SMALL GROUP LESSONS

Grade 5, Mission 5

Volume, Area, and Shapes

Lessons

YOUR NOTES

Topic A: Concepts of Volume

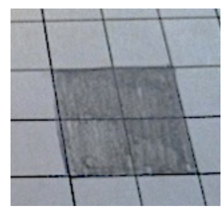
In Topic A, students extend their spatial structuring to three dimensions through an exploration of volume. Students come to see volume as an attribute of solid figures and understand that cubic units are used to measure it.

Lesson 1

Explore volume by building with and counting unit cubes.

Materials: (T) 20 centimeter cubes (S) Ruler, 20 centimeter cubes, centimeter grid paper (Template 1), isometric dot paper (Template 2)

Problem 1: Build a solid from cubes.



T: Shade a square on your centimeter grid paper with an area of 4 square units. (Pause to allow students to do this.)

T: This is going to be the foundation for our structure. Place 4 cubes directly on top of that square.

S: (Do so.)

T: Think of the first 4 cubes as the ground floor of a building. Make a second floor by putting another 4 cubes on top of them. (Pause.) How many cubes are there now?

S: 8 cubes.

T: Did we change the ground floor? Why or why not? Turn and talk.

S: No. We just built on top of it. → The second layer of cubes doesn't make it take up more space on the ground. → We built up, not out, so the structure got taller, not longer or wider.

T: Put one more layer of 4. (Pause.) Explain to your partner how you know the total number of cubes.

S: I just counted up from 8 as I put each cube. → Each floor had 4 blocks, so it's 3 fours. → I thought of 3 times 4, 12.

T: What is the total number of cubes in your solid?

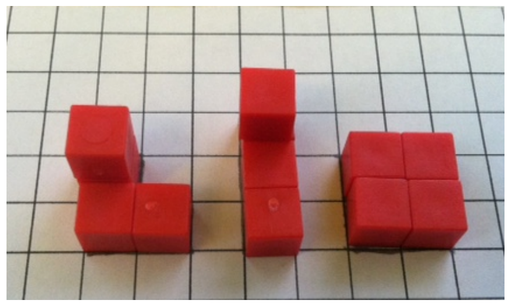
S: 12 cubes.

YOUR NOTES

Problem 2: Build solids with a given volume with cubic centimeters.

T: Since this is a cube with each edge measuring 1 centimeter, we call this a **cubic centimeter**.
T: (Hold up a centimeter cube.) These cubes can serve as a unit to measure the **volume** of your solid, the amount of space it takes up. What do we call this unit?
S: A cubic centimeter.
T: Just like we use squares to measure area in square units, we use cubes to measure volume in cubic units. (Write cubic unit, cubic centimeter, and cm^3 on the board.)
T: (Hold up 2 cubes.) How many cubes?
S: 2 cubes.
T: How many cubic centimeters?
S: 2 cubic centimeters.
T: (Hold up 4 cubes in a square formation.)
T: What is the volume of these 4 units together?
S: 4 cubic centimeters.
T: Work with a partner. On your grid paper, build three different solids with a volume of 4 cubic centimeters.

Possible Solutions



Give students time to build the structures. Move on to do likewise with five and then six cubes as time allows. While circulating, encourage students to use the words *volume* and *cubic centimeters* when answering questions.

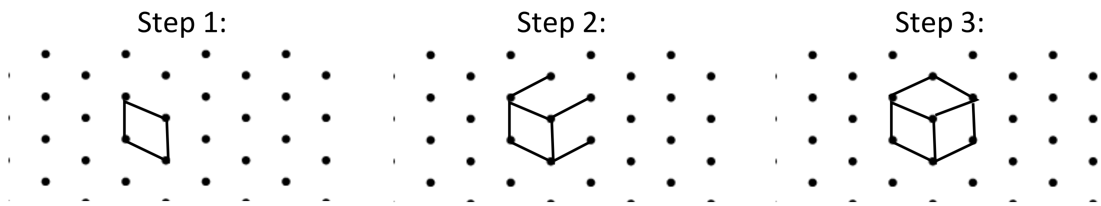
Problem 3: Represent solids on isometric dot paper.

T: We are going to build some other structures, but we want to draw what we build. Let's learn how to use our isometric dot paper to draw our structures.
T: We will start by drawing 1 cube. (Demonstrate while directing students in each step.)

Explain the process for drawing 1 cubic centimeter using the dot paper.

Step 1: Connect four dots to make a parallelogram. This will represent one square face of the cube, viewed at an angle.
Step 2: Draw three straight segments to the right from the two vertices on the top and the one on the bottom right.
Step 3: Draw two segments to represent the missing edges.

YOUR NOTES



T: Now we will put two cubes next to each other.

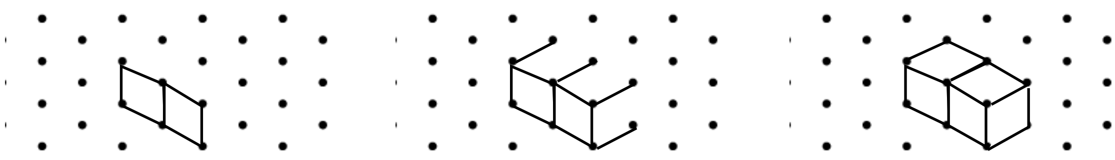
Explain the process for drawing 2 cubic centimeters.

Step 1: Connect four dots to make a parallelogram.

Step 2: Add another parallelogram that shares its right edge, just like your cubes.

Step 3: Draw four straight segments to the right from the three vertices on the top and the one on the bottom right.

Step 4: Draw three segments to represent the missing edges.



Allow students to practice several times. Then, choose examples of several students' work to show the class.

T: With a partner, build a structure with no more than 10 cubes each. Then, draw your partner's structure on dot paper. Help each other figure out if it matches what you built.

Circulate and help students draw their figures.

NOTES

Debrief Questions

- When we built different solids with 4 centimeter cubes, how were your shapes different? How were they the same?
- What do you need to think about when counting cubic centimeters in drawings?
- How is counting cubic centimeters in drawings different from counting them in person? Is it possible for a drawing to fool you?
- Might some cubes be hidden, or might there be gaps that you cannot see?

Multiple Means of Representation

If only 1-inch cubes are available, adapt the lesson to work with 1-inch cubes. Try to obtain 1-inch grid paper from the Internet, or create it on the computer and print it for students to use.

Multiple Means of Engagement

The spatial reasoning required to draw centimeter cubes on isometric dot paper may be difficult for some students. Pattern block rhombuses may help students orient their drawings. Three rhombuses may be laid on paper (with or without dots) and traced to draw a cube.

Students may also trace the yellow hexagon block and simply add three interior lines to create the cube.

YOUR NOTES

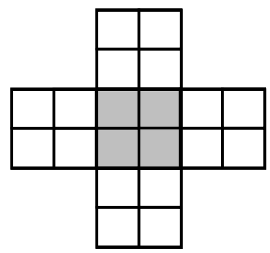
Lesson 2

Find the volume of a right rectangular prism by packing with cubic units and counting.

Note: Today's lesson uses the Problem Set. Solutions for each problem are included below.

Materials: (S) Pencil, centimeter grid paper (Lesson 1 Template 1), scissors, tape, 50 centimeter cubes, net (Template), Problem Set (see Appendix)

Problem 1(a)



Project the image from Problem 1(a) from the Problem Set.

T: To make a box, copy this image on grid paper by first shading the bottom of the box and then outlining the figure. (If necessary, model how to draw onto grid paper.)

T: Now, cut around the outside. The bottom is shaded, so fold up the flaps to make the sides of the box. Crease well, and tape to make the edges of the box. (Model cutting and folding as necessary.)

T: Fill the box with cubic centimeters to find how many cubes fill the box up.

S: Eight cubes.

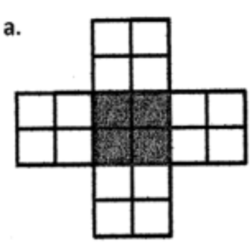
T: What is the volume of the box?

S: 8 cubic centimeters. → 8 centimeters cubed.

T: Talk to your partner about different ways to pack and count the cubic centimeters.

S: You can just count one by one. → You can put in a row of two on the bottom and then another row on top of that to have four. It looks like a slice. Another slice makes four more. → You can put four on the bottom and another four on top of that. 2 × 4 is 8. → You can count by two or by four.

a.



Number of cubes: 8

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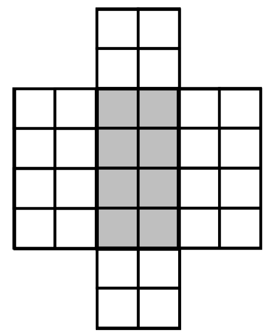
Problem 1(b)

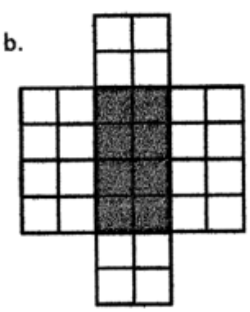
T: Let's fold to make another box with rectangular sides, or a **rectangular prism**.

Follow the same procedure as with Problem 1(a) to have students make the prism and pack the cubes into the box.



T: What is the volume of this box?
S: 16 cubic centimeters.
T: How does its volume compare with the volume of the first rectangular prism?
S: It doubled.
T: Interesting. Why do you think that might be? Turn and talk.
S: It was like two of the first box laid side by side. → The bottom was twice as long, but it had the same number of layers, so it was 16. → The sides of the box were the same height. Just the cubic centimeters on the bottom doubled.
T: Look at the image on the board. Talk to your partner about how you might find the volume without packing it.

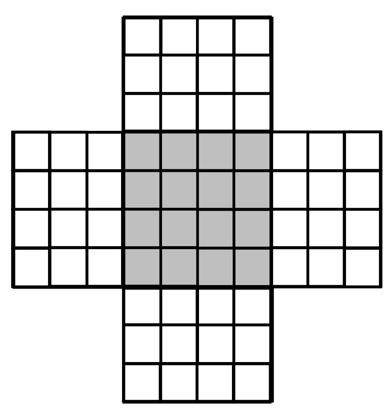
b.



Number of cubes: 16

Problem 1(c)

T: (Project or show the next figure.) This time, put in the first layer of cubes on the bottom without cutting to actually make the box. (Pause.) What do you think the volume of this box will be?
S: The flaps show the number of layers. → The bottom has 16 cubes, and there are 3 layers. → The bottom is 4 by 4, and it looks like it will be 3 layers, so ... 4 × 4 is 16, and double is 32, and another 16 is 48, so 48. → I think it will be 48 cubic centimeters because 16 × 3 is 48.



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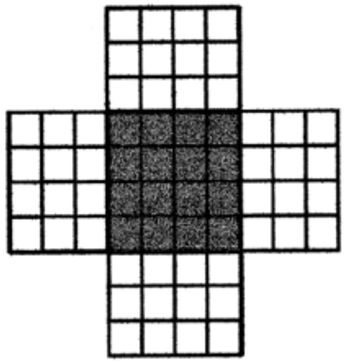
c. Number of cubes: 48

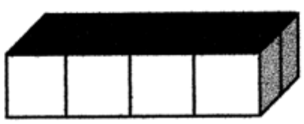


Allow students to answer Problems 2(a) and 2(b) independently. Check the answers and students' thinking together following the sequence above. Then, distribute the box patterns on the Template for students to cut out, and have them work independently on Problem 3.

Problem 2(a, b, c)

Predict how many centimeter cubes will fit in each box, and briefly explain your predictions. Use cubes to find the actual volume. (The figures are not drawn to scale.)

a.

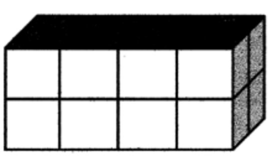


Prediction: $8 cm^3$

Actual: $8 cm^3$

It's 4 cubes across and 2 deep, so 8 cubes altogether.

b.

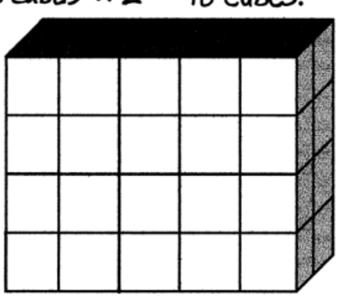


Prediction: $16 cm^3$

Actual: $16 cm^3$

There are 2 layers, top and bottom. Each layer has 8 cubes. 8 cubes × 2 = 16 cubes.

c.



Prediction: $40 cm^3$

Actual: $40 cm^3$

There are 4 layers. Each layer has 10 cubes, 10 cubes × 4 = 40 cubes.

Problem 3(a, b, c)

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Cut out the net in the template, and fold it into a cube. Predict the number of 1-centimeter cubes that would be required to fill it.

a. **Prediction:** 24 cubes

b. **Explain your thought process as you made your prediction.**

I saw that the net had six faces, So I Knew it would not be an open box. When I folded the net, I discovered it had the shape of a cube.

c. **How many 1-centimeter cubes are used to fill the figure? Was your prediction accurate?**

It took 27-centimeter cubes to fill the figure. My Prediction of 24 cubes wasn't 100% accurate but it was close.

NOTES

Debrief Questions

- How did you pack your boxes in Problem 1 (a), (b), and (c)? Cube by cube, row by row, or layer by layer? Did the way you packed your boxes change from problem to problem? If so, how and why did your thinking change?
- In Problem 2, how did you verify your prediction? Did your prediction change between 2 (a) and (c)? Why or why not?
- What did you discover in Problem 3? Did your discovery match your prediction? Could you have used fewer cubes to make your prediction? Why or why not?
- How has your understanding of the term *volume* changed from yesterday to today?

Multiple Means of Engagement

Breaking down a tissue or cereal box to show how the sides form a flat shape and then building it back into a box may be helpful for students to understand the figures used in the lesson to make the boxes.

Be aware that spatial skills and fine motor skills vary widely among fifth graders. Some may require more time to cut, fold, and tape the boxes. Proximity to the teacher and the demonstration can support students whose spatial skills are developing.

Multiple Means of Engagement

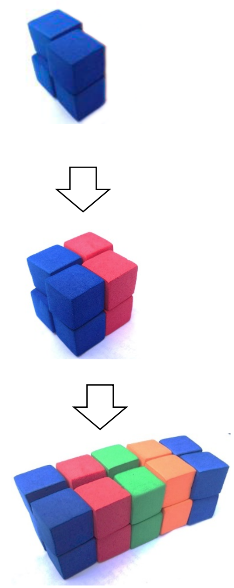
Encourage students who easily grasp this concept and move quickly to think about the results of the same problems if the units were 2 cm cubes, 3 cm cubes, and so on.

YOUR NOTES

Lesson 3

Compose and decompose right rectangular prisms using layers.

Materials: (T) 27 centimeter cubes (S) 27 centimeter cubes, rectangular prism recording sheet (Template)



T: Build this with your own cubes. (Show 4 cubes in a square formation stacked vertically—2 layers with 2 cubes in each layer.)

T: What's the volume of this rectangular prism?

S: 4 cubic centimeters.

T: Let's add layers horizontally. Add another layer *next* to the first one.

S: (Work.)

T: What is the volume?

S: 8 cubic centimeters.

T: Add 3 more layers next to the first two. (Pause for students to do this.)

T: What is the volume now?

S: 20 cubic centimeters.

T: How did you figure that out? Turn and talk.

S: I added the first 8 to the 12 more that I added. → I saw 5 along the bottom, and there were 2 layers going back, so that makes 10, and 2 layers going up makes 20. → I knew that I had 27 cubes to start, and I only have 7 left.

T: (Project a blank rectangular prism from the recording sheet, or draw one on the board.) Let's record how we built the layers. Use the first rectangular prism in the row of your recording sheet.

T: How many layers did we build in all?

S: 5.

T: Let's show that by partitioning the prism, vertically like bread slices, into 5 layers. (Partition the prism vertically into 5 equal sections.) Make your prism look like mine. How many cubes were in each layer?

S: 4 cubes.

T: Record that on each layer that we drew. (Write a 4 on each of the vertical layers.) Write a number sentence that expresses the volume of this prism using these layers. Turn and talk.

S: We could write $4\text{ cm}^3 + 4\text{ cm}^3 + 4\text{ cm}^3 + 4\text{ cm}^3 + 4\text{ cm}^3 = 20\text{ cm}^3$. → Since all the layers are the same, we could write 5 × 4 cubic cm = 20 cubic cm.

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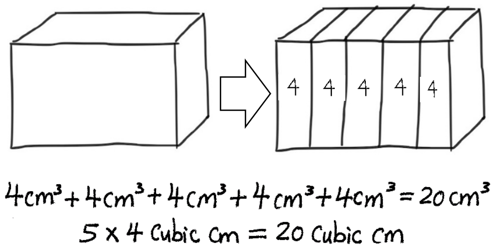
T: (Draw the table on the board.) I'll record that in a table. Now, imagine that we could partition this prism horizontally into layers like a cake, like our ice cube trays. What might that look like? Work with your partner to show the layers on the next prism in the row (point to the next prism on the recording sheet), and tell how many cubes would be in each. Use your cubes to help you.

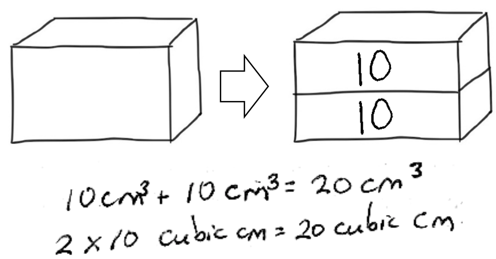


S: The prism is 2 units high, so we could cut the prism in half horizontally from left to right. That would be 10 cubes in each one. → We could make a top layer of 10 cubes and a bottom layer of 10 cubes.

