ and $\frac{1}{100}$ to reason about the location of the decimal point in a product of two decimals.

Let's take $24 \cdot (0.1)$ as an example. There are several ways to find the product:

- We can interpret it as 24 groups of 1 tenth (or 24 tenths), which is 2.4.
- We can think of it as $24 \cdot \frac{1}{10}$, which is equal to $\frac{24}{10}$ (and also equal to 2.4).
- Multiplying by $\frac{1}{10}$ has the same result as dividing by 10, so we can also think of the product as $24 \div 10$, which is equal to 2.4.

Similarly, we can think of $(0.7) \cdot (0.09)$ as 7 tenths times 9 hundredths, and write:

$$\left(7 \cdot \frac{1}{10}\right) \cdot \left(9 \cdot \frac{1}{100}\right)$$

We can rearrange whole numbers and fractions:

$$(7 \cdot 9) \cdot \left(\frac{1}{10} \cdot \frac{1}{100}\right) = 63 \cdot \frac{1}{1000} = \frac{63}{1000})$$

This tells us that $(0.7) \cdot (0.09) = 0.063$.

Here is another example: To find $(1.5) \cdot (0.43)$, we can think of 1.5 as 15 tenths and 0.43 as 43 hundredths. We can write the tenths and hundredths as fractions and rearrange the factors.

Multiplying 15 and 43 gives us 645, and multiplying $\frac{1}{10}$ and $\frac{1}{100}$ gives us $\frac{1}{1000}$. So $(1.5) \cdot (0.43)$ is $645 \cdot \frac{1}{1000}$, which is 0.645.

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GRADE 6 / MISSION 5 / LESSON 5

Exit Ticket

1. Use what you know about decimals or fractions to explain why $(0.2) \cdot (0.002) = 0.0004$.

2. A rectangular plot of land is 0.4 kilometer long and 0.07 kilometer wide. What is its area in square kilometers? Show your reasoning.

Lesson 6: Methods for Multiplying Decimals

Let's look at some ways we can represent multiplication of decimals.

Concept Exploration

ACTIVITY 1 TASK 1

1 Elena and Noah used different methods to compute $(0.23) \cdot (1.5)$. Both computations were correct.

Elena's method	Noah's method
$(0.23) \cdot 100 = 23$	$0.23 = \frac{23}{100}$
$(1.5) \cdot 10 = 15$	$1.5 = \frac{15}{10}$
$23 \cdot 15 - 345$	$\frac{23}{100} \cdot \frac{15}{100} = \frac{345}{1,000}$
$345 \div 1,000 = 0.345$	$\frac{345}{1,000} = 0.345$

Analyze the two methods, then discuss these questions with your partner.

- Which method makes more sense to you? Why?
- What might Elena do to compute $(0.16) \cdot (0.03)$? What might Noah do to compute $(0.16) \cdot (0.03)$? Will the two methods result in the same value?

ACTIVITY 1 TASK 2

2 Compute each product using the equation $21 \cdot 47 = 987$ and what you know about fractions, decimals, and place value. Explain your reasoning.

a) $(2.1) \cdot (4.7)$

b) $21 \cdot (0.047)$

c) $(0.021) \cdot (4.7)$

Digital Lesson

Place the decimal in the product. Explain how you know where to place it using estimation, fractions, or the rule you learned today.

$$452 \cdot 1{,}219 = 550{,}988$$

$$4.52 \cdot 12.19 =$$

Lesson Summary

Here are three other ways to calculate a product of two decimals such as $(0.04) \cdot (0.07)$.

- First, we can multiply each decimal by the same power of 10 to obtain whole-number factors.

  $(0.04) \cdot 100 = 4$
  $(0.07) \cdot 100 = 7$
  $4 \cdot 7 = 28$

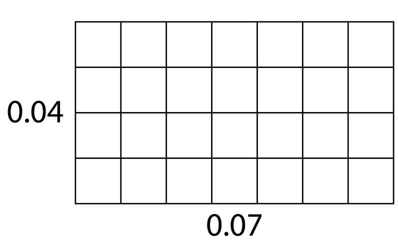
  Because we multiplied both 0.04 and 0.07 by 100 to get 4 and 7, the product 28 is $(100 \cdot 100)$ times the original product, so we need to divide 28 by 10,000.

$$28 \div 10{,}000 = 0.0028$$

- Second, we can write each decimal as a fraction, $0.04 = \frac{4}{100}$ and $0.07 = \frac{7}{100}$, and multiply them.

$$\frac{4}{100} \cdot \frac{7}{100} = \frac{28}{10{,}000} = 0.0028$$

- Third, we can use an area model. The product $(0.04) \cdot (0.07)$ can be thought of as the area of a rectangle with side lengths of 0.04 unit and 0.07 unit.



In this diagram, each small square is 0.01 unit by 0.01 unit. Its area, in square units, is therefore $(\frac{1}{100} \cdot \frac{1}{100})$, which is $\frac{1}{10{,}000}$.

Because the rectangle is composed of 28 small squares, its area, in square units, must be:

$$28 \cdot \frac{1}{10{,}000} = \frac{28}{10{,}000} = 0.0028$$

All three calculations show that $(0.04) \cdot (0.07) = 0.0028$.

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GRADE 6 / MISSION 5 / LESSON 6

Exit Ticket

1. Use the equation $135 \cdot 42 = 5{,}670$ and what you know about fractions, decimals, and place value to explain how to place the decimal point when you compute $(1.35) \cdot (4.2)$.

2. Which of the following is the correct value of $(0.22) \cdot (0.4)$? Show your reasoning.
   a) 8.8
   b) 0.88
   c) 0.088
   d) 0.0088

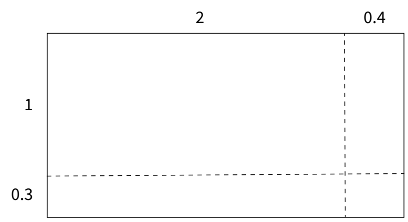
Lesson 7: Using Diagrams to Represent Multiplication

Let's use area diagrams to find products.

Concept Exploration

ACTIVITY 1

1 You can use area diagrams to represent products of decimals. Here is an area diagram represents (2.4) · (1.3).



a) Which region represents (0.4) · (0.3)? Label that region with its area of 0.12.

b) Label each of the other regions with their respective areas.

c) Find the value of (2.4) · (1.3). Show your reasoning.

2 Here are two ways of calculating (2.4) · (1.3). Analyze the calculations and discuss the following questions with a partner.

$$\begin{array}{rl} & 2.4 \\ \times & 1.3 \\ \hline & 0.12 \\ & 0.6 \\ & 0.4 \\ + & 2 \\ \hline & 3.12 \end{array} \quad \text{partial products}$$

Calculation A

$$\begin{array}{rl} & 2.4 \\ \times & 1.3 \\ \hline & 0.72 \\ + & 2.4 \\ \hline & 3.12 \end{array}$$

Calculation B

Analyze the calculations and discuss with a partner:

a) Which two numbers are being multiplied to get 0.12 in Calculation A? Which numbers are being multiplied to get 0.72 in Calculation B? How are the other numbers in blue calculated?

b) In each calculation, why are the numbers in grey lined up vertically the way they are?

3 Find the product of (3.1) · (1.5) by drawing and labeling an area diagram. Show your reasoning.

4 Show how to calculate (3.1) · (1.5) using numbers without a diagram. Be prepared to explain your reasoning. If you are stuck, use the examples in a previous question to help you.

Digital Lesson

How are the numbers in the vertical calculations below related to those in the area diagram? Use specific examples in the models below to prove your thinking.

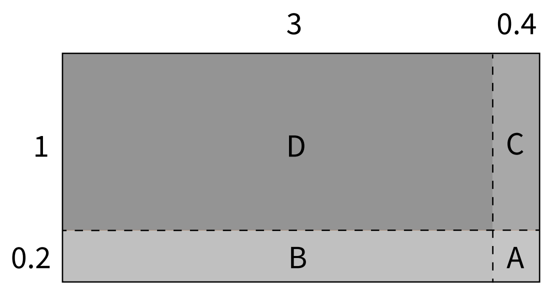
	2	0.7
0.3	0.6	0.21
1	2	0.7

$$\begin{array}{r} 2.7 \\ \times \quad 1.3 \\ \hline 0.81 \\ + \; 2.70 \\ \hline \end{array}$$

Lesson Summary

Suppose that we want to calculate the product of two numbers that are written in base ten. To explain how, we can use what we know about base-ten numbers and areas of rectangles.

Here is a diagram of a rectangle whose side lengths are 3.4 units and 1.2 units. Its area, in square units, is the product $(3.4) \cdot (1.2)$. To calculate this product and find the area of the rectangle , we can decompose each side length into its base-ten units, $3.4 = 3 + 0.4$ and $1.2 = 1 + 0.2$, decomposing the rectangle into four smaller sub-rectangles.



We can rewrite the product and expand it twice:

$$
\begin{aligned}
(3.4) \cdot (1.2) &= (3 + 0.4) \cdot (1 + 0.2) \\
&= (3 + 0.4) \cdot 1 + (3 + 0.4) \cdot 0.2 \\
&= 3 \cdot 1 + 3 \cdot (0.2) + (0.4) \cdot 1 + (0.4) \cdot (0.2)
\end{aligned}
$$

In the last expression, each of the four terms is called a partial product. Each partial product gives the area of a sub-rectangle in the diagram. The sum of the four partial products gives the area of the entire rectangle.

We can show the horizontal calculations above as two vertical calculations.

	3.4	
×	1.2	
	$\overset{1}{0}$.0 8	A
	0.6	B
	0.4	C
+	3	D
	4.0 8	

	3.4	
×	1.2	
	$\overset{1}{0}$.6 8	A + B
+	3.4	C + D
	4.0 8	

The vertical calculation on the left is an example of the partial products method. It shows the values of each partial product and the letter of the corresponding sub-rectangle. Each partial product gives an area:

- A is 0.2 unit by 0.4 unit, so its area is 0.08 square unit.
- B is 3 units by 0.2 unit, so its area is 0.6 square unit.
- C is 0.4 unit by 1 unit, so its area is 0.4 square unit.
- D is 3 units by 1 unit, so its area is 3 square units.
- The sum of the partial products is 0.08 + 0.6 + 0.4 + 3, so the area of the rectangle is 4.08 square units.

The calculation on the right shows the values of two products. Each value gives a combined area of two sub-rectangles:

- The combined regions of A and B have an area of 0.68 square units; 0.68 is the value of $(3 + 0.4) \cdot 0.2$.
- The combined regions of C and D have an area of 3.4 square units; 3.4 is the value of $(3 + 0.4) \cdot 1$.
- The sum of the values of two products is 0.68 + 3.4, so the area of the rectangle is 4.08 square units.

Name: ______________________ Date: ______________________

GRADE 6 / MISSION 5 / LESSON 7

Exit Ticket

Find (4.2) · (1.6) by drawing an area diagram or using another method. Show your reasoning.

Lesson 8: Calculating Products of Decimals

Let's multiply decimals.

Concept Exploration

ACTIVITY 1

1 A common way to find a product of decimals is to calculate a product of whole numbers, then place the decimal point in the product. Here is an example for $(2.5 \cdot 1.2)$. Use what you know about decimals and place value to explain why the decimal point of the product is placed where it is.

$$\begin{array}{r} 25 \\ \times \quad 12 \\ \hline 50 \\ + \quad 250 \\ \hline 300 \end{array}$$

$25 \cdot 12 = 300$

$(2.5) \cdot (1.2) = 3.00$

2 Use the method shown in the first question to calculate each product.

a) $(4.6) \cdot (0.9)$

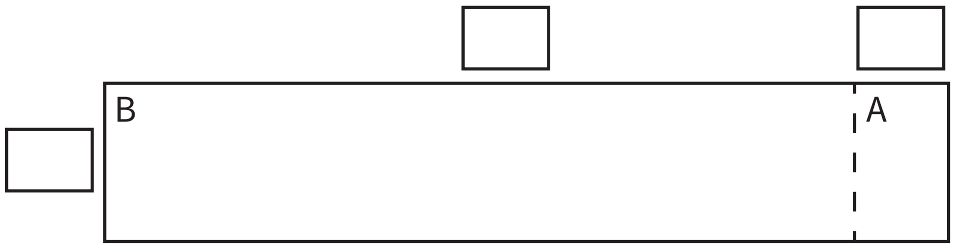
b) $(16.5) \cdot (0.7)$

3 Use area diagrams to check your earlier calculations.

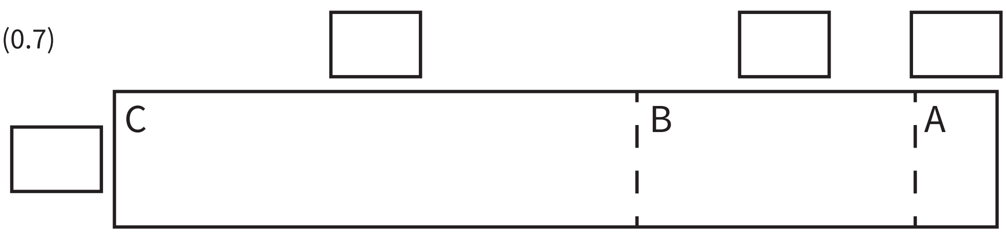
For each problem:

- Decompose each number into its base-ten units and write them in the boxes on each side of the rectangle.
- Write the area of each lettered region in the diagram. Then find the area of the entire rectangle. Show your reasoning.

a) (4.6) · (0.9)



b) (16.5) · (0.7)



4 About how many centimeters are in 6.25 inches if 1 inch is about 2.5 centimeters? Show your reasoning.

Digital Lesson

$$7.5 \cdot 3.8 = ?$$

Choose a strategy you just practiced to solve the problem. Then explain how you used the strategy to help you solve.