Number of Layers	Cubes in Each Layer	Volume
5	4	20 cm^3
2	10	20 cm^3
2	10	20 cm^3

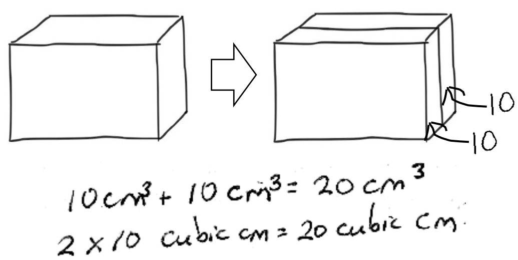
T: Let's record your thinking. (Draw the figure to the right.) Write a number sentence that expresses the volume of the prism using these layers.

S: 10 cm^3 + 10 cm^3 = 20 cm^3. → 2 × 10 cubic cm = 20 cubic cm.



T: Let's record that information in our table. (Record.) Work with your partner to find one last way that we can partition this prism into layers. Use the third prism on your recording sheet to label the layers, and write the number of cubes in each layer. Then, write a number sentence to explain your thinking.

S: (Work to draw the third figure and write the number sentences.)



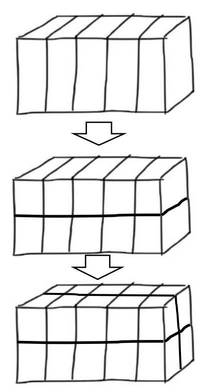
T: I'll record this last bit of information in our table. (Record.)

T: Now, let's draw the different layers together. Use the last prism in the row of your recording sheet.

Step 1: Draw vertical lines to show the 5 layers of 4 cubes each that remind us of bread slices. (Point to the table's first line.)

Step 2: Draw a horizontal line to show the two layers of 10 cubes each that remind us of layers of cake. (Point to the table's second line.)

Step 3: Draw both a horizontal and a vertical line to show the front and back layers of 10 each. (Point to the table's last line.)



YOUR NOTES

T: What is the volume of the prism?

S: 20 cubic centimeters.

T: Build a prism with a partner that has one 3 cube by 3 cube layer. (Demonstrate building this with the cubes.)

T: What is the volume?

S: 9 cubic centimeters.

T: Add another layer of cubes on top that completely cover the first layer.

T: What is the volume now? How do you know?

S: It's 18 cubic centimeters because now, we have 2 groups of 9 cubic centimeters. → Two layers with 9 cubes each is 18 cubic centimeters.

T: Now, add another layer. What is the volume?

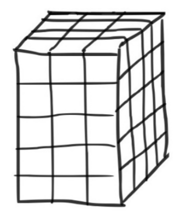
S: 27 cubic centimeters.

T: What is the overall shape of your rectangular prism?

S: A cube!

T: Use the set of cubes on your recording sheet to show the three ways of layering using the same system we just did with our 2 by 2 by 5 rectangular prism.

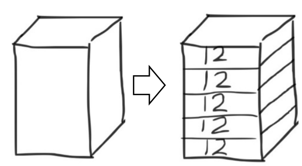
S: (Work.)



T: (Project or draw an image of a 3 × 4 × 5 rectangular prism. Direct students to the set of vertical prisms on the rectangular prism recording sheet.) Imagine what the bottom layer of this prism would look like. Describe it to your partner, and then build it.

S: There would be 3 rows with 4 cubes in each row. → There would be 12 cubes in all. It would be 3 cubes wide and 4 cubes long and 1 cube high. → This would be like a 4 by 3 rectangle, but it is 1 centimeter tall. (Build.)

T: Here's the same prism but without the unit cubes drawn. How might we represent the bottom layer on this picture? Use your recording sheet, and talk to your partner.



S: We could draw a horizontal slice toward the bottom and label it with 12. → I can see in the drawing that there are 5 layers in all, so I'll need to make the bottom about 1 fifth of the prism and put 12 on it.

$12\text{ cm}^3 + 12\text{ cm}^3 + 12\text{ cm}^3 + 12\text{ cm}^3 + 12\text{ cm}^3 = 60\text{ cm}^3$

5×12 cubic cm $= 60$ cubic cm

T: What is the volume of the single layer?

S: 12 cubic centimeters.

T: What is the volume of the prism with 5 of these layers?

S: I know there are 5 layers that are the same, so 12 cm^3 + 12 cm^3 + 12 cm^3 + 12 cm^3 + 12 cm^3, so 60 cm^3. → It's 5 × 12 cubic cm, so 60 cubic cm.

T: What other ways could we partition this prism into layers? Turn and talk, and then draw a picture of your thinking on the recording sheet.

S: (Draw.)

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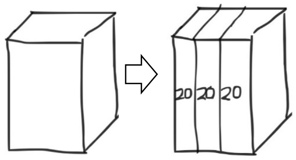
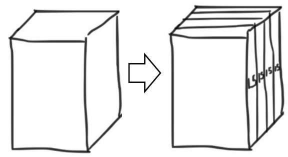
Possible Solutions



$20 cm^3 + 20 cm^3 + 20 cm^3 = 60 cm^3$

3 x 20 cubic cm = 60 cubic cm



$15 cm^3 + 15 cm^3 + 15 cm^3 + 15 cm^3 = 60 cm^3$

4 x 15 cubic cm = 60 cubic cm

NOTES

Debrief Questions

- How did you decide to decompose the prisms? Is there a different way or order in which you could have done it?
- Do you have a hard time visualizing prisms? Which layers are easier for you to visualize? Which are the hardest? How can you make the hardest layers easier to see?
- At what point did you not need to model with the physical cubes anymore?

Multiple Means of Engagement

Challenge students who quickly grasp the decompositions by asking them to determine a "rule" for finding the volume and test it for different rectangular prisms. They might also be asked to calculate the volume of the prisms as if they were built from 2 cm cubes. Ask them to explain what would happen to the volume if the dimensions of the cubes were doubled or tripled.

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Topic B: Volume and the Operations of Multiplication and Addition

Concrete understanding of volume and multiplicative reasoning come together in Topic B as the systematic counting from Topic A leads naturally to formulas for finding the volume of a right rectangular prism.

Lesson 4

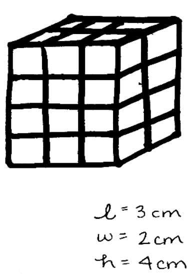
Use multiplication to calculate volume.

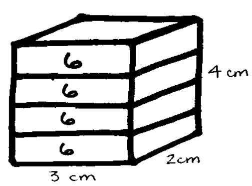
Materials: (T) Images of rectangular prisms to project (S) Personal white board, rectangular prism recording sheet (Lesson 3 Template)

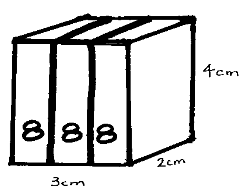
Part 1: Find the volume of multilayer prisms using multiplication.

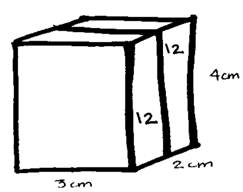
T: (Project the leftmost image on the next page.) Record the length, width, and height of this rectangular prism on your recording sheet. Then, decompose the prism into layers three different ways to find the volume like we did together yesterday.

S: (Work on the recording sheet to show the three different decompositions pictured.)









T: Let's record some information about our prism in this table. Look at this layer on the top. How many cubes are in each layer? How do you know?

S: There are 6 cubes. It is 3 cubes by 2 cubes. → I counted them. → It's like an array, 3 × 2 = 6.

T: (Record in the table as 3 × 2.)

Cubes in Each Layer	Number of Layers	Volume
(3 × 2)	4	24 cm^3
(2 × 4)	3	24 cm^3
(3 × 4)	2	24 cm^3

Follow a similar sequence to record

YOUR NOTES

the other decompositions.

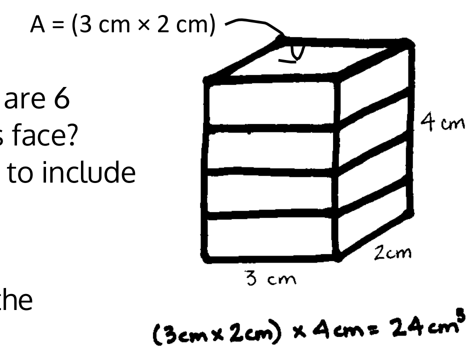
T: How do we use this information to find the volume of the prism? Turn and talk.

S: With 4 layers, that's 4 copies of the same array of cubes, 4 times 6. That's 24 cubic centimeters. → I see 3 layers that each have 8 cubes in them. Eight cubes 3 times is 24 cubes. That's 24 cubic centimeters. → Three times 4 shows the cubes in the first layer on the front, but I need 2 of those, so 2 twelves make 24 cubic centimeters. → Count the layers. Four layers, and each layer is a 3 cm by 2 cm by 1 cm prism; 6 fours is 24. The volume is 24 cubic centimeters.

T: (Record the number of layers and volumes in the table.)

T: (Hold up a cube.) We know that this is 1 cubic centimeter. Look at one face of this cube (point to one face); what is the area of this face?

S: 1 square centimeter.



T: (Point to the face on the top of the first prism.) If 1 square unit is the area of one cube's face, and there are 6 cubes that make up this face, what is the area of this face? Write a number sentence to show the area. Be sure to include the units.

S: $3 \text{ cm} \times 2 \text{ cm} = 6 \text{ cm}^2$.

T: What do you notice about the area of this face and the number of cubes in this layer?

S: They are the same.

T: A moment ago, we said that to find the volume, we had to account for the number of layers in the prism. How many layers are under this face?

S: 4.

T: Which dimension of the prism gives us that number?

S: The height.

T: How many centimeters is the height? Give me the unit, too.

S: 4 centimeters.

T: So, we can find the volume by multiplying the area of this face by the height. (Write (3 cm × 2 cm).) The height, 4 cm, happens to tell us the number of the layers. Show me the multiplication sentence you can use to find the volume of this prism that matches this way of seeing the layers.

S: V = (3 cm × 2 cm) × 4 cm = 24 cubic cm. → $V = 6 \text{ cm}^2 \times 4 \text{ cm} = 24 \text{ cm}^3$.

T: (Write $V = (3 \text{ cm} \times 2 \text{ cm}) \times 4 \text{ cm} = 24 \text{ cm}^3$ and $6 \text{ cm}^2 \times 4 \text{ cm} = 24 \text{ cm}^3$ on the board.) I notice some of you wrote $6 \text{ cm}^2 \times 4$ cm, and others multiplied centimeters by centimeters by centimeters. What happens to the square units when you multiply them by the third factor? Why? Talk with a partner.

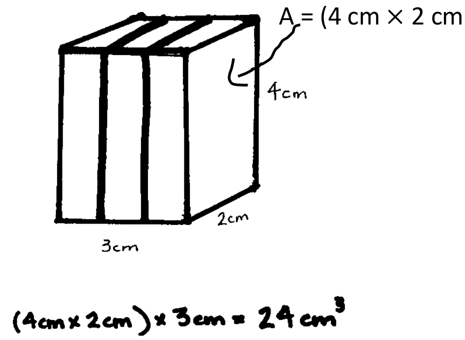
S: When multiplying a square unit times one more unit, it becomes a cubic unit. → You start out with length units, the second factor makes them square units, and the third

YOUR NOTES

factor makes them cubic units. → To measure area, we use squares. To measure volume, we use cubes. The third factor means we don't just have flat squares, but cubes.

T: Is this the same volume we found when we counted by the number of cubes in each layer?

S: Yes.



T: Let's use this method again, but I'd like to use the area of this face. (Point to the layer on the end.) Write a multiplication expression that shows how to find the area of this face.

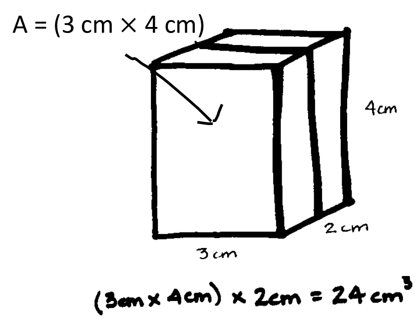
S: 4 cm × 2 cm.

T: (Write (4 cm × 2 cm).) To find the volume, we need to know how many layers are to the left of this face. What dimension of this prism tells us how many layers this time? How many centimeters is that? Turn and talk.

S: This time there are 3 layers. → The length is the one that shows how many layers this time. It's 3 centimeters. → The prism is 3 centimeters long. This shows the layers beside this face.

T: (Write (4 cm × 2 cm) × 3 cm.) Multiply to find the volume.

S: (Work to find 24 cm^3.)



T: (Project the image of the prism shown to the right.) Now, let's look at this last decomposition. Find the area of the front face. Tell which dimension shows the layers, and work with your partner to write an expression to find the volume. Turn and talk.

S: The area of this face is 3 cm times 4 cm. That's 12 square centimeters. There are 2 layers that are each 1 cm. 3 × 4 × 2 = 24. The volume is 24 cubic centimeters. → The area is 12 square centimeters, and the width is 2 cm. Twelve square centimeters times 2 centimeters is 24 cubic centimeters.

T: This is the same volume as before. Look at all three multiplication sentences. What patterns do you notice? Turn and talk.

S: The volume is the same every time. → We are multiplying all the sides together, but they are in a different order. → When we multiply the length of the sides together, we get the same volume as when we counted the layers.

T: So, does centimeters times centimeters times centimeters give us centimeters cubed? Why or why not? Turn and talk.

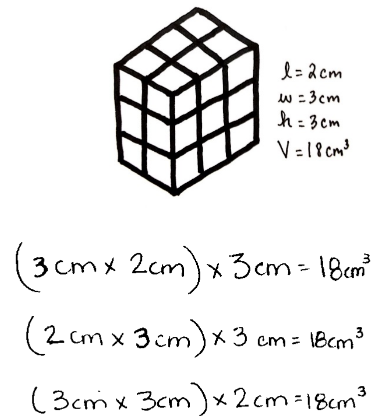
S: Yes. There are three measurements that are centimeters, and then the answer is in cubic units. → True. There are three factors that have centimeter units. So, the product has to be cubic units because cubes measure space in three dimensions!

T: Let's see if this pattern holds. (Display the image of the prism shown below.) Record the dimensions of this prism. What's different about it?

YOUR NOTES

S: It's the same width and length, but now, the height is 1 cm shorter. → There are 6 fewer cubic centimeters in this one. → There are still some 2 × 3 layers in this one.

T: How would you find its volume? Turn and talk.



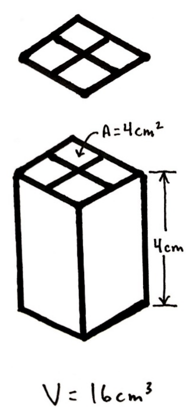
S: I can subtract 6 cubic units from the 24 cubic units in the 4-layer prism. That makes the volume 18 cubic centimeters. → I can multiply the 6 cubes in the top layer by 3 layers. That's 18 cubic centimeters. → I can multiply 2 cm times 3 cm times 3 cm, which is 18 cubic centimeters. → The end has a 6 cm^2 area and 3 layers, so 6 cm^2 × 3 cm = 18 cm^3. → The front face is different now. It is 3 cm by 3 cm. There is 1 layer behind the 3 cm x 3 cm face for a total of 2 layers.
3 cm × 3 cm × 2 cm = 18 cm^3.

T: Let's record this. (Record.)

T: What can we conclude about finding volume from these examples?

S: We can multiply the sides to find the volume. → If we know the area of one face and multiply by the number of those layers, we can find volume. → Yes, but the number of layers is just the length of the remaining side.

Part 2: Calculate the volume when the area of one side is given.



T: (Post image illustrated to the right.) This image represents the top face of a rectangular prism. If the prism is made of 1 cm cubes, what is the area of this face?

S: 4 square centimeters.

T: (Write A = 4 cm^2. Then, post the image of the prism with a height of 4 cm.) If the rectangular prism that sits below this face is built of centimeter cubes and has a height of 4 cm, how many layers of centimeter cubes are in the prism?

S: 4 layers.

T: How can we use the layers to find the volume? Turn and talk.

S: I can see that the length is 2 cm and the width is also 2 cm, so if the height is 4 cm, I can multiply 2 by 2 by the number of layers, which is 4, to get the volume. → Since the area of the top is 2 cm times 2 cm, which is 4 cm^2, we can just multiply the area of the top by the height to find the volume.

T: Show me the multiplication sentence you can use to find the volume of this prism.

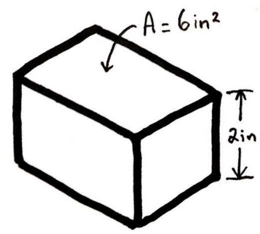
S: V = (2 cm × 2 cm) × 4 cm = 16 cubic cm. → V = 4 cm^2 × 4 cm = 16 cm^3.

T: (Write V = 16 cm^3 on the board.)

T: (Post the image to the right on the board.) What's different about this prism?

S: We can't see the individual cubes in the face with the area. → We don't know the dimensions of the top face, just the area.

YOUR NOTES



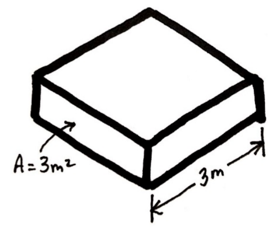
T: Do we need the dimensions of that top face to find the volume? Why or why not?

S: No. We can use the area. → We don't need to know how many cubes are in each layer. We just want the total volume. The area of the top and the height are enough to find volume.

T: Work with a neighbor to find the volume of this prism.

S: (Work to show V = 6 in^2 × 2 in = 12 in^3.)

T: (Post the final image, to the right, on the board.) Compare this prism to the last one. Turn and talk.



S: This one shows just the area again. → This one shows the area of the front and the width. We can still just multiply them. → This time, we have the area of a different face. We have the area of the front face and the width of the prism, which tells how many layers are behind the front face.

T: Find the volume of this prism.

S: (Work to show V = 3 m^2 × 3 m = 9 m^3.)

NOTES

Debrief Questions

- Explain how we get cubic units when we multiply to find volume.
- (Connect the term *face* with the term *base*. Discuss with students that these two terms may be used interchangeably when dealing with right rectangular prisms.) Why could we think of any face as the base of our prism? (Discuss the fact that if we imagine rotating the prism so that the chosen face lies at the bottom, or what we typically think of as the base, the remaining dimension can be thought of as the height of the prism.)
- What would happen to the volume of a box if you doubled the height? If you halved the length? If you doubled the height while halving the length?
- Compare your earlier strategies for finding volume to the method we learned today. How is the formula for finding the volume of rectangular prisms helpful?

Vocabulary

While it is true that any face of a rectangular prism may serve as the base, it is not true for other prisms or cylinders. For example, a right triangular prism has two triangular bases, but the remaining rectangular faces are not bases.

YOUR NOTES

Lesson 5

Use multiplication to connect volume as *packing* with volume as *filling.*

OPTIONAL FOR FLEX DAY: ALL OF LESSON 5

Note: Today's lesson uses the Problem Set. Solutions for each problem are included below.

Note: Before class, prepare a large poster or sheet for groups to record their findings. Be sure to use cubes that are denser than water for the displacement exploration.

Materials: (S) Per group: centimeter cubes, several small watertight containers (preferably right rectangular prisms) marked with a horizontal line for measuring, small pitcher of water, graduated cylinder labeled with mL, class data recording sheet poster, ruler or tape measure, Problem Set (Problems 1-3, see Appendix)

Problem 1

Investigate 1 cm^3 = 1 mL.

T: What are some ways that we can determine the volume of the box you've been given using the materials on your table?

S: We can pack it with cubes and count. → We can pack the bottom layer and then use the cubes to find how many layers. → We could find the area of any base and then count the layers. → We can measure the sides and then multiply the three dimensions.

T: Measure the inside dimensions of your box using the line that's drawn as the height, and multiply to find the volume. Then, confirm the measurement by packing the box to the line that's drawn. Record the volume in cubic centimeters on Problem #1 of your Problem Set.

S: (Work.)

T: Now I would like you to find the amount of liquid your container will hold. Any ideas how you might do this using the materials on your table? Turn and talk.

S: We could pour in some water and then measure the water with the graduated cylinder. → We could fill the container with water and then use a measuring cup to measure the water. That would tell us the amount it will hold.

T: What units are used on the graduated cylinder?

YOUR NOTES

S: Milliliters.

T: Pour the water to the fill line. Then, measure the amount of water by carefully pouring it into the graduated cylinder. Record the liquid volume on Problem #2 of your Problem Set. Once your group is done, have a member of your group record your data onto the class poster.

S: (Work and record.)

T: (Circulate, asking students to describe what they're doing. Encourage use of the terms *volume*, *capacity*, and other unit language.)

T: Now that we've recorded our findings, let's look at the volume data. What do you notice about the volume as measured by the cubes and the liquid volume?

S: They are the same. → Our box packed 36 cubes, and it held 36 mL of water. → Although our prism was a different size than the first group's, our packing and filling amounts were the same. → Ours was really close—just one cubic centimeter more than the milliliters.

T: What can you say about the relationship of 1 milliliter to 1 cubic centimeter?

S: They seem to be the same. → I think they are equal.

T: There's a way we can show that these two measurements are equal. Put water into your graduated cylinder to any measuring point other than the fill line. Be careful to fill it exactly to the line you choose. For example, you might fill your graduated cylinder to 15 mL.

S: (Pour.)

T: Now, pour in 1 more milliliter of water, and describe what happens to the water level.

S: The water went up one more line. → The water rose because we put more in.

T: Record the new amount of water on Problem #3 of your Problem Set. What will happen to the water level if we place 1 cube in the graduated cylinder? Tell your partner.

S: It will go up again. → The water will rise because the cube pushes some of the water out of the way.

T: Let's find out how far the water will rise. Place 1 centimeter cube into the water. Describe what happens to your partner.

S: (Work and discuss.)

T: How did the water level change?

S: The water rose. → It looks like there's more water in the graduated cylinder. → The water went up 1 mL.

T: We didn't actually put more water in, and yet the cube caused a rise in the water level equal to when we put 1 mL of water in the graduated cylinder. From this investigation and from our work with the boxes, what can we say about the relationship between 1 mL of water and 1 cubic cm?

S: They are equal. → I know they are equal because I measured my box and got the same number of cubes as milliliters. → I know they are equal because one cube made the water go up 1 milliliter.

YOUR NOTES

T: Yes. We have seen that 1 cm3 = 1 mL. (Write 1 cm3 = 1 mL on the board.) This is an important relationship that will help us solve problems.

Problem 2

A rectangular tank measures 30 cm by 20 cm by 40 cm. How many milliliters of water are in the tank when it is full? How many liters is that?

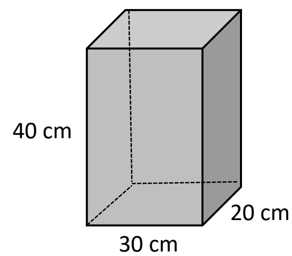
T: Let's use what we've learned about volume as filling to solve this problem. We need to find the volume of the water in the tank. What do we know about the tank that can help us?

S: We know the size of the tank. → Since the water is filled to the top, the volume of the tank will be the same as the volume of the water.

T: Find the volume of the tank.

S: (Work to find 24,000 cubic centimeters.)

T: We discovered today that 1 cubic centimeter is equal to 1 mL. Since this is true, how many milliliters of water are in the tank when it is full?



S: 24,000 mL.

T: How many liters is that?

S: 24 liters.

Problem 3

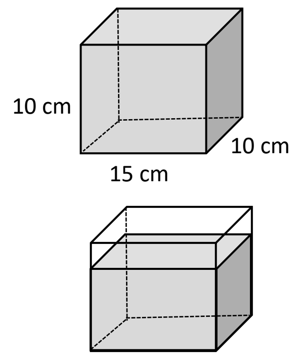
a. A small fish tank is filled to the top with water. If the tank measures 15 cm by 10 cm by 10 cm, what is the volume of water in the tank? Express your answer in mL.
b. After a week, water evaporates out of the tank, so the water is 9 cm high. What is the volume of the water in the tank?

T: (Project Problem 3(a) and the accompanying image onto the board.) Using what we've talked about today, turn and talk to your partner, and find the volume of water in the tank in cubic centimeters and in milliliters.

S: All we need to do is multiply the sides because the water is all the way to the top. → Since the water fills the whole tank, we can just multiply 15 × 10 × 10. That's 1,500 cubic centimeters. → It's easy to find the volume. It's 1,500 cubic centimeters. We have to say it in milliliters. That's exactly the same number, so it's 1,500 mL.

T: (Project Problem 3(b).) Let's imagine that some of the water evaporated out of the tank. Now the water is only 9 cm deep. Does this change the height of the tank? Why or why not?

YOUR NOTES



S: No, because the tank doesn't change size.

T: Does this change the area of the bottom of the tank?

S: No. The tank is still the same size.

T: Will the volume of the water change? Why or why not?

S: The volume in the tank will be less because some of the water is gone. → The water won't be as high. → The water level will go down by 1 cm, so that's like pouring out a layer of 15 by 10 centimeters.

T: Find the volume of the water in the tank now. Turn and talk.

S: The bottom of the tank is the same, so the water is spread out on the bottom the same way as before. The only thing that is different is the height of the water. I'll multiply 15 and 10 and then multiply by 9. That's 1,350 cubic centimeters of water. → The part of the water that is gone is 15 × 10 × 1. That's 150 cubic centimeters. I can subtract that from 1,500. That will be 1,350 cubic centimeters still in the tank.

T: What is the volume of the water in the tank now?

S: 1,350 cubic centimeters.

T: Express that in milliliters.

S: 1,350 mL.

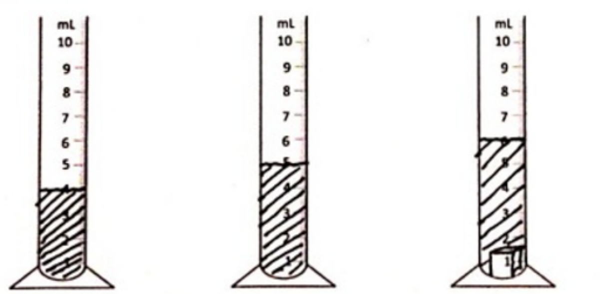
1. Determine the volume of two boxes on the table using cubes and then confirm by measuring and multiplying.

Box Number:	Number of cubes packed:	Measurements: Length	Width	Height	Volume:
1	32	4 cm	4 cm	2 cm	32 cm^3
2	20	2 cm	5 cm	2 cm	20 cm^3

2. Using the same boxes from #1, record the amount of liquid that your box can hold.

Box Number:	Liquid the box can hold
1	32 mL
2	20 mL

3. Shade to show the water in the beaker.



At first: 4 mL

After 1 mL water added: 5 mL

After 1 cm cube added: 6 mL

NOTES

Debrief Questions

- Discuss the connection between these two terms. How is height like depth? When might you use the word *height* to describe a figure, and when might *depth* be more appropriate? Can the words be used interchangeably?
- If 1 cubic centimeter is equal to 1 milliliter, to what liquid measure is 1 cubic meter equivalent? How could you find out? (Students can draw to investigate using 100 cm = 1 m. Building a cubic container from meter sticks in the classroom helps students visualize the actual volume of 1 kiloliter. They might also imagine pouring 1,000 liter bottles of water or 500 2-liter soft drinks into that single container.)
- (Ask students to generate as many rectangular prisms with whole number sides as they can that would hold 1 liter of liquid. Although the dimensions are all factors of 1,000, the shapes of the containers may be drastically different. Students might even be encouraged to draw the containers on isometric dot paper for comparison.) What do the sides of all these containers have in common? (All are factors of 1,000.) Because they all have the same factors, are they all the same shape? Why or why not?

Multiple Means of Representation and Materials

Fancy toothpicks, straight pins, and some office supplies come in clear rectangular boxes suitable for this activity. The horizontal fill line for the water can be drawn at a height that matches the number of cubes that can be packed into the box. If these are not readily available, small rectangular breath mint boxes or metal spice boxes can be used. However, students may have to estimate if the corners are rounded. It is best to test the boxes and volumes before implementing this lesson.

An alternative approach is to gather a collection of small gift boxes and use salt rather than water to fill them. While salt is not a liquid, it does behave like one for the purposes of the first activity.

The second activity must be done with water and a centimeter cube that sinks. If a dense cube is not available, students should use a drinking straw or coffee stirrer to submerge the cube completely.

If resources are limited, this may be done as a demonstration and then as a center over the course of a few days in small groups.

Multiple Means of Engagement

While the relationship of 1 cm^3 = 1 mL seems a simple one numerically, the concept behind this relationship—that of volume as *filling* as well as *packing* (especially when comparing a rectangular container to a cylindrical one)—is more complex. Be sure to offer many opportunities for students to encounter this concept beyond today's lesson. Ask often if an amount of liquid will fit into rectangular containers and how it might be confirmed without pouring.

YOUR NOTES

Lesson 6

Find the total volume of solid figures composed of two non-overlapping rectangular prisms.

Materials: (T) Drawing of rectangular prism figures (S) 15 centimeter cubes, dot paper

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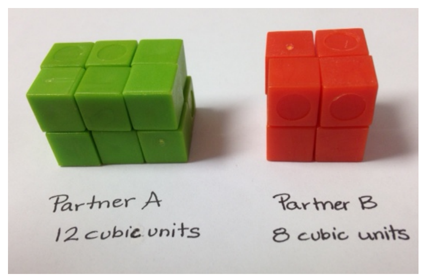
Problem 1

Build and combine structures, and then find the total volume.

T: Partner A, use one color cube to build a structure that is 3 cm by 2 cm by 2 cm. Partner B, use a different color to build a cube that is 2 cm long on every side. Record the volume of your structures.



S: (Work.)

T: Keeping their original dimensions, how could you combine the two structures you've built? Turn and talk. Then, find the volume of your new structures.

S: We could put the cube on top of the rectangular prism. → We could put them beside each other on the end. → We could make an L. → The volume is 20 cubic units.

T: Now, build a different structure using the two prisms, and find the volume.

S: (Work.)

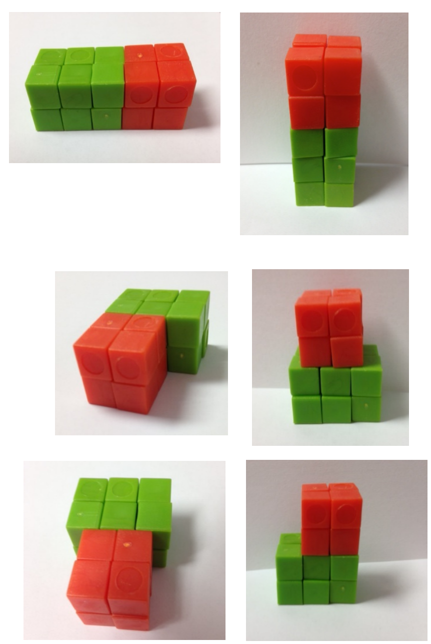
T: How did you find the volume of your new structures?

S: We counted all the blocks. → We knew that one was 12 cubic units and the other one was 8 cubic units. We just added that together to get 20 cubic units.

T: When you built the second structure, did the volume change? Why or why not?

S: It did not change the volume. There were still 20 cubic units. → It doesn't matter how we stacked the two prisms together. The volume of each one is the same every time, and the volume of the whole thing is still 20 cubic units. → The total volume is always going to be the volume of the red structure plus the volume of the green structure, no matter how we stack them.

Possible Combinations

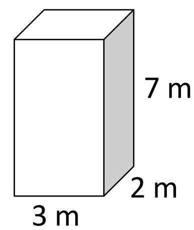


YOUR NOTES

Problem 2

T: (Project or draw on the board the 3 m × 2 m × 7 m prism on the right.) What is the volume of this prism?

S: 42 m^3.



T: Imagine another prism identical to this one. If we glued them together to make a bigger prism, how could we find the volume? Turn and talk. Then, find the volume.

S: We already know that the volume of the first one is 42 m^3. We could just add another 42 m^3 to it. That would be 84 m^3. → We could multiply 42 by 2 since they are just alike. That's 84 m^3.

Problem 3

T: (Project or draw on the board the composite structure.) How is this drawing different from the last one?

S: There are two different-size boxes this time. → The little box on top only has measurements on the length and the height.

T: There are a lot of markings on this figure. We'll need to be careful that we use the right ones when we find the volume. Find the volume of the bottom box.

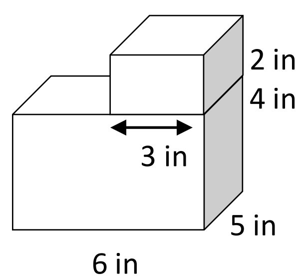
S: (Work to find 120 cubic inches.)

T: What about the one on the top? I heard someone say that there isn't a width measurement on the drawing. How will we find the volume? Turn and talk.

S: The boxes match up exactly in the drawing on the width. That means the width of the top box is the same as the bottom one, so it's still 5 inches wide. → You can tell the top and bottom box are the same width, so just multiply 3 × 5 × 2.

T: What is the volume of the top box?

S: 30 cubic inches.



T: How could we find the total volume?

S: Add the two together.

T: Say the number sentence with the units.

S: 120 cubic inches + 30 cubic inches = 150 cubic inches.

Problem 4

T: (Project or draw the figure given on the right.) Compare this figure to the last one.

S: There are two different boxes again. → There's a little one and a big one, like last time. → This time, there's a bracket on the height of both boxes. → There's no length or width or height measurement on the top box this time.

YOUR NOTES

T: If there are no measurements on the top box alone, how might we still calculate the volume? Turn and talk.

S: We can tell the length of the top box by looking at the 6 meters along the bottom. The other box has 4 meters sticking out on the top of the box. That means the box must be 2 meters long. → The length is 6 minus 4. That's 2. → The width is easy. It's the same as the bottom box, so that's 2 meters. → The height of both boxes is 4 meters. If the bottom box is 2 meters, then the top box must also be 2 meters.

T: What is the volume of the top prism? Say the number sentence.

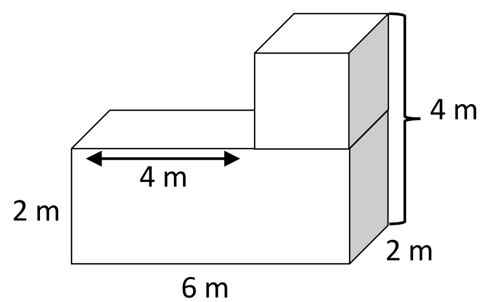
S: 2 m × 2 m × 2 m = 8 cubic meters.