Lesson Summary

We can use $84 \cdot 43$ and what we know about place value to find $(8.4) \cdot (4.3)$.

Since 8.4 is 84 tenths and 4.3 is 43 tenths, then:

$$(8.4) \cdot (4.3) = \frac{84}{10} \cdot \frac{43}{10}$$

$$(8.4) \cdot (4.3) = \frac{84 \cdot 43}{100}$$

That means we can compute and then divide by 100 to find $(8.4) \cdot (4.3)$.

$$84 \cdot 43 = 3612$$

$$(8.4) \cdot (4.3) = 36.12$$

Using fractions such as $\frac{1}{10}$, $\frac{1}{100}$, and $\frac{1}{1{,}000}$ allows us to find the product of two decimals using the following steps:

- Write each decimal factor as a product of a whole number and a fraction.
- Multiply the whole numbers.
- Multiply the fractions.
- Multiply the products of the whole numbers and fractions.

We know multiplying by fractions such as $\frac{1}{10}$, $\frac{1}{100}$, and $\frac{1}{1{,}000}$ is the same as dividing by 10, 100, and 1,000, respectively. This means we can move the decimal point in the whole-number product to the left the appropriate number of spaces to correctly place the decimal point.

Name: ______________________ Date: ______________________

GRADE 6 / MISSION 5 / LESSON 8

Exit Ticket

Calculate (1.6) · (0.215). Show your reasoning.

Lesson 9: Using the Partial Quotients Method

Let's divide whole numbers.

Concept Exploration

ACTIVITY 1

1 Andre calculated 657 ÷ 3 using a method that was different from Elena's.

Andre's method:

He started by writing the dividend (657) and the divisor (3).

```
3 | 6 5 7
```

He then subtracted 3 groups of different amounts from 657,

```
    2 0 0
3 | 6 5 7
  - 6 0 0
  -------
      5 7
```

starting with 3 groups of 200...

...then 3 groups of 10, and then 3 groups of 9.

```
        9
      1 0
    2 0 0
3 | 6 5 7
  - 6 0 0
  -------
      5 7
  -   3 0
  -------
      2 7
  -   2 7
  -------
        0
```

Andre calculated 200 + 10 + 9 and then wrote 219.

```
   [2 1 9]
        9
      1 0
    2 0 0
3 | 6 5 7
  - 6 0 0
  -------
      5 7
  -   3 0
  -------
      2 7
  -   2 7
  -------
        0
```

1. Discuss the following questions with a partner:
   a) Andre subtracted 600 from 657. What does the 600 represent?

   b) Andre wrote 10 above the 200, and then subtracted 30 from 57. How is the 30 related to the 10?

   c) What do the numbers 200, 10, and 9 represent?

   d) What is the meaning of the 0 at the bottom of Andre's work?

2. How might Andre calculate 896 ÷ 4? Explain or show your reasoning.

ACTIVITY 2

2 Solve the following problems. Show your reasoning.

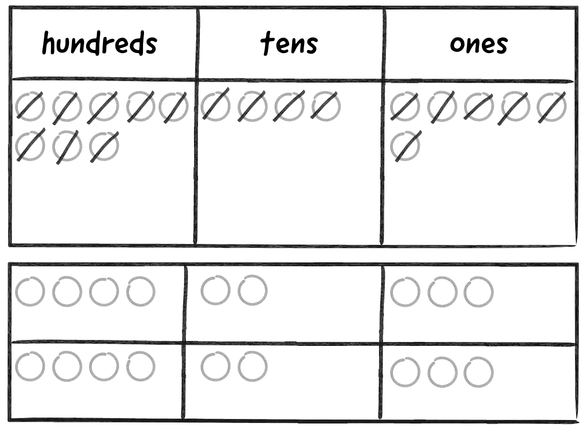
1. Find the quotient of 1,332 ÷ 9 using one of the methods you have seen so far. Show your reasoning.

2. Find each quotient and show your reasoning. Use the partial quotients method at least once.
   a) 1,115 ÷ 5
   b) 665 ÷ 7
   c) 432 ÷ 16

Digital Lesson

The following place value chart shows how Melei divided 846 by 2.

846 ÷ 2 = ____



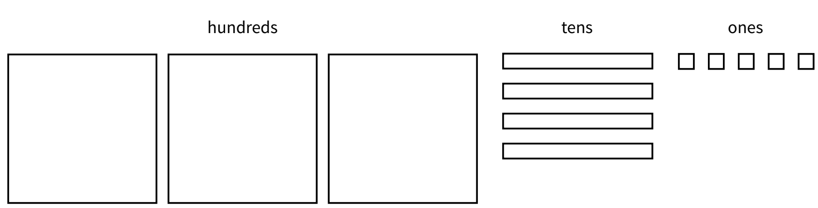
Describe the steps Melei took to divide.

Explain what the answer means.

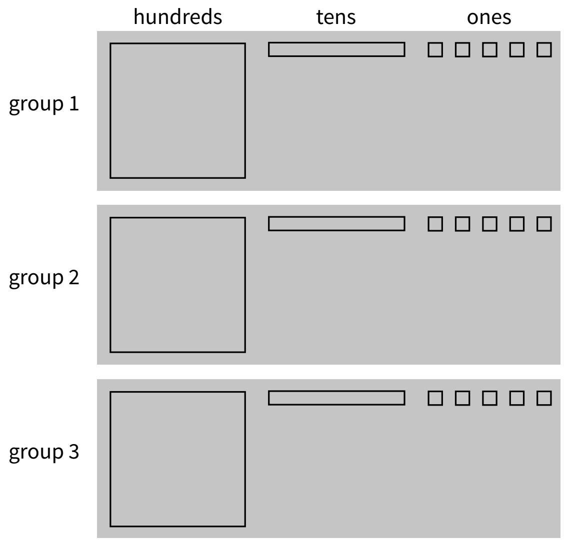
Lesson Summary

We can find the quotient for 345 ÷ 3 in different ways.