T: What is the volume of the bottom prism? Say the number sentence.

S: 6 m × 2 m × 2 m = 24 cubic meters.

T: What's the total volume of both? Say the number sentence.

S: 8 cubic meters + 24 cubic meters = 32 cubic meters.



Problem 5

Two rectangular prisms have a combined volume of 135 cubic meters. Prism A has double the volume of Prism B.

a. What is the volume of each prism?

b. If one face of Prism A has an area of 10 square meters, what is its height?

T: Let's use a tape diagram to help us with this problem. Read it with me.

T/S: (Read.)

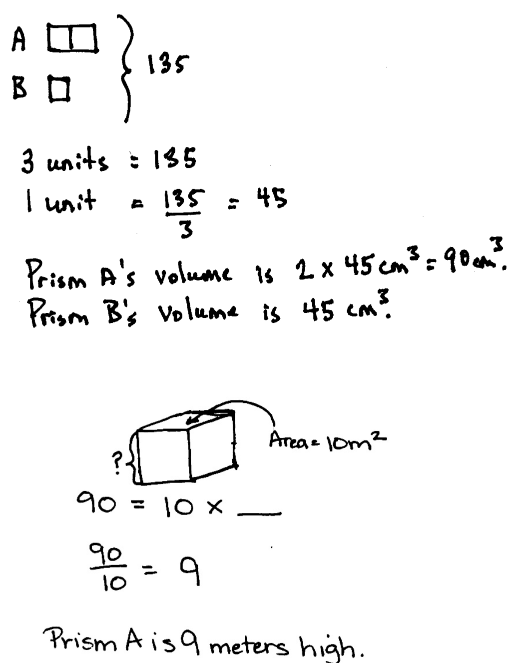
T: What can we draw from the first sentence?

S: A tape diagram for each prism. → Two tape diagrams labeled *Prism A's volume* and *Prism B's volume*. → A bracket on both to show they are 135 cubic meters total. → Their total volume is 135 cubic meters.

T: What does the next sentence tell us, and how can we represent it?

S: Prism A is double the volume of Prism B. → We need 2 units for Prism A. → Prism A's tape should be twice as long as Prism B's.

T: Show that in your diagram. Then, use this



YOUR NOTES

information to solve for the volumes of both prisms.

S: (Work.)

T: What is the volume of each prism?

S: Prism A is 90 cubic meters, and Prism B is 45 cubic meters.

T: To find the height of Prism A, what do we need to think about? Turn and talk, and then solve.

S: We know the area of one face. If we multiply the area by something, we should get the volume of 90 m^3. The area is 10 m^2, and 10 times 9 is 90. It is 9 meters tall. → We can divide 90 by 10 and get 9 meters tall.

NOTES

Debrief Questions

- What advice would you give to a friend who was having trouble picturing the dimensions on a composite figure? What helps you to figure out missing dimensions?
- Is a shorter container always a smaller volume? Give some examples of prisms to support your answer.

Multiple Means of Engagement

Challenge students whose spatial skills allow them to see these figures easily by having them draw a figure consisting of three different prisms on dot paper with just enough information given to calculate the volume of the figure. They should calculate the volume of their own figures and then exchange figures with a partner.

Students can write about the minimum information necessary to calculate the volume of a composite figure.

YOUR NOTES

Lesson 7

Solve word problems involving the volume of rectangular prisms with whole number edge lengths.

OPTIONAL FOR FLEX DAY: ALL OF LESSON 7

Note: Today's lesson uses the Problem Set. Solutions for each problem are included below.

Materials: (S) Problem Set (see Appendix)

Suggested Delivery of Instruction for Solving Lesson 7's Word Problems

1. Model the problem.

Have two pairs of students who can successfully model the problem work at the board while the others work independently or in pairs at their seats. Review the following questions before beginning the first problem:

- Can you draw something?
- What can you draw?
- What conclusions can you make from your drawing?

As students work, circulate. Reiterate the questions above. After two minutes, have the two pairs of students share only their labeled diagrams. For about one minute, have the demonstrating students receive and respond to feedback and questions from their peers.

2. Calculate to solve and write a statement.

Give everyone two minutes to finish work on that question, sharing their work and thinking with a peer. All students should write their equations and statements of the answer.

3. Assess the solution for reasonableness.

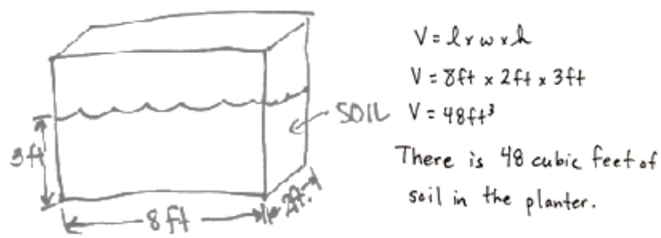
YOUR NOTES

Give students one to two minutes to assess and explain the reasonableness of their solutions.

Note: Geoffrey builds rectangular planters. All of the inside dimensions of the planters are whole numbers.

Problem 1

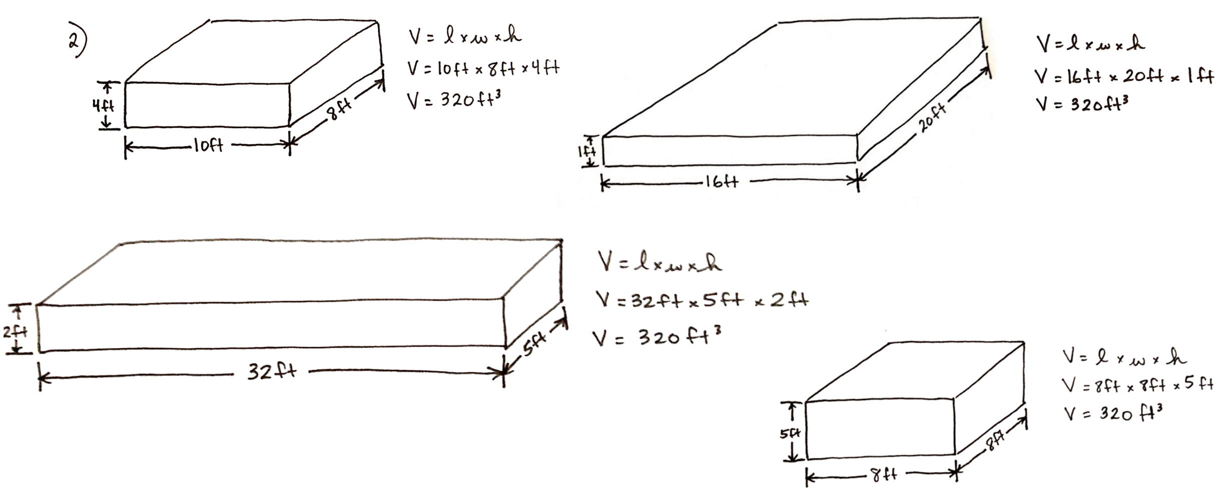
Geoffrey's first planter is 8 feet long and 2 feet wide. The container is filled with soil to a height of 3 feet in the planter. What is the volume of soil in the planter? Explain your work using a diagram.



In this problem, students are given three dimensions (length, width, and height) and are asked to find the volume of the soil in the planter. The use of the volume formula allows students to find the number of cubic feet of soil in the planter. A non-scaled illustration of the planter is the most logical diagram to accompany this work.

Problem 2

Geoffrey wants to grow some tomatoes in four large planters. He wants each planter to have a volume of 320 cubic feet, but he wants them all to be different. Show four different ways Geoffrey can make these planters, and draw diagrams with the planters' measurements on them.



YOUR NOTES

In Problem 2, students are asked to come up with four sets of differing dimensions that all result in a volume of 320 cubic feet. This problem requires students to think in terms of whole to part. They need to find factors of 32 tens to generate the dimensions. The illustrations are just four such examples. Encourage students to come up with different values for each dimension rather than just changing the name of the same dimensions (length, width, and height) repeatedly, although that method would result in the same volume. Simply changing the shape of the planter can provide opportunities for discussion of which shape might be best to use for different situations. For example, in a small yard, planting a tree that would need more depth than length might be the best option. There are a wide variety of dimensions that would be acceptable here, but be sure to have students check their multiplication using the volume formula.

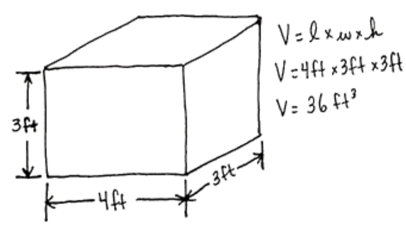
Problem 3

Geoffrey wants to make one planter that extends from the ground to just below his back window. The window starts 3 feet off the ground. If he wants the planter to hold 36 cubic feet of soil, name one way he could build the planter so it is not taller than 3 feet. Explain how you know.

Since Geoffrey wants to build a planter with a height of 3 feet & a volume of 36 cubic feet, the base of the planter should have an area of 12 sq. ft. I drew a planter with a length of 4 ft, a width of 3 ft, & a height of 3 ft.

$36 \div 3 = 12$

$12 = 4 \times 3$



This problem requires students to work backward and reason that since Geoffrey needs 36 cubic feet of volume and that the height of Geoffrey's planter is 3 feet, then division shows that the base of the planter must have an area of 12 square feet. From there, students have the freedom to design a planter with a base measuring 12 ft × 1 ft, 6 ft × 2 ft, or 4 ft × 3 ft.

(Note: These dimensions may represent either the length or the width.) Again, this presents an opportunity to discuss the connection between the areas of the base and the shape the planter takes. Extend the problem by asking students to choose the planter that would not extend beyond a 5-foot window. (Any of the previously mentioned planters could work in this case, depending on which side is turned

YOUR NOTES

toward the window.) These discussions are sure to invite construction of viable arguments and the opportunity to critique the reasoning of classmates.

Problem 4

After all of this gardening work, Geoffrey decides he needs a new shed to replace the old one. His current shed is a rectangular prism that measures 6 feet long by 5 feet wide by 8 feet high. He realizes he needs a shed with 480 cubic feet of storage.

a. Will he achieve his goal if he doubles each dimension? Why or why not?

4a) SHED: $V = 6\text{ ft} \times 5\text{ ft} \times 8\text{ ft}$

$V = 240\text{ ft}^3$

SHED DIMENSIONS DOUBLED: $V = 240\text{ ft}^3 \times 8$

$V = 1{,}920\text{ ft}^3$

Geoffrey's current shed has a volume of 240 cubic ft, which is half the volume he needs. By doubling each dimension of the shed, Geoffry will get a shed that is 8 times the current size (because $2 \times 2 \times 2 = 8$). To double the volume he needs only to double one dimension, not all three.

This part of Problem 4 gives students a chance to explore the exponential growth potential of doubling all three dimensions simultaneously. Doubling the length, width, and height of Geoffrey's shed results in a volume that is 8 times that of his current shed $(l \times 2) \times (w \times 2) \times (h \times 2) = (l \times w \times h) \times 8$. While this size shed certainly provides the 480 cubic feet he is looking for, students can reason that doubling each dimension would lead to a shed that is far larger than Geoffrey needs. This may lead to students trying to double only two of the dimensions and then realizing that simply doubling one of the dimensions of his shed gives Geoffrey double the volume. This discussion can also include an exploration of which dimension makes the most sense to double given that this is a garden shed. Would doubling the height give more usable space for gardening equipment? Does it make more sense to double either the length or the width? Challenge: Is there a way to change two dimensions and still simply double the space?

b. If he wants to keep the height the same, what other dimensions could he use to reach his target volume?

YOUR NOTES

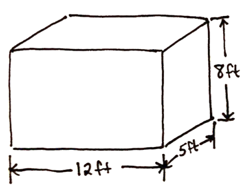
4b) Since Geoffrey wants to double the volume of his shed & keep the height the same, he could double the length & keep the width the same too. Or he could double the width & keep the length the same.

$l = 12$ ft, $w = 5$ ft, $h = 8$ ft OR $l = 6$ ft, $w = 10$ ft, $h = 8$ ft

This problem builds on students' thinking from the previous problem and asks them to identify dimensions that would yield a shed volume of 480 cubic feet while maintaining a height of 8 feet. Most students will correctly reason that they simply need to double one of the other dimensions (the length or the width) in order to create a doubled volume. However, there are additional ways to create a volume of 480 cubic feet with a height of 8 feet, including halving the length and quadrupling the width. Engage students in a discussion about why this is possible (think back to Problem 3), and have them share their alternate solutions.

c. If he uses the dimensions in part (b), what would be the area of the new shed's floor?

4c)



$A = l \times w$

$A = 12\text{ft} \times 5\text{ft}$

$A = 60\text{ ft}^2$

The floor would have an area of 60 square feet.

Part (c) requires students to remember their work from Lesson 4 and multiply the length times the width to find the area of the shed floor. Since students use their varied answers from part (b) to answer this question, expect to find variety in the responses here as well. However, this is another opportunity to engage students in a discussion about why the area must be 60 square feet, despite using different dimensions from part (b).

NOTES

Debrief Questions

- What effect does doubling one dimension have on the volume? Doubling two dimensions? Doubling all dimensions? Why?
- What effect would doubling one dimension while halving another have on the volume? Why?
- How many prisms can you think of that have a volume of 100 cm^3?
- If Geoffrey had been using fractional lengths for the dimensions for his planters, how would that have changed the possible answers to these questions?

Multiple Means of Engagement

The problems in today's lesson are focused on planters and gardening. The lesson could serve as a springboard for planning a school garden. Students could plan planters as Geoffrey does in the lesson given parameters of height or base area. Students might also research optimal soil depths (and thus volume) for particular plants and vegetables to incorporate into their designs.

Bringing their designs into reality is the ultimate real-world problem connection. If it is possible, let them try. Writing a proposal to present the designs (along with estimated costs) to the principal or school board encompasses many skills, but even cardstock planters with paper flowers can be a rewarding experience.

Multiple Means of Expression

If students have difficulty drawing the rectangular prisms freehand, isometric dot paper can be used as a scaffold. Students may also like to use a computer to draw their figures to be printed out. Calculations can be done alongside.

Lesson 8

Apply concepts and formulas of volume to design a sculpture using rectangular prisms within given parameters.

YOUR NOTES

Note: Today's lesson uses the Problem Set. Solutions for each problem are included below.

Materials: (S) Problem Set (see Appendix), project requirements (Template 1), box patterns (a–c) (Templates 2–4), lid patterns (Template 5) (at least three of each template per group), evaluation rubric (Template 6), scissors, tape, rulers

Copy the flattened boxes on the thickest paper available. Each student or group needs three to five copies of each box, but they may not all be used.

Students cut the templates to form boxes of a certain volume by adjusting the height of the sides. They should construct the boxes by taping edges together and then turn the boxes open side down to create their sculptures. They may also tape lids on the open ends of their boxes to make construction easier.

Please also note the evaluation rubric included in this lesson. The rubric can be shared with students so they understand how their work will be judged.

This activity can be done individually or in pairs.

T: Today, we will be putting our math sense and geometric skills to work as each of you designs a sculpture created from a collection of rectangular prisms. Read the requirements and the rubric with a neighbor.

Distribute the project requirements (Template 1), the Problem Set, and the evaluation rubric (Template 6). Allow students time to read all three.

T: Now that you've had an opportunity to read the requirements and the way your work will be evaluated, share your ideas about what you might like to design.

S: I want to make a shape using five prisms and make it as random as possible. → I want to see if I can do a capital F, for my name. → I was going to do a scaled version of my tree house, but I'm not sure if I'll be able to scale the dimensions right.

YOUR NOTES

T: Here are the boxes, like the ones we used in Lesson 2, that you can use to build your rectangular boxes. There are three different bases to choose from. You may adjust the volume by adjusting the height of the sides of your box. Watch me cut one of the box patterns and make a box. (Demonstrate cutting the 6 cm × 3 cm box pattern, Template 2.) If I want to build this first prism to have a volume of 36 cubic centimeters, what height will I need to measure and cut the sides?

S: They would need to be 2 cm high.

T: Yes. I'll measure all my side flaps 2 cm from the base. Then, I'll cut, fold, and tape them. (Demonstrate.) Talk with your neighbor about how you'll construct your first box and calculate the volume.

S: Cut the base with rectangles attached on each side that are the same height, and fold them up. Then, calculate the volume. → Decide on the volume, and find the area of the base. Then, cut the height to give the volume you need.

T: It might be a good idea to draw a very rough sketch of the design you're thinking of creating.

S: (Draw.)

T: Reread the third prompt with a friend. Share your ideas about how you'll meet its requirements.

S: (Share.)

T: What were your ideas?

S: I'm going to make Prism A and then try to cut one of the other prisms to make it half the volume of Prism A and call it Prism D. → I can make the biggest prism possible and then divide the volume in half and try to make another prism one-half of that volume. → I'll make a prism and then use the same base to make another prism but cut the height in half. That will give me half of the original volume.

T: Reread the fourth prompt with a friend. Share your ideas about how you'll meet its requirements.

S: I will take Prism B and cut it up to create a prism with one-third the volume. → I can use the same big prism as before and divide the volume by 3 and find a prism with dimensions that will equal that number. → I can take one of the first three prisms and make the height one-third of the original height, and this will give me one-third of the volume.

T: The final prompt says that the total volume of your design must not exceed 1,000 cubic centimeters. Share your ideas.

S: What's the biggest prism I can do? → We can do a total of five prisms, but they can't be more than 200 cubic centimeters. One has to be half of another. Another has to be a third of another; this is going to require some thinking! → Let's just say we do one prism that's 420 cubic cm and one that's half of that (210 cubic cm) and then one that's a third of that (140 cubic cm). That's 770 cubic centimeters. That means I still have 230 cubic units and two prisms to play with. I can make one 3 cm by 6 cm by 4 cm and one 5 cm x 5 cm x 4 cm for a total of 942 cubic centimeters.

T: Once you've finalized your boxes, cut a lid with tabs that will fit, and tape it to your box.

YOUR NOTES

This will give your boxes stability, so they'll be easier to tape together. I can tell that you're excited to get started. Be sure to check your math as you progress, and feel free to share your ideas with a neighbor as you work. (Circulate around the room to ask clarifying questions or provide support to struggling learners as students work.)

1.	My sculpture has 5 to 7 rectangular prisms.	Number of prisms: 5
2.	Each prism is labeled with a letter, dimensions, and volume.	

Prism		by		by		Volume =
Prism A	10 cm	by	7 cm	by	4 cm	Volume = $280 cm^3$
Prism B	5 cm	by	5 cm	by	6 cm	Volume = $150 cm^3$
Prism C	6 cm	by	3 cm	by	5 cm	Volume = $90 cm^3$
Prism D	10 cm	by	7 cm	by	2 cm	Volume = $140 cm^3$
Prism E	5 cm	by	5 cm	by	2 cm	Volume = $50 cm^3$
Prism __		by		by		Volume =
Prism __		by		by		Volume =

3.	Prism D has $\frac{1}{2}$ the volume of prism A.	Prism D Volume = $140 cm^3$ Prism A Volume = $280 cm^3$
4.	Prism E has $\frac{1}{3}$ the volume of prism B.	Prism E Volume = $50 cm^3$ Prism B Volume = $150 cm^3$
5.	The total volume of all the prisms is 1,000 cubic centimeters or less. Yes!	Total volume: $710 cm^3$ Show calculations: 280 150 90 140 + 50 710

NOTES

Debrief Questions

- What was your thought process as you designed your sculpture? Was your sculpture inspired by something you have seen or owned?
- Which figure did you cut into halves or thirds when creating another shape? What was your thought process as you created a shape one-half or one-third the size? Did you cut one dimension into halves or thirds, or did you scale the entire volume first and then select dimensions to meet that volume?
- What was your biggest challenge in designing your sculpture? Explain.

Materials

When printing the box and lid patterns, be sure the printer is set to *actual size*.

Multiple Means of Engagement

Students who struggle may be encouraged to use only three prisms or be given more latitude in total volume or in the relationships between the prisms. Alternatively, those whose spatial skills are well developed may be given additional restrictions to meet or requirements to fulfill or may be asked to use more prisms to construct their designs.

Multiple Means of Engagement

Some students may be overwhelmed by the amount of reading and interpretation of directions required for the project. Reading the requirements as a class and having discussion after each one can be helpful. Or place accomplished readers with those who struggle.

Some students may benefit from having cubes to actually construct a model of their structures first.

Multiple Means of Expression

Students whose fine motor skills are less developed may enjoy producing a virtual version of this project. They might use computer-based drawing tools to draw the prisms (e.g., Google's SketchUp). These can be printed and then measured to fulfill the requirements of the project.

YOUR NOTES

Lesson 9

Apply concepts and formulas of volume to design a sculpture using rectangular prisms within given parameters.

Note: Today's lesson uses the Problem Set. Solutions for each problem are included below.

Note: Before class, the projects should be labeled only with a number and no student names. The review process in today's lesson should proceed anonymously.

Materials: (T) Copy of student work from Lesson 8, evaluation rubric (Lesson 8 Template 6) (S) Rulers, 2 copies of Problem Set (1 for use during the lesson and 1 for independent work, see Appendix)

T: (Post the image of the shape below on the board.) Here is a student's project designed according to the directions we used yesterday. I've measured the boxes, and the measurements that you see on the diagram are correct. The volume of Prism A is given. (Distribute a copy of the Problem Set to each student.) Your job is to use the rubric to see if this student met all the requirements of the assignment.

T: Before we can do that, we must confirm the volumes that the student recorded are correct. Work with a neighbor to check the work this student did to find the volumes of the prisms. (Allow students time to work and share their results.)

T: What did you find? Are the recorded volumes correct?

S: They are correct. → Prisms A and C have volumes of 36 cm^3. → Prism B has a volume of 420 cm^3. → Prism E has a volume of 140 cm^3. → Prism D's volume is 18 cm^3.

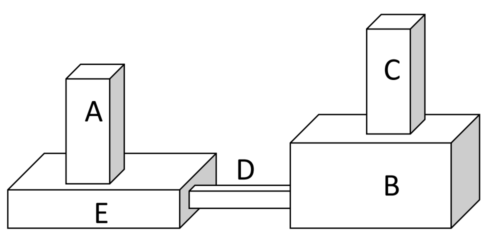
T: Now we are ready to begin our review. Look at the first item on the list. How many prisms are in this design?

S: 5.

T: Check the Element Present? box, and record the number of prisms used under Specifics of Element.

S: (Check the box and record 5 prisms.)

T: The Notes box is for any positive comments we might like to give to this student on this particular element. This is also the place to tell him anything that might be missing in the design. Since this student has met this requirement, turn and talk to your partner about what



Prism A: $6 \text{ cm} \times 3 \text{ cm} \times 2 \text{ cm} = 36 \text{ cm}^3$
Prism B: $10 \text{ cm} \times 7 \text{ cm} \times 6 \text{ cm} = 420 \text{ cm}^3$
Prism C: $6 \text{ cm} \times 3 \text{ cm} \times 2 \text{ cm} = 36 \text{ cm}^3$
Prism D: $6 \text{ cm} \times 3 \text{ cm} \times 1 \text{ cm} = 18 \text{ cm}^3$
Prism E: $10 \text{ cm} \times 7 \text{ cm} \times 2 \text{ cm} = 140 \text{ cm}^3$

YOUR NOTES

positive comment you might make.

S: I like the way the prisms are sort of symmetrical. → The way the boxes are stacked from big to little looks good. → Putting the skinny box in the middle makes the design look really big even though he only used 5 boxes.

T: Let's look at our next requirement. Are all the parts labeled with a letter? Record your answer.

S: Yes. They all have letters.

T: Are all prisms labeled with their dimensions and volume? Record your response.

S: Yes. All the prisms have dimensions and volume recorded.

T: Do all recorded measurements have the correct units? What are the units for the dimensions and volume?

S: Yes. All dimensions are in centimeters, and the volumes are in cubic centimeters.

T: Write that down. What's next on our list? How will we find out if this student met the requirements? Turn and talk.

S: We need to find out if Prism D is one-half of one of the other prisms and if Prism E is one-third of another prism. → We need to calculate the volume of all of the prisms first and then check if Prism D has one-half the volume of one of the other prisms and if Prism E has one-third the volume of one prism. → Prism D has a volume of 18 cm^3, which is one-half of Prism A's volume. → Prism E has a volume of 140 cm^3, which is one-third of Prism B's volume.

T: Record your findings. Check the Requirement boxes, and use the second page to record your calculations. (Circulate to make sure students are using the correct parts of the rubric to record the information.)

S: (Record.)

T: What is the total volume of this shape?

S: (Work and show that 36 cm^3 + 36 cm^3+ 18 cm^3 + 140 cm^3 + 420 cm^3 = 650 cm^3.)

T: Did this student meet all the requirements of the assignment? Tell me how you know.

S: Yes, she did. The volume is 650 cm^3, which is less than 1,000 cm^3. → There are 5 prisms, and they had to have 5 to 7 prisms. → The volume of Prism D is one-half the volume of Prism A. → The volume of Prism E is one-third the volume of Prism B.

T: Remember, if there's something that doesn't meet a requirement in the project that you review, you will record that in the Notes column. You may also use the Notes box to say something that you notice about their work.

T: I'm assigning each of you the sculpture of a fellow classmate (or pair) to review independently just as we did this one. Write the number of the project that you review on your Problem Set. Begin by confirming the measurements and volumes calculated by your classmate. (Distribute one sculpture to each student, and circulate to answer questions that arise.)

YOUR NOTES

I reviewed project number 21.

Use the rubric below to evaluate your friend's project. Ask questions and measure the parts to determine whether your friend has all the required elements. Respond to the prompt in italics in the third column. The final column can be used to write something you find interesting about that element if you like.

Space is provided beneath the rubric for your calculations.

	Requirement	Element present? (✓)	Specifics of Element	Notes
1	Sculpture has 5 to 7 prisms.	✓	*# of prisms:* 5	
2	All prisms are labeled with a letter.	✓	*Write letters used:* A – D	
3	All prisms have correct dimensions with units written on the top.	✓	*List any prisms with incorrect dimensions or units:*	
4	All prisms have correct volume with units written on top.	✓	*List any prism with incorrect dimensions or units:*	
5	Prism D has $\frac{1}{2}$ the volume of another prism.	✓	*Record on next page:*	
6	Prism E has $\frac{1}{3}$ the volume of another prism.	✓	*Record on next page:*	
7	The total volume of all the parts together is 1,000 cubic units or less.	✓	*Total volume:* $650 cm^3$	

Calculations:

$36 cm^3 + 420 cm^3 + 36 cm^3 + 18 cm^3 + 140 cm^3 = 650 cm^3$

8. Measure the dimensions of each prism. Calculate the volume of each prism and the total volume. Record that information in the table below. If your measurements or volume differ from those listed on the project, put a star by the prism label in the table below, and record on the rubric.

Prism	Dimensions	Volume
A	6cm by 3cm by 2cm	$36 cm^3$
B	10cm by 7cm by 6cm	$420 cm^3$
C	6cm by 3cm by 2cm	$36 cm^3$
D	6cm by 3cm by 1cm	$18 cm^3$
E	10cm by 7cm by 2cm	$140 cm^3$
	____ by ____ by ____	
	____ by ____ by ____	

9. Prism D's volume is $\frac{1}{2}$ that of Prism A or C.
Show calculations below.

$36 cm^3 \div 2 = 18 cm^3$. Both Prisms A and C have the same volume of $36 cm^3$.

Prism D's volume of $18 cm^3$ is $\frac{1}{2}$ of Prism A or C's volume.

10. Prism E's volume is $\frac{1}{3}$ that of Prism B.
Show calculations below.

$420 cm^3 \div 3 = 140 cm^3$

Prism E's volume of $140 cm^3$ is $\frac{1}{3}$ of Prism B's volume.

11. Total volume of sculpture: $650 cm^3$
Show calculations below.

$36 + 420 = 456$ $456 + 36 = 492$ $492 + 18 = 510$ $510 + 140 = 650$

NOTES

Debrief Questions

- How was the student work you assessed similar to and different from the design you created?
- If the work that you assessed did not meet the requirements, what feedback did you provide to help the student be successful?
- How was assessing student work different from creating your design yesterday? If you could go back and change your design, would you? In what ways?
- Students might enjoy investigating the sculptures of David Smith, particularly his *Cubi* series. Many in this series of sculptures are composed exclusively of rectangular prisms.

Multiple Means of Engagement

The high number of measurements recorded on the diagram may be overwhelming to students with visual acuity difficulties. These students may benefit from a second diagram with figures slightly separated and units listed on each dimension or a larger print version of the diagram.

Multiple Means of Engagement

Have students who easily grasp this concept and move quickly through the Problem Set double one or more dimensions and calculate the new volume of the figure. Another option is to ask them if the units were centimeters instead of inches, how many liters of liquid the structure would hold.

YOUR NOTES

Topic C: Area of Rectangular Figures with Fractional Side Lengths

In Topic C, students extend their understanding of area as they use rulers and set squares to construct and measure rectangles with fractional side lengths and find their areas.

Lesson 10

Find the area of rectangles with whole-by-mixed and whole-by-fractional number side lengths by tiling, record by drawing, and relate to fraction multiplication.

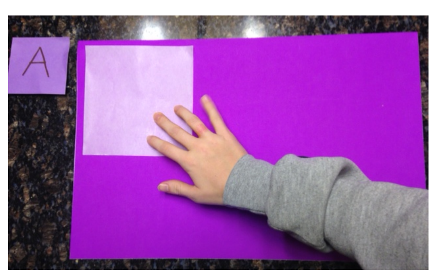
Note: Today's lesson uses the Problem Set. Solutions for each problem are included below.

Note: The lesson is written such that the length of one standard patty paper ($5\frac{1}{2}$ inches by $5\frac{1}{2}$ inches) is one unit. Hamburger patty paper (available from big box discount stores in boxes of 1,000) is the ideal square unit for this lesson due to its translucence and size. Measurements for the mystery rectangles are given in generic units so that any size square unit may be used to tile, as long as the tiling units can be folded. Any paper may be used if patty paper is not available. Consider color-coding Rectangles A–E for easy reference.

Materials: (T) 3-unit × 2-unit rectangle, patty paper (units for tiling), large chart paper (for recording dimensions of rectangles), (S) personal white board, 5 large mystery rectangles lettered A–E (1 of each size per group), patty paper (units for tiling), Problem Set (see Appendix)

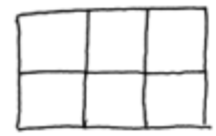
Preparation: Each group needs one copy of Rectangles A–E. The most efficient way of producing these rectangles is to use the patty paper to measure and trace the outer dimensions of one rectangle. Then, use that rectangle as a template to cut the number required for the class. Rectangles should measure as follows:

Demo Rectangle A: 3 units × 2 units
Rectangle B: 3 units × $2\frac{1}{2}$ units
Rectangle C: $1\frac{1}{2}$ units × 5 units
Rectangle D: 2 units × $1\frac{3}{4}$ units
Rectangle E: $\frac{3}{4}$ unit × 5 units



YOUR NOTES

T: We want to determine the areas of some mystery rectangles today. Find the rectangle at your table labeled A. (Allow students time to find the rectangle.)

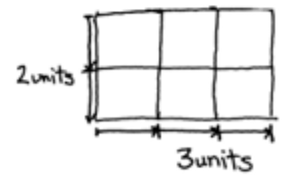


Rectangle A

T: If we want to find the area of this mystery rectangle, what kind of units would we use to measure it?

S: Square units.

T: (Hold up a patty paper tile.) This will be the square unit we will use to find the area of Rectangle A. Work with your partner to find the number of squares that will cover this rectangle with no space between the units and no overlaps. Please start at the top left-hand corner to place your first tile. (Allow students time to work.)



T: How many square units covered the rectangle?

S: 6 square units.

T: Let's sketch a picture of what our tiling looks like. Draw the outside of your rectangle first. (Model as students draw.)

T: Now, show the six tiles. (Allow students time to draw.)

T: Look at the longest side of your rectangle. If we wanted to measure this side with a piece of string, how many units long would the string need to be? Explain how you know to your partner.

S: It is 3 units long. I can look at the edge of the units and count. → To measure the length of the side, I'm not looking at the whole tile; I only need to count the length of each unit. There are 3 equal units on the edge, so the string would need to be 3 units long.

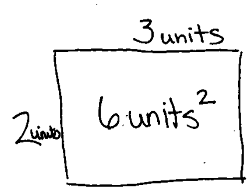
T: Let's record that. (Write the length of Rectangle A in the chart.) What is the length of the shorter side?

S: 2 units.

T: Let's record that in our chart.

T: What is the area of Rectangle A?

S: 6 square units.



T: If we had only labeled the length and the width in our sketch, could we still know the area? Why or why not?

Rectangle	Length	Width	Area
A	3 units	2 units	6 units^2
B	3 units	$2\frac{1}{2}$ units	$7\frac{1}{2}$ units^2

S: Yes. We know the square units are there even if we do not draw them all. → We still just multiply the sides together. We can imagine the tiles.

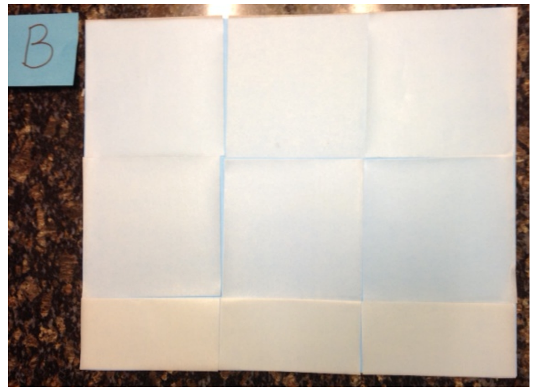
T: What would a sketch of this look like? Draw it with your partner. (Allow students time to draw.)

T: Now, find Rectangle B. Compare its size to Rectangle A. Will its area be greater than or less than that of Rectangle A?

S: Greater.

T: We see that Rectangles A and B are the same length. What about the width?

YOUR NOTES



S: Rectangle B is wider than two tiles but not as wide as three tiles.

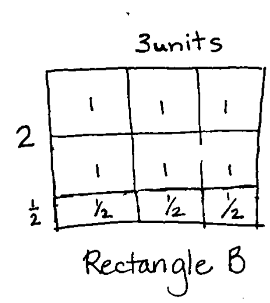
T: Fold your tiles to decide what fraction of another tile we need to cover the extra width. Work with your partner. (Allow students time to fold.)