One way is to use a base-ten diagram to represent the hundreds, tens, and ones and to create equal-sized groups.



We can think of the division by 3 as splitting up 345 into 3 equal groups.



Each group has 1 hundred, 1 ten, and 5 ones, so 345 ÷ 3 = 115. Notice that in order to split 345 into 3 equal groups, one of the tens had to be unbundled or decomposed into 10 ones.

Another way to divide 345 by 3 is by using the partial quotients method, in which we keep subtracting 3 groups of some amount from 345.

```
  115
    5
   10
  100
3|345
 -300  ←  3 groups of 100
   45
  -30  ←  3 groups of 10
   15
  -15  ←  3 groups of 5
    0
```

```
  115
   50
   50
   15
3|345
 - 45  ←  3 groups of 15
  300
 -150  ←  3 groups of 50
  150
 -150  ←  3 groups of 50
    0
```

In the calculation on the left, first we subtract 3 groups of 100, then 3 groups of 10, and then 3 groups of 5. Adding up the partial quotients (100 + 10 + 5) gives us 115.

The calculation on the right shows a different amount per group subtracted each time (3 groups of 15, 3 groups of 50, and 3 more groups of 50), but the total amount in each of the 3 groups is still 115. There are other ways of calculating 345 ÷ 3 using the partial quotients method.

Both the base-ten diagrams and partial quotients methods are effective. If, however, the dividend and divisor are large, as in 1,248 ÷ 26, then the base-ten diagrams will be time consuming.

Name: ______________________ Date: ______________________

GRADE 6 / MISSION 5 / LESSON 9

Exit Ticket

Calculate 4,235 ÷ 11 using any method.

Lesson 10: Using Long Division

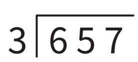
Let's use long division.

Concept Exploration

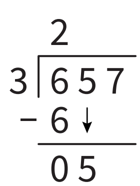
ACTIVITY 1

Lin has a method of calculating quotients that is different from Elena's method and Andre's method. Here is how she found the quotient of 657 ÷ 3.

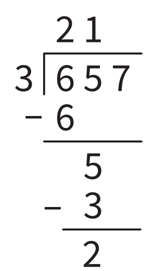
Lin arranged the numbers for vertical calculations. Her plan was to divide each digit of 657 into 3 groups, starting with the 6 hundreds.



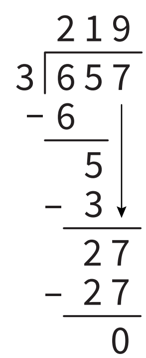
There are 3 groups of 2 in 6, so Lin wrote 2 at the top and subtracted 6 from the 6, leaving 0. Then, she brought down the 5 tens of 657.



There are 3 groups of 1 in 5, so she wrote 1 at the top and subtracted 3 from 5, which left a remainder of 2.



She brought down the 7 ones of 657 and wrote it next to the 2, which made 27. There are 3 groups of 9 in 27, so she wrote 9 at the top and subtracted 27, leaving 0.



Discuss with your partner how Lin's method is similar to and different from drawing base-ten diagrams or using the partial quotients method.

- Lin subtracted $3 \cdot 2$ then $3 \cdot 1$, and lastly $3 \cdot 9$. Earlier, Andre subtracted $3 \cdot 200$, then $3 \cdot 10$, and lastly $3 \cdot 9$. Why did they have the same quotient?

- In the third step, why do you think Lin wrote the 7 next to the remainder of 2 rather than adding 7 and 2 to get 9?

2 Lin's method is called long division. Use this method to find the following quotients. Check your answer by multiplying it by the divisor.

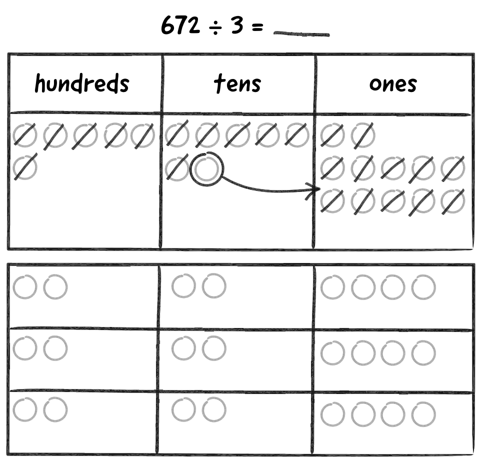
a) 846 ÷ 3

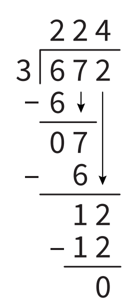
b) 1,816 ÷ 4

c) 768 ÷ 12

Digital Lesson

How is the division algorithm similar to and different from dividing on the place value chart? Use these two examples to support your thinking.





Lesson Summary

Long division is another method for calculating quotients. It relies on place value to perform and record the division.

When we use long division, we work from left to right and with one digit at a time, starting with the leftmost digit of the dividend. We remove the largest group possible each time, using the placement of the digit to indicate the size of each group. Here is an example of how to find 948 ÷ 3 using long division.

```
    316
 3 |948
  - 9       ←  3 groups of 3 (hundreds)
    ---
     4
   - 3      ←  3 groups of 1 (ten)
    ---
     18
   - 18     ←  3 groups of 6 (ones)
    ---
      0
```

- We start by dividing 9 hundreds into 3 groups, which means 3 hundreds in each group. Instead of writing 300, we simply write 3 in the hundreds place, knowing that it means 3 hundreds.
- There are no remaining hundreds, so we work with the tens. We can make 3 groups of 1 ten in 4 tens, so we write 1 in the tens place above the 4 of 948. Subtracting 3 tens from 4 tens, we have a remainder of 1 ten.
- We know that 1 ten is 10 ones. Combining these with the 8 ones from 948, we have 18 ones. We can make 3 groups of 6, so we write 6 in the ones place.
- In total, there are 3 groups of 3 hundreds, 1 ten, and 6 ones in 948, so 948 ÷ 3 = 316.

TERMINOLOGY

Long division

Name: ______________________ Date: ______________________

GRADE 6 / MISSION 5 / LESSON 10

Exit Ticket

Use long division to find the value of 1,875 ÷ 15. Show your work.