T: What fraction of the tile do you need to cover this part of the rectangle? How do you know?

S: I need half a tile. I laid a whole tile over the extra part, and it looked like half to me. → After I folded up the part of the tile that was hanging off the rectangle, I could see that the fold split the tile into two equal parts. That means it is halves.

T: Finish folding enough tiles to completely cover the width of Rectangle B.

S: (Fold to cover the rectangle completely.)



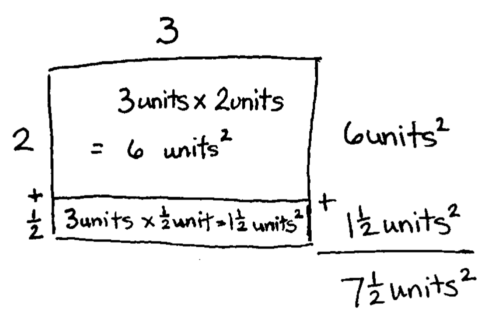
T: Let's record by sketching and filling in the blanks on the Problem Set. I will record in the chart. What is the length of Rectangle B?

S: 3 units. (Record on the Problem Set.)

T: What is the width?

S: $2\frac{1}{2}$ units.

T: What is the area? How do you know?



S: The area is $7\frac{1}{2}$ units squared. I counted all of the whole square units first and then added on the halves. → I knew it was at least 6 square units, and then we had 3 more halves, so that's $7\frac{1}{2}$ square units. → $3 \times 2\frac{1}{2} = 6\frac{3}{2} = 7\frac{1}{2}$.

T: When we record our tiling, is it necessary to sketch each tile? Why or why not?

S: No. We can just write down how many there are. → We can show just the side lengths of 3 and $2\frac{1}{2}$. I'll know that means three squares across and two and a half squares down. → It is like the area model with whole numbers. If I know the sides, I can show the total area by just multiplying.

T: Let's sketch this rectangle again but without the individual tiles. Draw the rectangle, and label the length. (Allow students time to draw.)

T: Now, let's decompose the $2\frac{1}{2}$ units on the width as $2 + \frac{1}{2}$. (Label and draw a horizontal line across the rectangle as pictured. Allow students time to draw.)

T: Let's record the first partial product. (Point.) Three units long by 2 units wide is what area?

S: 6 square units.

YOUR NOTES

T: Let's record the second partial product. (Point.) What is the length of this portion?

S: 3 units.

T: What is the width of this portion?

S: 1 half unit.

T: What is the area of this part? How do you know?

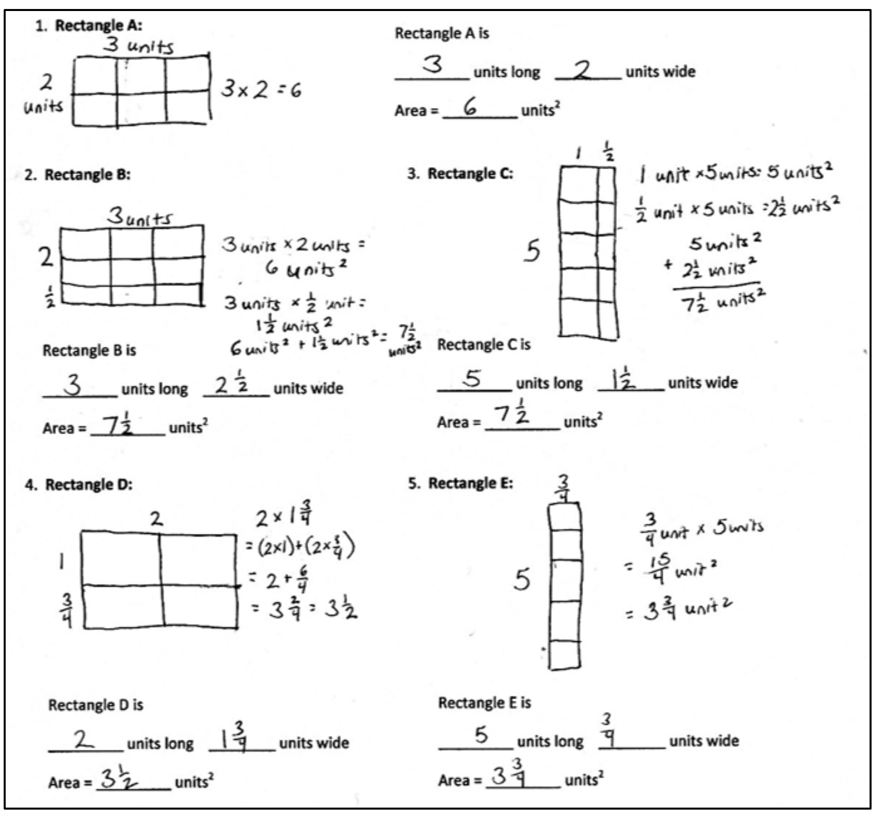
S: The area is $1\frac{1}{2}$ square units because 3 copies of $\frac{1}{2}$ is 3 halves. → 3 units long by $\frac{1}{2}$ unit wide is $1\frac{1}{2}$ square units.

T: Does this $7\frac{1}{2}$ unit squared area make sense given our prediction? Why or why not?

S: It does make sense. It is only a little wider than the first rectangle, and $7\frac{1}{2}$ is not that much more than 6. → You can see the first rectangle inside this one. There was a part that was 3 units by 2 units, and then a smaller part was added on that was 3 units by just half another unit. That is where the extra $1\frac{1}{2}$ square units come from. → Three times two was easy, and then I know that half of 3 is $1\frac{1}{2}$. By decomposing the mixed number, it was easy to find the total area.

T: Work with your partner to find the length, width, and area of Rectangles C, D, and E using the patty paper and recording with the area model. Record your findings on your Problem Set, and then answer the last two questions in the time remaining. You may record your tiling without drawing each tile if you wish.

S: (Work.)



1. Rectangle A:

3 units, 2 units, $3 \times 2 = 6$

Rectangle A is 3 units long 2 units wide

Area = 6 units²

2. Rectangle B:

3 units, 2, $\frac{1}{2}$

3 units × 2 units = 6 units²

3 units × $\frac{1}{2}$ unit = $1\frac{1}{2}$ units²

6 units² + $1\frac{1}{2}$ units² = $7\frac{1}{2}$ units²

Rectangle B is 3 units long $2\frac{1}{2}$ units wide

Area = $7\frac{1}{2}$ units²

3. Rectangle C:

1, $\frac{1}{2}$, 5

1 unit × 5 units = 5 units²

$\frac{1}{2}$ unit × 5 units = $2\frac{1}{2}$ units²

5 units² + $2\frac{1}{2}$ units² = $7\frac{1}{2}$ units²

Rectangle C is 5 units long $1\frac{1}{2}$ units wide

Area = $7\frac{1}{2}$ units²

4. Rectangle D:

2, 1, $\frac{3}{4}$

$2 \times 1\frac{3}{4}$

$= (2\times1)+(2\times\frac{3}{4})$

$= 2+\frac{6}{4}$

$= 3\frac{2}{4} = 3\frac{1}{2}$

Rectangle D is 2 units long $1\frac{3}{4}$ units wide

Area = $3\frac{1}{2}$ units²

5. Rectangle E:

$\frac{3}{4}$, 5

$\frac{3}{4}$ unit × 5 units

$= \frac{15}{4}$ unit²

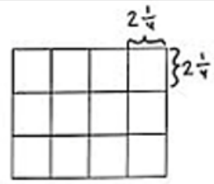
$= 3\frac{3}{4}$ unit²

Rectangle E is 5 units long $\frac{3}{4}$ units wide

Area = $3\frac{3}{4}$ units²

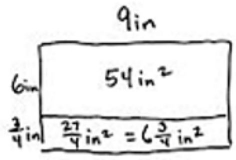
YOUR NOTES

6. The rectangle to the right is composed of squares that measure $2\frac{1}{4}$ inches on each side. What is its area in square inches? Explain your thinking using pictures and numbers.



l: $2\frac{1}{4}$ in × 4 = 9 in

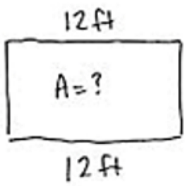
w: $2\frac{1}{4}$ in × 3 = $6\frac{3}{4}$ in



$A = 54 \text{ in}^2 + 6\frac{3}{4} \text{ in}^2$

$A = 60\frac{3}{4} \text{ in}^2$

7. A rectangle has a perimeter of $35\frac{1}{2}$ feet. If the length is 12 feet, what is the area of the rectangle?



Perimeter: $35\frac{1}{2}$ ft

$35\frac{1}{2} \text{ ft} - 24 \text{ ft} = 11\frac{1}{2} \text{ ft}$

$11\frac{1}{2} \text{ ft} \div 2 = \frac{23}{2} \text{ ft} \times \frac{1}{2} = \frac{23}{4} \text{ ft} = 5\frac{3}{4} \text{ ft}$

Area: $12 \text{ ft} \times 5\frac{3}{4} \text{ ft}$

$= 60 \text{ ft}^2 + \frac{12 \times 3}{4} \text{ ft}^2$

$= 60 \text{ ft}^2 + 9 \text{ ft}^2$

$= 69 \text{ ft}^2$

The area of the rectangle is 69 ft².

NOTES

Debrief Questions

- What relationship did you notice between the areas of Rectangle C and Rectangle E? What accounts for this relationship?
- How was Rectangle E different from the other rectangles you tiled? Describe how you tiled it.
- How did you determine the area of Rectangle E? Did you count the single units? Add repeatedly? Multiply the sides?
- Could you place these rectangles in order of greatest to least area by using relationships among the dimensions but without actually performing the calculations? Why or why not?
- How did you determine the area of the rectangle in Problem 6?
- Analyze and compare different solution strategies for Problem 7.

Multiple Means of Representation

Folding the square units allows students to clearly see the relationship of the fractional square unit while maintaining the relationship to the whole square unit. Consequently, if students become confused about the size of the fractional square unit, the paper may be easily unfolded as a reminder.

Multiple Means of Engagement

The spatial and visualization skills involved in Lessons 10 and 11 are quite natural for some students and quite challenging for others. Consequently, the time needed to accomplish the tasks varies, but all students should be given the opportunity to tile all the rectangles. Both lessons offer two challenging questions at the end of the Problem Sets for those who finish the tiling quickly.

Multiple Means of Action and Expression

Students may use the tiles to measure the outside dimensions of the rectangle before tiling. For some, marking the length and width with tick marks to show the lengths of the units may help them visualize the linear measurement more easily.

The dimensions can then be recorded on the Problem Set prior to drawing the rectangle and partial products.

YOUR NOTES

Lesson 11

Find the area of rectangles with mixed-by-mixed and fraction-by-fraction side lengths by tiling, record by drawing, and relate to fraction multiplication.

Note: Today's lesson uses the Problem Set. Solutions for each problem are included below.

Note: Today's lesson parallels the structure of Lesson 10. Rectangles for each group should be prepared in advance following yesterday's instructions.

Materials: (T) Rectangles, patty paper units for tiling, personal white board (S) 1 demonstration and 5 mystery rectangles lettered A–E (1 of each size per group), patty paper units for tiling, Problem Set (see Appendix)

The dimensions of today's rectangles are given below.

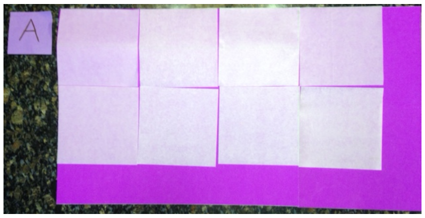
Rectangle A: $4\frac{1}{2}$ units × $2\frac{1}{2}$ units

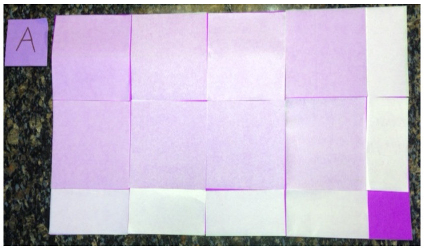
Rectangle B: $1\frac{3}{4}$ units × $3\frac{3}{4}$ units

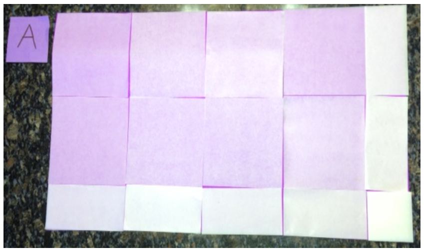
Rectangle C: $\frac{3}{4}$ unit × $1\frac{1}{2}$ units

Rectangle D: $\frac{3}{4}$ unit × $\frac{1}{2}$ unit

Rectangle E: $\frac{2}{3}$ unit × $\frac{2}{3}$ unit







The added complexities of today's lesson involve the inclusion of two mixed number or fractional side lengths. This is an application of the fraction multiplication lessons of Mission 4. Students record partial products rather than draw individual tiles.

T: Let's start with Rectangle A. Work with your partner to place as many whole tiles on Rectangle A as you can. Remember to start at the top left corner of the rectangle.

S: (Place whole tiles on Rectangle A.)

T: How many whole tiles fit?

YOUR NOTES

S: 8.

T: Is this the area of the rectangle?

S: No.

T: Fold some of your square units to cover the rest of the rectangle's length. (Allow time for students to work.)

T: What fractional unit do you need to do this? How many?

S: I needed 2 half units. → The unit was halves. I needed 2 of them.

T: Now, fold some units to cover the rest of the rectangle's width. (Allow time for students to work.)

T: What fractional unit did you use this time, and how many?

S: I needed halves again. This time, it was 4. → It was 4 half units.

T: I see that we have covered almost all of the rectangle, but there seems to be a part at the bottom that is even smaller than the halves we just placed. How can we find the fractional unit that will fit here? Turn and talk.

S: I can see that if I fold a square unit in half, it fits in one direction, but it is too long in the other direction. Maybe if I fold it again, it will fit. → If I fold it in half, it fits the length. Then, if I fold that half in half again, it fits perfectly in the space. → The part is half the size of half a square unit. Half of a half is 1 fourth of a square unit.

T: Unfold the paper that you have made to fit in this part. What fraction of a whole square unit covers this part?

S: 1 fourth of a square unit.

T: Work with your partner to count the tiles to determine the area.

S: (Count the tiles with partners.)

T: What is the area? How did you count it?

S: I counted the 8 squares first. I added 6 halves or 3 more squares. Then, I added $\frac{1}{4}$ to 11. That's $11\frac{1}{4}$ square units. → I could see 2 rows of $4\frac{1}{2}$ units. That is 9. Then, there were 4 halves and $\frac{1}{4}$ in a row. That is $2\frac{1}{4} + 9 = 11\frac{1}{4}$. The area is $11\frac{1}{4}$ square units.

T: Let's record our thinking. I will work on the board. You record on your Problem Set. Sketch the rectangle first. Decompose the length and width into ones and fractional parts.

S: (Sketch and decompose the length and width.)

T: How did you decompose the length?

S: $4 + \frac{1}{2}$.

T: (Record in the algorithm.) The width?

S: $2 + \frac{1}{2}$.

T: (Record in the algorithm.) Let's use multiplication to confirm the area we found with counting. Let's start with the ones. (Point, and then record each partial product in the rectangle and in the algorithm.) 2 units × 4 units equals ...?

YOUR NOTES

S: 8 square units.

T: (Point and record.) 2 units × $\frac{1}{2}$ unit equals…?

S: 2 half square units. → 1 square unit.

T: (Point and record.) $\frac{1}{2}$ unit × 4 units equals…?

S: 4 half square units. → 2 square units.

T: (Point and record.) $\frac{1}{2}$ unit × $\frac{1}{2}$ unit equals…?

S: $\frac{1}{4}$ square unit.

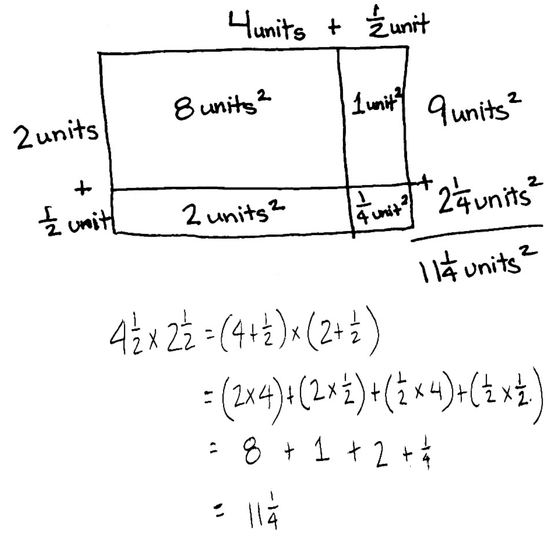
T: Find the sum.