Lesson 11: Dividing Numbers that Result in Decimals

Let's find quotients that are not whole numbers.

Concept Exploration

ACTIVITY 1

1 Here is how Lin calculated 62 ÷ 5.

Lin set up the numbers for long division.

She subtracted 5 times 1 from the 6, which leaves a remainder of 1.

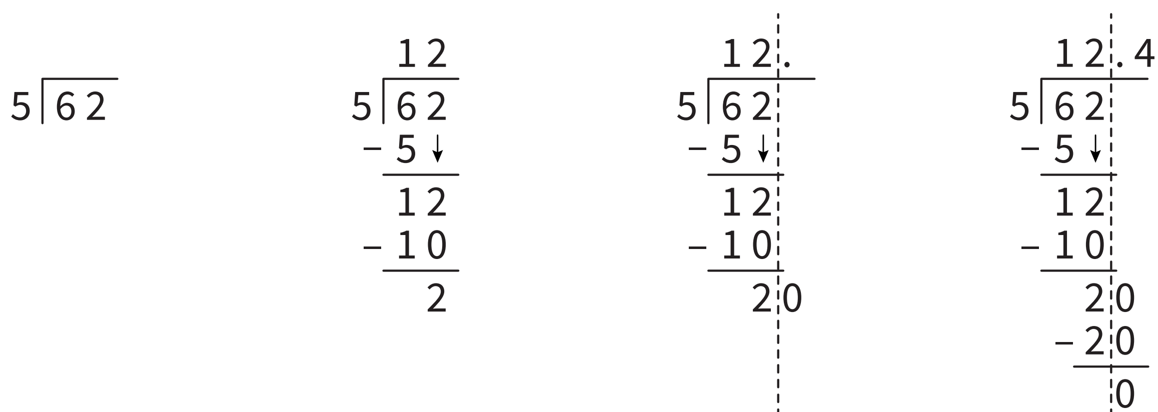
She wrote the 2 from 62 next to the 1, which made 12, and subtracted 5 times 2 from the 12.

Lin drew a vertical line and a decimal point, separating the ones and tenths place.

12 - 10 is 2. She wrote 0 to the right of 2, which made 20.

Lastly, she subtracted 5 times 4 from 20, which left no remainder.

At the top she wrote 4, next to the decimal point.



Discuss with your partner:

- Lin put a 0 after the remainder of 2. Why? Why does this 0 not change the value of the quotient?
- Lin subtracted 5 groups of 4 from 20. What value does the 4 in the quotient represent?
- What value did Lin find for 62 ÷ 5?

2 Use your understanding of long division to solve the following problems.

1. Use long division to find the value of each expression.
   a) $126 \div 8$
   b) $90 \div 12$

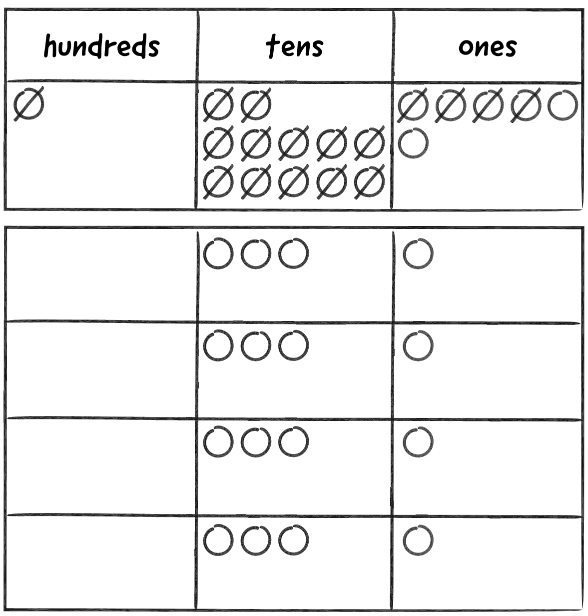
2. Use long division to show that:
   a) $5 \div 4$, or $\frac{5}{4}$, is 1.25.
   b) $4 \div 5$, or $\frac{4}{5}$, is 0.8.
   c) $1 \div 8$, or $\frac{1}{8}$, is 0.125.
   d) $1 \div 25$, or $\frac{1}{25}$, is 0.04.

3. Noah said we cannot use long division to calculate 10 ÷ 3 because there will always be a remainder.

   a) What do you think Noah meant by "there will always be a remainder"?

   b) Do you agree with his statement? Why or why not?

Digital Lesson



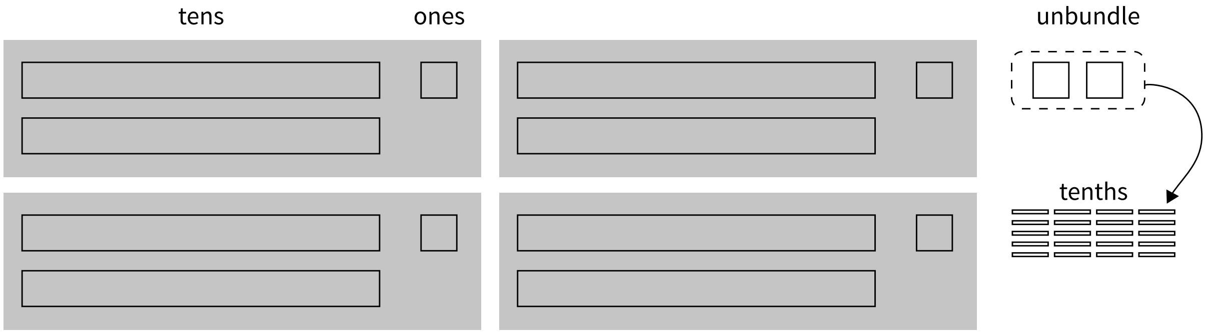

```
    3 1
4 ) 1 2 6
  − 1 2 ↓
      0 6
    −   4
        2
```

Tamaya thinks she is done dividing.

Explain how you could continue dividing on the place value chart and the division algorithm.

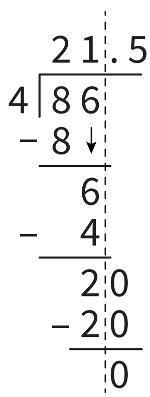
Lesson Summary

Dividing a whole number by another whole number does not always produce a whole-number quotient. Let's look at 86 ÷ 4, which we can think of as dividing 86 into 4 equal groups.



We can see in the base-ten diagram that there are 4 groups of 21 in 86 with 2 ones left over. To find the quotient, we need to distribute the 2 ones into the 4 groups. To do this, we can unbundle or decompose the 2 ones into 20 tenths, which enables us to put 5 tenths in each group.

Once the 20 tenths are distributed, each group will have 2 tens, 1 one, and 5 tenths, so 86 ÷ 4 = 21.5.



We can also calculate 86 ÷ 4 using long division.

The calculation shows that, after removing 4 groups of 21, there are 2 ones remaining. We can continue dividing by writing a 0 to the right of the 2 and thinking of that remainder as 20 tenths, which can then be divided into 4 groups.

To show that the quotient we are working with now is in the tenth place, we put a decimal point to the right of the 1 (which is in the ones place) at the top. It may also be helpful to draw a vertical line to separate the ones and the tenths.

There are 4 groups of 5 tenths in 20 tenths, so we write 5 in the tenths place at the top. The calculation likewise shows 86 ÷ 4 = 21.5.

Name: ______________________ Date: ______________________

GRADE 6 / MISSION 5 / LESSON 11

Exit Ticket

Use long division to find each quotient. Show your computation and write your answer as a decimal.

1. $22 \div 5$

2. $7 \div 8$

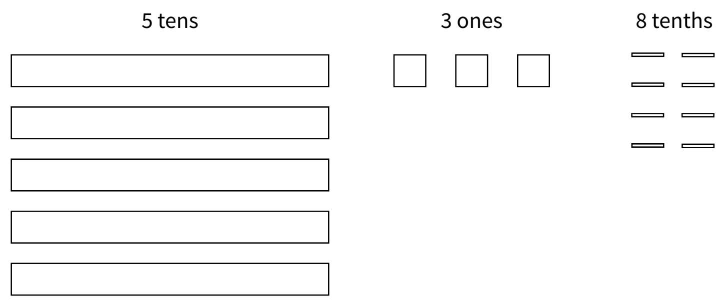
Lesson 12: Dividing Decimals by Whole Numbers

Let's divide decimals by whole numbers.

Concept Exploration

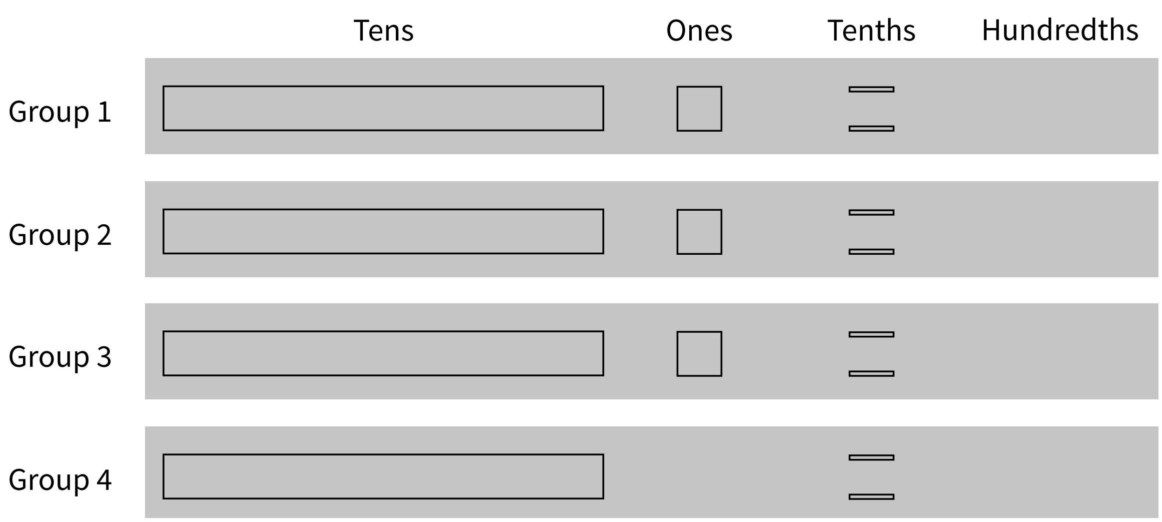
ACTIVITY 1 LAUNCH

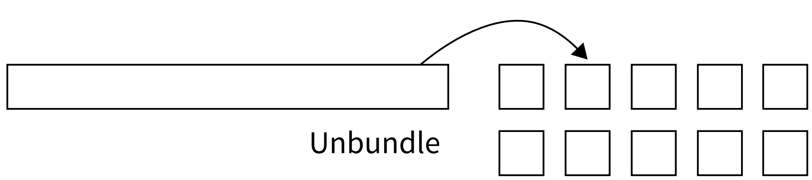
Elena is finding 53.8 ÷ 4 using diagrams. Elena began by representing 53.8.



She placed 1 ten into each group, unbundled the remaining 1 ten into 10 ones, and went on distributing the units.

This diagram shows Elena's initial placement of the units and the unbundling of 1 ten.





ACTIVITY 1

1 Use your understanding of division to solve problems 1-4 in your notes.

1. Complete the diagram by continuing the division process. How would you use the available units to make 4 equal groups?

   As the units get placed into groups, show them accordingly and cross out those pieces from the bottom. If you unbundle a unit, draw the resulting pieces.

2. What value did you find for 53.8 ÷ 4? Be prepared to explain your reasoning.

3. Use long division to find 53.8 ÷ 4. Check your answer by multiplying it by the divisor 4.

4. Use long division to find 77.4 ÷ 5. If you get stuck, you can draw diagrams or use another method.

ACTIVITY 2

2 Analyze the dividends, divisors, and quotients in the calculations, then answer the questions.

24	24	24	24
3 ⟌ 72	30 ⟌ 720	300 ⟌ 7200	3000 ⟌ 72000
− 6	− 60	− 600	− 6000
12	120	1200	12000
− 12	− 120	− 1200	− 12000
0	0	0	0

1. Complete each sentence. In the calculations above:
   - Each dividend is ____________ times the dividend to the left of it.
   - Each divisor is ____________ times the divisor to the left of it.
   - Each quotient is ____________ the quotient to the left of it.

2. Suppose we are writing a calculation to the right of 72,000 ÷ 3,000. Which expression has a quotient of 24? Be prepared to explain your reasoning.

   a) 72,000 ÷ 30,000

   b) 720,000 ÷ 300,000

   c) 720,000 ÷ 30,000

   d) 720,000 ÷ 3,000

3. Suppose we are writing a calculation to the left of 72 ÷ 3. Write an expression that would also give a quotient of 24. Be prepared to explain your reasoning.