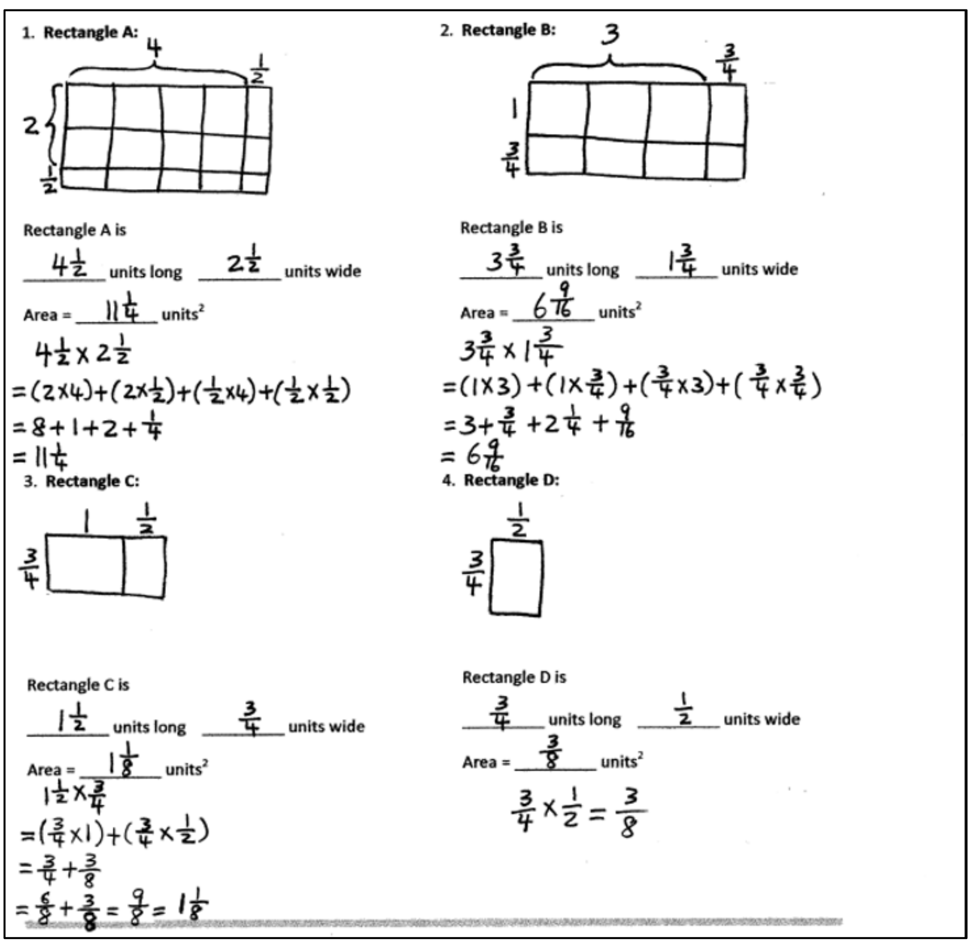
S: (Work to find 11$\frac{1}{4}$ units2.)

T: Was the area the same using multiplication and the area model?

S: Yes!

T: Use your tiles to determine the area and dimensions of the other rectangles. Record your findings on your Problem Set. Then, multiply to confirm the area.





1. Rectangle A:

Rectangle A is $4\frac{1}{2}$ units long $2\frac{1}{2}$ units wide

Area = $11\frac{1}{4}$ units²

$4\frac{1}{2} \times 2\frac{1}{2}$

$= (2\times4)+(2\times\frac{1}{2})+(\frac{1}{2}\times4)+(\frac{1}{2}\times\frac{1}{2})$

$= 8+1+2+\frac{1}{4}$

$= 11\frac{1}{4}$

2. Rectangle B:

Rectangle B is $3\frac{3}{4}$ units long $1\frac{3}{4}$ units wide

Area = $6\frac{9}{16}$ units²

$3\frac{3}{4} \times 1\frac{3}{4}$

$= (1\times3)+(1\times\frac{3}{4})+(\frac{3}{4}\times3)+(\frac{3}{4}\times\frac{3}{4})$

$= 3+\frac{3}{4}+2\frac{1}{4}+\frac{9}{16}$

$= 6\frac{9}{16}$

3. Rectangle C:

Rectangle C is $1\frac{1}{2}$ units long $\frac{3}{4}$ units wide

Area = $1\frac{1}{8}$ units²

$1\frac{1}{2}\times\frac{3}{4}$

$= (\frac{3}{4}\times1)+(\frac{3}{4}\times\frac{1}{2})$

$= \frac{3}{4}+\frac{3}{8}$

$= \frac{6}{8}+\frac{3}{8} = \frac{9}{8} = 1\frac{1}{8}$

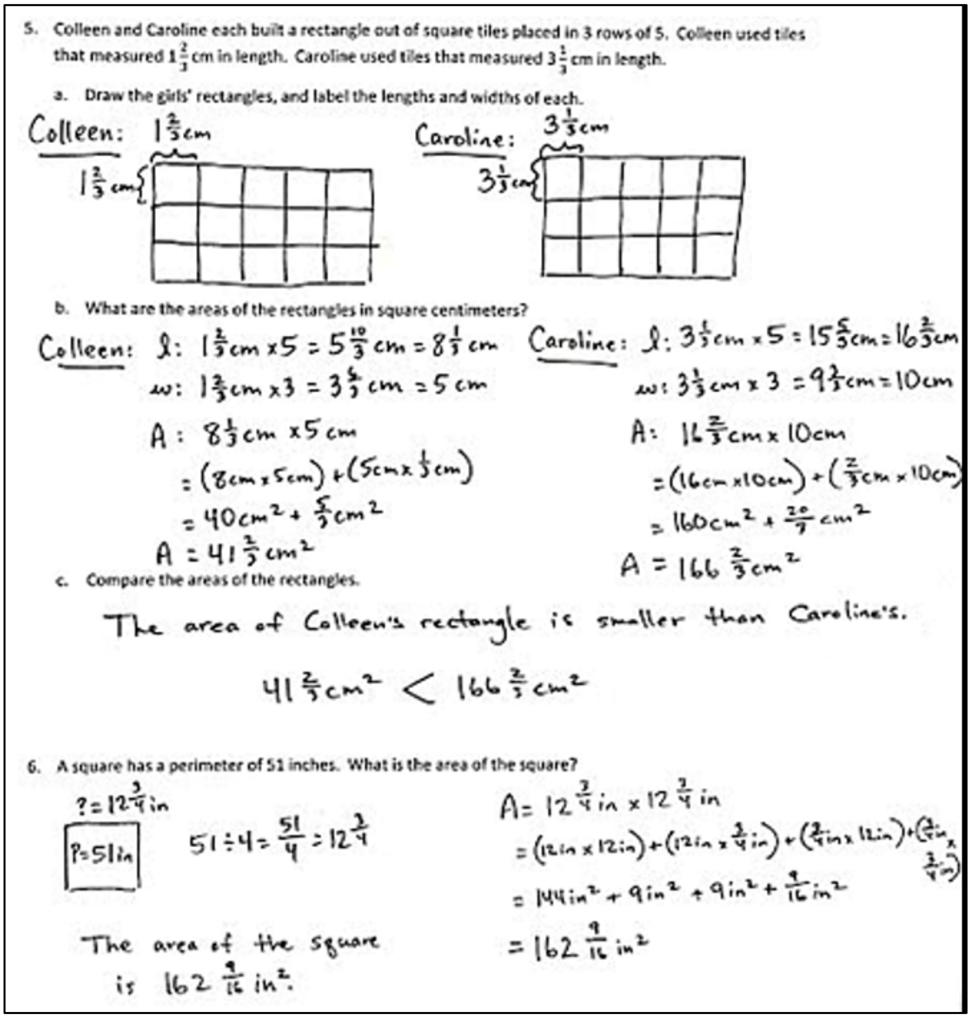
4. Rectangle D:

Rectangle D is $\frac{3}{4}$ units long $\frac{1}{2}$ units wide

Area = $\frac{3}{8}$ units²

$\frac{3}{4}\times\frac{1}{2} = \frac{3}{8}$

YOUR NOTES



5. Colleen and Caroline each built a rectangle out of square tiles placed in 3 rows of 5. Colleen used tiles that measured $1\frac{2}{3}$ cm in length. Caroline used tiles that measured $3\frac{1}{3}$ cm in length.

a. Draw the girls' rectangles, and label the lengths and widths of each.

b. What are the areas of the rectangles in square centimeters?

Colleen:
l: $1\frac{2}{3}\text{ cm} \times 5 = 5\frac{10}{3}\text{ cm} = 8\frac{1}{3}\text{ cm}$
w: $1\frac{2}{3}\text{ cm} \times 3 = 3\frac{6}{3}\text{ cm} = 5\text{ cm}$
A: $8\frac{1}{3}\text{ cm} \times 5\text{ cm}$
$= (8\text{ cm} \times 5\text{ cm}) + (5\text{ cm} \times \frac{1}{3}\text{ cm})$
$= 40\text{ cm}^2 + \frac{5}{3}\text{ cm}^2$
$A = 41\frac{2}{3}\text{ cm}^2$

Caroline:
l: $3\frac{1}{3}\text{ cm} \times 5 = 15\frac{5}{3}\text{ cm} = 16\frac{2}{3}\text{ cm}$
w: $3\frac{1}{3}\text{ cm} \times 3 = 9\frac{3}{3}\text{ cm} = 10\text{ cm}$
A: $16\frac{2}{3}\text{ cm} \times 10\text{ cm}$
$= (16\text{ cm} \times 10\text{ cm}) + (\frac{2}{3}\text{ cm} \times 10\text{ cm})$
$= 160\text{ cm}^2 + \frac{20}{3}\text{ cm}^2$
$A = 166\frac{2}{3}\text{ cm}^2$

c. Compare the areas of the rectangles.

The area of Colleen's rectangle is smaller than Caroline's.

$41\frac{2}{3}\text{ cm}^2 < 166\frac{2}{3}\text{ cm}^2$

6. A square has a perimeter of 51 inches. What is the area of the square?

$? = 12\frac{3}{4}$ in

P = 51 in

$51 \div 4 = \frac{51}{4} = 12\frac{3}{4}$

$A = 12\frac{3}{4}\text{ in} \times 12\frac{3}{4}\text{ in}$
$= (12\text{ in} \times 12\text{ in}) + (12\text{ in} \times \frac{3}{4}\text{ in}) + (\frac{3}{4}\text{ in} \times 12\text{ in}) + (\frac{3}{4}\text{ in} \times \frac{3}{4}\text{ in})$
$= 144\text{ in}^2 + 9\text{ in}^2 + 9\text{ in}^2 + \frac{9}{16}\text{ in}^2$
$= 162\frac{9}{16}\text{ in}^2$

The area of the square is $162\frac{9}{16}\text{ in}^2$.

NOTES

Debrief Questions

- Compare the rectangles we tiled today to the rectangles we tiled yesterday. What do you notice? How did that change the way we had to tile?
- Which rectangle was the easiest to tile? Which was the hardest? Why?
- Explain your strategy for tiling Rectangle D (and Rectangle E, where applicable). How was finding the area of this rectangle similar to the fraction multiplication we did in Mission 4? How was it different?
- Explain your strategy for finding the areas of the rectangles in Problem 5 and how you compared them.
- How is Problem 6 in today's Problem Set like Problem 7 in yesterday's Problem Set (Lesson 10), and how is it different? Yesterday's problem read: A rectangle has a perimeter of $35\frac{1}{2}$ feet. If the length is 12 feet, what is the area of the rectangle?

Multiple Means of Engagement

Include Rectangle E as an optional challenge. Folding and tiling Rectangle E requires students to fold thirds and to reason about another area less than 1 square tile. Recording for Rectangle E should be done on a separate piece of paper, as it is not included on the Problem Set.

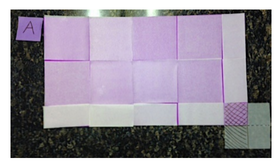
The last two problems on the Problem Set also offer extensions for students who finish the tiling and multiplication quickly.

Multiple Means of Representation

Please note that the algorithm is provided so that students are exposed to a more formal representation of the distribution. However, students should not be required to be as formal in their own calculations. Using an area model to keep track of students' thinking is sufficient.

Multiple Means of Action and Expression

For some students, it may be more effective to place a whole square unit over the last corner of the rectangle and then trace the outline or shade the corner of the rectangle on the patty paper. (Because the patty paper is translucent, the edge of the rectangle is clearly visible.) Students may then fold until only the outlined portion of the paper is visible. When the paper is unfolded, only 1 of the 4 equal parts is bordered (or shaded).



Also, guide students to isolate the last corner of the rectangle and use a single piece of patty paper to model the multiplication of a fraction by a fraction to produce a double-shaded area (as in Mission 4). This double-shaded portion can then be laid on top of the rectangle's corner, fitting perfectly.

YOUR NOTES

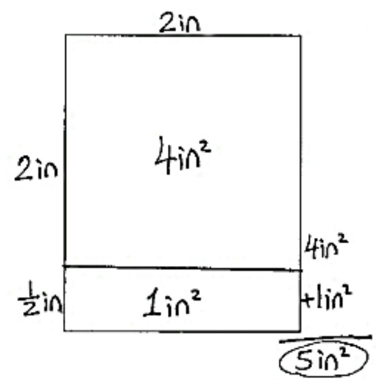
Lesson 12

Measure to find the area of rectangles with fractional side lengths.

Note: Today's lesson uses the Problem Set. Solutions for each problem are included below.

Materials: (T) Ruler, projector (S) Ruler, Problem Set (see Appendix)

Problem 1(a)



Project the first rectangle in the Problem Set.

T: We will find the areas of more mystery rectangles today. What was the relationship between the areas we found using square tiles and the areas we found using multiplication?

S: We got the same answers. → Tiling or finding partial products using multiplication will always give the same area because the rectangle we are using is the same.

T: Today, we will use a ruler to help us find area. Turn and talk to your partner about how you think a ruler might be useful in finding the area of a rectangle.

S: It's not square units, but we can measure the edges. → The ruler lets us measure the sides to find out the lengths we need to multiply.

T: Work with your partner to measure in inches the lengths of the first rectangle of the Problem Set. Compare your measurements.

S: (Measure the first rectangle.)

T: What are the lengths of the sides?

S: 2 inches and $2\frac{1}{2}$ inches.

T: Estimate the area of this rectangle. Turn and talk.

S: If this was just a 2-inch square, the area would be 4 square inches. It's a little longer than that, so it will be a little more than 4. → The longer side is between 2 and 3 inches, so the area should be somewhere between 4 square inches and 6 square inches.

T: Let's find the actual area. Decompose the longer side by marking the end of the 2 whole inches and labeling the wholes and the half inch on our rectangles. (Model on the board as shown.)

S: (Decompose and label.)

T: Now, let's use this decomposition to find the area of smaller parts of the rectangle. Using your ruler, draw a line separating the ones from the fractional units. (Model.)

S: (Separate the ones with a line.)

YOUR NOTES

T: Now, let's multiply to find the areas of these sections. Let's start with the ones by ones part. Talk with your partner. What is the area of the part that is 2 inches by 2 inches? If it helps, imagine or draw tiles in your rectangle.

S: There are 2 going across and 2 rows of them, so 4 altogether. → I remember that I can multiply the sides, so 2 inches × 2 inches is 4 square inches.

T: What is the area?

S: 4 square inches.

T: Record that.

T: Turn and talk. What is the area of the smaller part? How do you know?

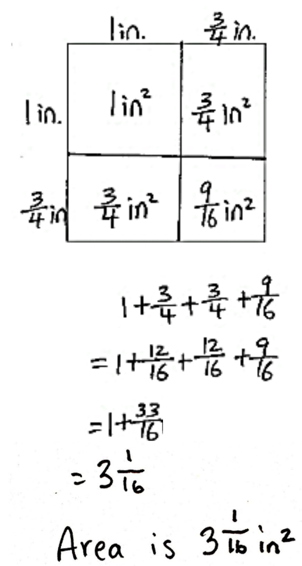
S: Half of 2, so 1. → Two times $\frac{1}{2}$. Two halves make 1, so 1. → 1 square inch.

T: Yes, the area is 1 square inch. Let's write that, too. (Model as shown in the image on the previous page.)

T: What is the total area of the rectangle? Does our answer make sense?

S: 5 square inches. → It makes sense because we said the area should be between 4 and 6 square inches, and it is.

Problem 1(b)



Project the second rectangle in the Problem Set.

T: Measure the next rectangle with your ruler.

T: What is the length?

S: $1\frac{3}{4}$ inches.

T: And the width?

S: $1\frac{3}{4}$ inches. → This is a square, so the width is also $1\frac{3}{4}$ inches.

T: Estimate the area with your partner.

S: It's almost 2 inches by 2 inches. The area should be less than 4 square inches. → The area will be between 1 square inch and 4 square inches but closer to 4 because the sides are almost 2 inches long.

T: Decompose the sides into ones and fractional parts, and record that on your Problem Set.

Circulate and assist students. Then, project a student's work, or record on the board as shown.

T: Work with your partner to find the area of each of these four parts.

S: (Find the area of each of the four parts.)

YOUR NOTES

T: What is the area of the part that is 1 inch on each side?

S: 1 square inch.

T: Then, we have two parts with 1 inch on one side and $\frac{3}{4}$ inch on the other. What is the area of each of those parts? How do you know?

S: It is not a whole square inch. → A whole tile would not fit in either of these places. We would have to fold it to make it fit. → Three-fourths of a square inch because 3 fourths times 1 is 3 fourths.

T: (Record the measurements in each part of the area model.) Now we are left with the last little square. It is $\frac{3}{4}$ of an inch on each side. Is this area greater or less than the other parts? How do you know?

S: It is smaller because both sides are shorter than the other parts. → It is only part of an inch on each side, so it will be less area. → The area is a fraction of a fraction. We want 3 fourths of 3 fourths. It is a fraction of an inch on each side. Three-fourths of a square inch would be like splitting a whole into 4 parts and taking 1 part off.