4. Decide which of the following expressions would have the same value as 250 ÷ 10. Be prepared to share your reasoning.

   a) 250 ÷ 0.1

   b) 25 ÷ 1

   c) 2.5 ÷ 1

   d) 2.5 ÷ 0.1

   e) 2,500 ÷ 100

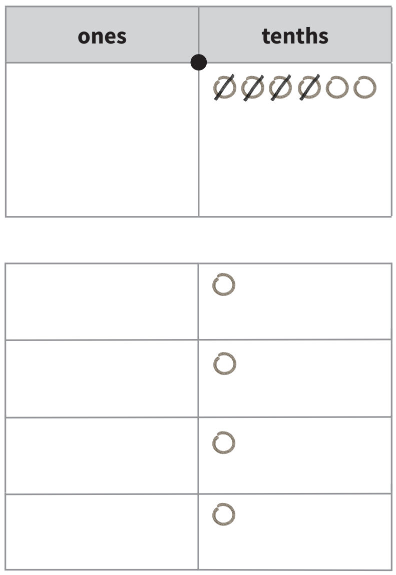
   f) 0.25 ÷ 0.01

ACTIVITY 2 RECAP

3 What happens to the value of the quotient when both the divisor and the dividend are multiplied by the same power of 10? Use examples to show your thinking.

Digital Lesson

In this example of 0.6 ÷ 4. what can you do with the remainder of 2 tenths in order to get a more precise answer?




```
    0 . 1
4 | 0 . 6
 −    . 4
   -------
      . 2
```

Lesson Summary

We know that fractions such as $\frac{6}{4}$ and $\frac{60}{40}$ are equivalent because:

- Both the numerator and denominator of $\frac{60}{40}$ have a factor of 10, so it can be written as $\frac{6}{4}$.
- Both fractions can be simplified to $\frac{3}{2}$.
- 600 divided by 400 is 1.5, and 60 divided by 40 is also 1.5.

Just like fractions, division expressions can be equivalent. For example, the expressions 540 ÷ 90 and 5,400 ÷ 900 are both equivalent to 54 ÷ 9 because:

- They all have a quotient of 6.
- The dividend and the divisor in 540 ÷ 90 are each 10 times the dividend and divisor in 54 ÷ 9. Those in 5,400 ÷ 900 are each 100 times the dividend and divisor in 54 ÷ 9. In both cases, the quotient does not change.

This means that an expression such as 5.4 ÷ 0.9 also has the same value as 54 ÷ 9. Both the dividend and divisor of 5.4 ÷ 0.9 are $\frac{1}{10}$ of those in 54 ÷ 9.

In general, multiplying a dividend and a divisor by the same number does not change the quotient. Multiplying by powers of 10 (e.g., 10, 100, 1,000, etc.) can be particularly useful for dividing decimals, as we will see in an upcoming lesson.

Name: ______________________ Date: ______________________

GRADE 6 / MISSION 5 / LESSON 12

Exit Ticket

1. Use long division to find the value of $43.5 \div 3$.
   If you get stuck, you can draw base-ten diagrams. Be sure to say what each type of figure represents in your diagrams.

2. Explain why all of these expressions have the same value.

   $100 \div 5$ $10 \div 0.5$ $1 \div 0.05$

Lesson 13: Dividing Decimals by Decimals

Let's divide decimals by decimals.

Warm-Up

1 Use long division to find the value of $5.04 \div 7$.

2 Which of the following quotients has the same value as $5.04 \div 7$? Be prepared to explain how you know.

a) $5.04 \div 70$

b) $50.4 \div 70$

c) $504{,}000 \div 700$

d) $504{,}000 \div 700{,}000$

Concept Exploration

ACTIVITY 1

3 Think of one or more ways to find $3 \div 0.12$. Show your reasoning.

4 Solve the problems about dividing with decimals.

1. Find 1.8 ÷ 0.004. Show your reasoning. If you get stuck, think about what equivalent division expression you could write to help you divide.

2. Diego said, "To divide decimals, we can start by moving the decimal point in both the dividend and divisor by the same number of places and in the same direction. Then we find the quotient of the resulting numbers."

   Do you agree with Diego's statement? Use the division expression 7.5 ÷ 1.25 to support your answer.

ACTIVITY 2

5 Find each quotient using a method of your choice. Then discuss your calculations with your group and agree on the correct answers.

1. 106.5 ÷ 3
2. 58.8 ÷ 0.7
3. 257.4 ÷ 1.1

4. Mai is making friendship bracelets. Each bracelet is made from 24.3 cm of string. If she has 170.1 cm of string, how many bracelets can she make? Explain or show your reasoning.

Digital Lesson

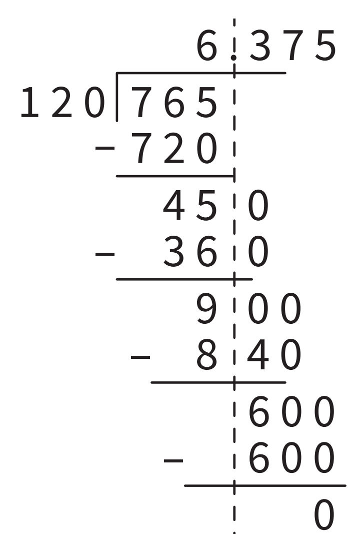
A student said “To find the value of 96 divided by 0.06, I can divide 960 by 6.” Do you agree with this statement? Why or why not?

Lesson Summary

One way to find a quotient of two decimals is to multiply each decimal by a power of 10 so that both products are whole numbers.

If we multiply both decimals by the same power of 10, this does not change the value of the quotient. For example, the quotient 7.65 ÷ 1.2 can be found by multiplying the two decimals by 10 (or by 100) and instead finding 76.5 ÷ 12 or 765 ÷ 120.

To calculate 765 ÷ 120, which is equivalent to 76.5 ÷ 12, we could use base-ten diagrams, partial quotients, or long division. Here is the calculation with long division:



Name: ______________________ Date: ______________________

GRADE 6 / MISSION 5 / LESSON 13

Exit Ticket

1. Write two division expressions that have the same value as $36.8 \div 2.3$.

2. Find the value of $36.8 \div 2.3$. Show your reasoning.

Lesson 14: Using Operations on Decimals to Solve Problems

Let's solve some problems using decimals.

Concept Exploration

ACTIVITY 1

There are 10 equally-spaced hurdles on a race track. The first hurdle is 13.72 meters from the start line. The final hurdle is 14.02 meters from the finish line. The race track is 110 meters long.

1. Draw a diagram that shows the hurdles on the race track. Label all known measurements.

2. How far are the hurdles from one another? Explain or show your reasoning.

3. A professional runner takes 3 strides between each pair of hurdles. The runner leaves the ground 2.2 meters *before* the hurdle and returns to the ground 1 meter *after* the hurdle. About how long are each of the runner's strides between the hurdles? Show your reasoning.

Digital Lesson

In this lesson, you reviewed adding, subtracting, multiplying and dividing using a standard algorithm. In your notes, choose two of the following problems and solve using an algorithm.

3.187 + 2.654

9.321 – 4.162

1.5 · 3.2

4.5 ÷ 0.5

Lesson Summary

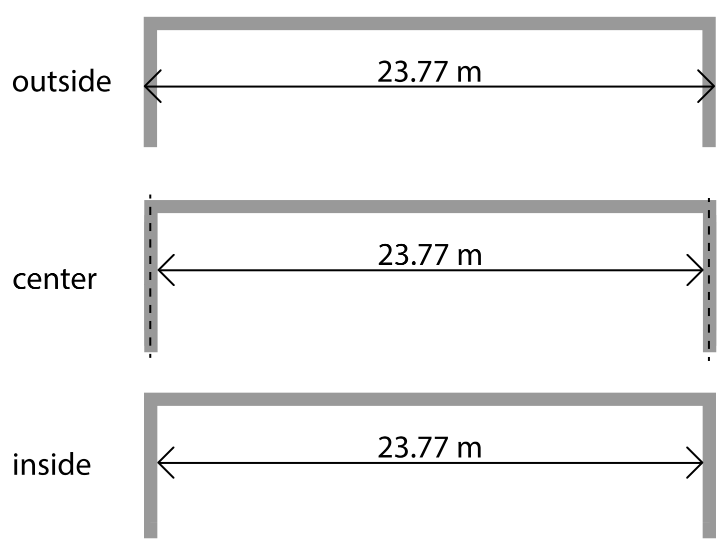
Diagrams can help us communicate and model mathematics. A clearly-labeled diagram helps us visualize what is happening in a problem and accurately communicate the information we need.

Sports offer great examples of how diagrams can help us solve problems. For example, to show the placement of the running hurdles in a diagram, we needed to know what the distances 13.72 and 14.02 meters tell us and the number of hurdles to draw. An accurate diagram not only helped us set up and solve the problem correctly, but also helped us see that there are only *nine* spaces between ten hurdles.

To communicate information clearly and solve problems correctly, it is also important to be precise in our measurements and calculations, especially when they involve decimals.

In tennis, for example, the length of the court is 23.77 meters. Because the boundary lines on a tennis court have a significant width, we would want to know whether this measurement is taken between the inside of the lines, the center of the lines, or the outside of the lines. Diagrams can help us attend to this detail, as shown here.

The accuracy of this measurement matters to the tennis players who use the court, so it matters to those who paint the boundaries as well. The tennis players practice their shots to be on or within certain lines. If the tennis court on which they play is not precisely measured, their shots may not land as intended in relation to the boundaries. Court painters usually need to be sure their measurements are accurate to within $\frac{1}{100}$ of a meter or one centimeter.



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GRADE 6 / MISSION 5 / LESSON 14

Exit Ticket

Andre is running in an 80-meter hurdle race. There are 8 equally-spaced hurdles on the race track. The first hurdle is 12 meters from the start line and the last hurdle is 15.5 meters from the finish line.

1. Estimate how far the hurdles are from one another. Explain your reasoning.

2. Calculate how far the hurdles are from one another. Show your reasoning.

Lesson 15: Making and Measuring Boxes

Let's use what we know about decimals to make and measure boxes.

Warm-Up

1

Your teacher will demonstrate how to make an open-top box by folding a sheet of paper. Your group will receive 3 or more sheets of square paper. Each person in your group will make 1 box. Before you begin folding:

1. Record the side lengths of your papers, from the smallest to the largest.
    - Paper for Box 1: ________ cm
    - Paper for Box 2: ________ cm
    - Paper for Box 3: ________ cm
2. Compare the side lengths of the square sheets of paper. Be prepared to explain how you know.
    a) The side length of the paper for Box 2 is ________ times the side length of the paper for Box 1.
    b) The side length of the paper for Box 3 is ________ times the side length of the paper for Box 1.
3. Make some predictions about the measurements of the three boxes your group will make:
    - The surface area of Box 3 will be ________ as large as that of Box 1.
    - Box 2 will be ________ times as tall as Box 1.
    - Box 3 will be ________ times as tall as Box 1.

Now you are ready to fold your paper into a box!

Lesson

ACTIVITY 1

2

Now that you have made your boxes, you will measure them and check your predictions about how their heights and surface areas compare.

1.

a) Measure the length and height of each box to the nearest tenth of a centimeter. Record the measurements in the table.

	side length of paper (cm)	length of box (cm)	height of box (cm)	surface area (sq cm)
Box 1				
Box 2				
Box 3				

b) Calculate the surface area of each box. Show your reasoning and decide on an appropriate level of precision for describing the surface area (Is it the nearest 10 square centimeters, nearest square centimeter, or something else?). Record your answers in the table.

2. To see how many times as large one measurement is when compared to another, we can compute their quotient. Divide each measurement of Box 2 by the corresponding measurement for Box 1 to complete the following statements.
   a) The length of Box 2 is ________ times the length of Box 1.

   b) The height of Box 2 is ________ times the height of Box 1.

   c) The surface area of Box 2 is ________ times the surface area of Box 1.

3. Find out how the dimensions of Box 3 compare to those of Box 1 by computing quotients of their lengths, heights, and surface areas. Show your reasoning.
   a) The length of Box 3 is ________ times the length of Box 1.

   b) The height of Box 3 is ________ times the height of Box 1.

   c) The surface area of Box 3 is ________ times the surface area of Box 1.

4. Record your results in the table.

	side length of paper	length of box	height of box	surface area
Box 2 compared to Box 1				
Box 3 compared to Box 1				

5. Earlier, in the warm-up, you made predictions about how the heights and surface areas of the two larger boxes would compare to those of the smallest box. Discuss with your group:
   - How accurate were your predictions? Were they close to the results you found by performing calculations?
   - Let's say you had another piece of square paper to make Box 4. If the side length of this paper is 4 times the side length of the paper for Box 1, predict how the length, height, and surface area of Box 4 would compare to those of Box 1. How did you make your prediction?

Terminology

Long division

Long division is a way to show the steps for dividing numbers in decimal form. It finds the quotient one digit at a time, from left to right.

For example, here is the long division for 57 ÷ 4.

```
    14.25
  4|57.00
  - 4
    17
  - 16
      10
     - 8
       20
     - 20
        0
```