T: What do we need to do to find the area of this last section of our square?

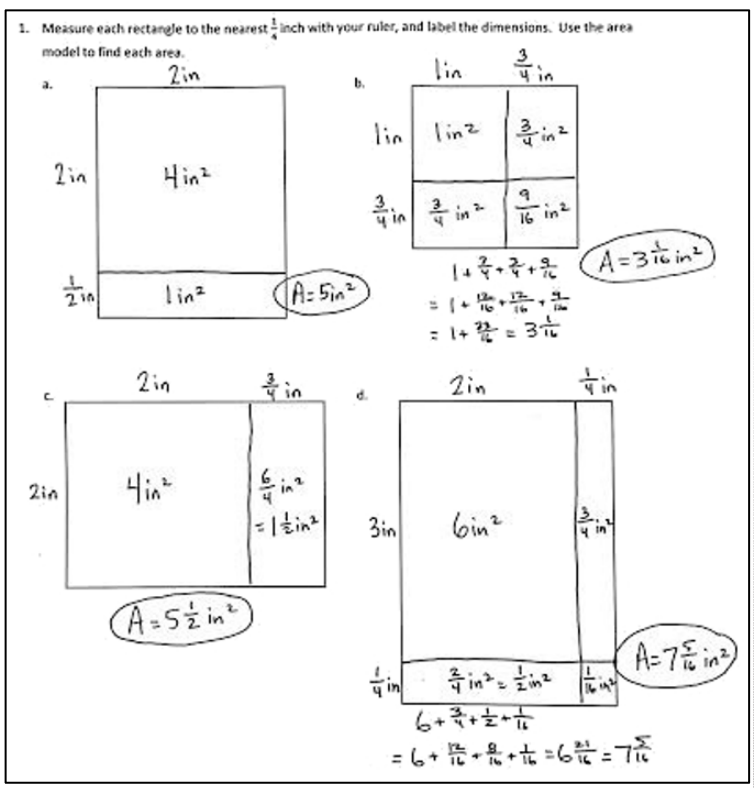
S: Just like before, we need to multiply the length times the width. → We need to multiply $\frac{3}{4}$ by $\frac{3}{4}$.

T: What is the area of the small square?

S: $\frac{9}{16}$ square inch.

T: How will we find the total area?

S: Add all the parts. → Add across each row, and then add the rows together.



Circulate and support students as they add the partial products. Review the need for common denominators as necessary.

T: What is the total area of the square?

S: $3\frac{1}{16}$ square inches!

Repeat this sequence of questioning with each problem as necessary. As students understand the concept, release them to work independently.

YOUR NOTES

e.

$\frac{3}{4}$ in; 2 in; $\frac{1}{2}$ in; $\frac{6}{4}$ in²; $\frac{3}{8}$ in²

$\frac{6}{4}+\frac{3}{8}$
$=\frac{12}{8}+\frac{3}{8}=\frac{15}{8}$
$=1\frac{7}{8}$

$A=1\frac{7}{8}$ in²

f.

3 in; $\frac{3}{4}$ in; $\frac{1}{2}$ in; $\frac{3}{2}$ in²; $\frac{3}{8}$ in²

$\frac{3}{2}+\frac{3}{8}$
$=\frac{12}{8}+\frac{3}{8}=\frac{15}{8}$
$=1\frac{7}{8}$

$A=1\frac{7}{8}$ in²

2. Find the area of rectangles with the following dimensions. Explain your thinking using the area model.

a. $1 \text{ ft} \times 1\frac{1}{2}$ ft

1 ft; 1 ft; $\frac{1}{2}$ ft; 1 ft²; $\frac{1}{2}$ ft²

$A=1\frac{1}{2}$ ft²

b. $1\frac{1}{2} \text{ yd} \times 1\frac{1}{2}$ yd

1 yd; $\frac{1}{2}$ yd; 1 yd; $\frac{1}{2}$ yd; 1 yd²; $\frac{1}{2}$ yd²; $\frac{1}{2}$ yd²; $\frac{1}{4}$ yd²

$1+\frac{1}{2}+\frac{1}{2}+\frac{1}{4}=2\frac{1}{4}$

$A=2\frac{1}{4}$ yd²

c. $2\frac{1}{2} \text{ yd} \times 1\frac{3}{16}$ yd

2 yd; $\frac{1}{2}$ yd; 1 yd; $\frac{3}{16}$ yd; 2 yd²; $\frac{1}{2}$ yd²; $\frac{6}{16}$ yd²; $\frac{3}{32}$ yd²

$2+\frac{1}{2}+\frac{6}{16}+\frac{3}{32}$
$=2+\frac{16}{32}+\frac{12}{32}+\frac{3}{32}$
$=2+\frac{31}{32}=2\frac{31}{32}$

$A=2\frac{31}{32}$ yd²

3. Hanley is putting carpet in her house. She wants to carpet her living room, which measures $15 \text{ ft} \times 12\frac{1}{3}$ ft. She also wants to carpet her dining room, which is $10\frac{1}{4} \text{ ft} \times 10\frac{1}{3}$ ft. How many square feet of carpet will she need to cover both rooms?

living rm:
$15 \times 12\frac{1}{3}$
$=180\frac{15}{3}$
$=185$
$A=185$ ft²

dining rm:
$10\frac{1}{4} \times 10\frac{1}{3}$
$=(10\times10)+(\frac{1}{4}\times10)+(10\times\frac{1}{3})+(\frac{1}{4}\times\frac{1}{3})$
$=100+\frac{10}{4}+\frac{10}{3}+\frac{1}{12}$
$=100+\frac{30}{12}+\frac{40}{12}+\frac{1}{12}$
$=100+\frac{71}{12}=105\frac{11}{12}$
$A=105\frac{11}{12}$ ft²

$185+105\frac{11}{12}$
$=290\frac{11}{12}$

She will need $290\frac{11}{12}$ ft² to cover both rooms.

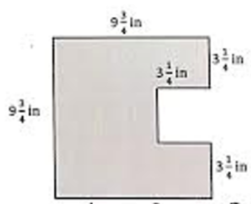
4. Fred cut a $9\frac{3}{4}$-inch square of construction paper for an art project. He cut a square from the edge of the big rectangle whose sides measured $3\frac{1}{4}$ inches. (See the picture below.)

a. What is the area of the smaller square that Fred cut out?

$A=l\times w$
$3\frac{1}{4}\times3\frac{1}{4}$
$=(3\times3)+(\frac{1}{4}\times3)+(3\times\frac{1}{4})+(\frac{1}{4}\times\frac{1}{4})$
$=9+\frac{3}{4}+\frac{3}{4}+\frac{1}{16}$
$=9+\frac{12}{16}+\frac{12}{16}+\frac{1}{16}=9+\frac{25}{16}=10\frac{9}{16}$

$A=10\frac{9}{16}$ in²

b. What is the area of the remaining paper?



$9\frac{3}{4}\times9\frac{3}{4}$
$=(9\times9)+(\frac{3}{4}\times9)+(9\times\frac{3}{4})+(\frac{3}{4}\times\frac{3}{4})$
$=81+\frac{27}{4}+\frac{27}{4}+\frac{9}{16}$
$=81+6\frac{3}{4}+6\frac{3}{4}+\frac{9}{16}$
$=93+\frac{12}{16}+\frac{12}{16}+\frac{9}{16}=93+\frac{33}{16}=95\frac{1}{16}$

$95\frac{1}{16}-10\frac{9}{16}=94\frac{17}{16}-10\frac{9}{16}$
$=84\frac{8}{16}=84\frac{1}{2}$

The area of the remaining paper is $84\frac{1}{2}$ in².

NOTES

Debrief Questions

- Look back at the area model we did together in Problem 1(b) ($1\frac{3}{4} \times 1\frac{3}{4}$). How many squares do you see in your area model? What patterns do you see whenever you have an area model of a square?
- What is the relationship between Problem 1(e) and Problem 1(f) in the Problem Set? (Both rectangles have the same area. The length of Problem 1(f) is 5 times the length of Problem 1(e). The width of Problem 1(f) is one-fifth the width of Problem 1(e).)
- Using mental math, how can you find $\frac{1}{2}$ times any fraction? (Double the denominator.)
- How is Problem 2(b) like the example we did together, $1\frac{3}{4} \times 1\frac{3}{4}$? (Both have two factors that are the same.)

Multiple Means of Representation

Some students will benefit from drawing each square inch as a tile, connecting back to the tiling process. Others may need to use inch tile manipulatives to understand this process. (Remember that concrete materials should be foldable.)

Encourage students to return to pictorial or concrete representations as needed during any lesson to scaffold understanding.

Multiple Means of Engagement

For students who need to review fraction multiplication, model the shaded area models from Mission 4 to show a fraction times a fraction or a fraction of a fraction.

YOUR NOTES

Lesson 13

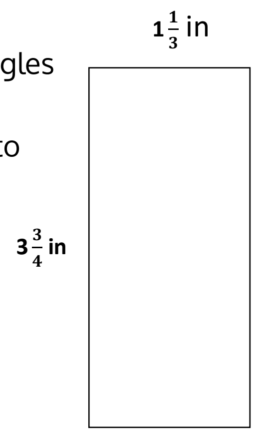
Multiply mixed number factors, and relate to the distributive property and the area model

Materials: (S) Personal white board

In this lesson, students reason about the most efficient strategy to use for multiplying mixed numbers: distributing with the area model or multiplying improper fractions and canceling to simplify.

Problem 1

Find the area of a rectangle $1\frac{1}{3}$ inches × $3\frac{3}{4}$ inches, and discuss strategies for solving.



T: (Project Rectangle 1.) How is this rectangle different from the rectangles we have been working with?

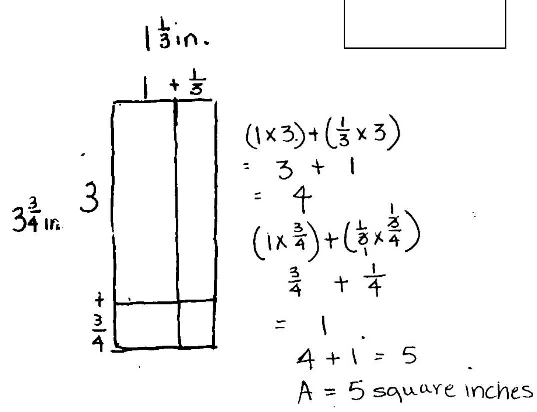
S: We know the dimensions of this one. → The side lengths are given to us, so we don't need to tile or measure.

T: Find the area of this rectangle. Use an area model to show your thinking.

S: (Find the area using a model.)

T: What is the area of this rectangle?

S: 5 inches squared.



T: We have used the area model many times in Grade 5 to help us multiply numbers with mixed units. How are these side lengths like multi-digit numbers? Turn and talk.

S: A two-digit number has two different size units in it. The ones are the smaller units, and the tens are the bigger units. These mixed numbers are like that. The ones are the bigger units, and the fractions are the smaller units. → Mixed numbers are another way to write decimals. Decimals have ones and fractions, and so do these.

T: (Point to the model and calculations.) When we add partial products, what property of multiplication are we using?

S: The distributive property.

YOUR NOTES

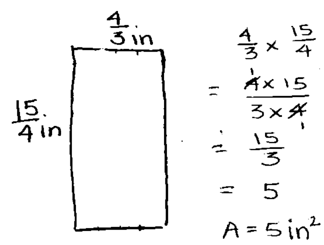
T: Let's find the area of this rectangle again. This time, let's use a single unit to express each of the side lengths. What is $1\frac{1}{3}$ expressed in thirds?

S: 4 thirds.

T: (Record on the rectangle.) Express $3\frac{3}{4}$ using only fourths.

S: 15 fourths.

T: (Record on the rectangle.) Multiply these fractions to find the area.



S: (Multiply to find the area.)

T: What is the area?

S: 5 in^2.

T: Which strategy did you find to be more efficient? Why?

S: I like the strategy of expressing the dimensions as a fraction greater than one. This way was a lot faster for me! → These fractions were easy to simplify before I multiplied, so there were fewer calculations to do to find the area.

T: Do you think it will always be true that multiplying the fractions will be the most efficient? Why or why not?

S: This seems easier because it's multiplying whole numbers. → I like the distributive property better because the numbers stay smaller doing one part at a time. → I'm not sure—some larger mixed numbers might be a lot more challenging.

T: There are lots of different viewpoints here. Let's try another example to test these strategies again.

OPTIONAL FOR FLEX DAY: PROBLEM 2

Problem 2

Determine when the distributive property or the multiplication of fractions is more efficient to solve for area.

T: (Draw a rectangle with side lengths $16\frac{1}{2}$ inches and $4\frac{1}{4}$ inches.) Which strategy do you think might be more efficient to find the area of this rectangle? Turn and talk.

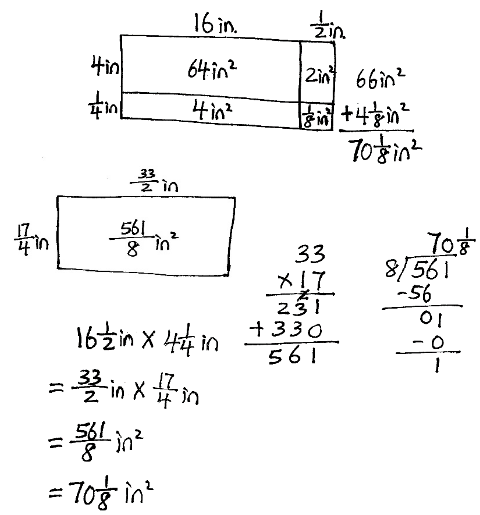
S: The fractions are pretty easy, so I think the distributive property will be quicker. → The numerators will be big. I think distributing will be easier. → I like to simplify fractions, so I think it would be easier to work with improper fractions.

T: Work with a partner to find the area of this rectangle. Partner A, use the distributive property with an area model. Partner B, express the sides using fractions greater than 1. (Allow students time to work.)

YOUR NOTES

T: What is the area? Which strategy was more efficient?

S: The improper fractions were messy. When I converted to improper fractions, the numerators were 33 and 17, and there weren't any common factors to help me simplify. The area is $\frac{561}{8}$ in^2, which is right, but it's weird. I had to use long division to figure out that the area was $70\frac{1}{8}$ square inches. → The distributive property was much easier on this one. The partial products were all easy to do in my head. I just added the sums of the rows and got $70\frac{1}{8}$ square inches.



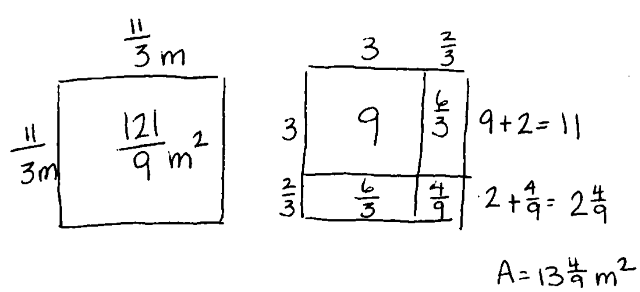
T: Does the method that you choose matter? Why or why not? Turn and talk.

S: Either way, we got the right answer. → Depending on the numbers, sometimes distributing is easier, and sometimes just multiplying the improper fractions is easier.

Repeat the process to find the area of a square with side length $3\frac{2}{3}$ m.

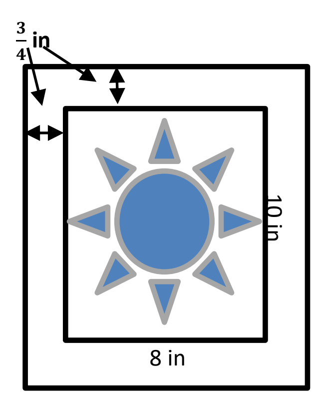
T: When should you use each strategy? Talk to your partner.

S: If the numbers are small, fraction multiplication might be better, especially if some factors can be simplified. → For large mixed numbers, I think the area model is easier, especially if some of the partial products are whole numbers or have common denominators. → You can always start with one strategy and change to the other if it gets too hard.



Problem 3

An 8-inch by 10-inch picture is resting on a mat. Three-fourths inch of the mat shows around the entire edge of the picture. Find the area of the mat not covered by the picture.



T: Compare this problem to others we have done. Turn and talk.

S: There are two rectangles to think about here. → We have to think about how to get just the part that is the mat—not the area of the whole thing. → It is a little bit of a mystery

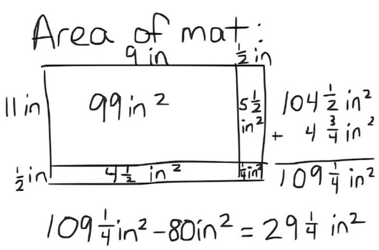
YOUR NOTES

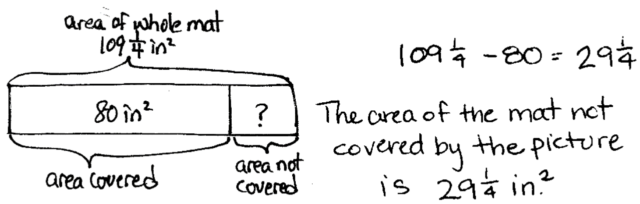
rectangle because they are asking about the mat. They gave us the measurements of the picture and only what we see of the mat.

T: Work with your partner, and use RDW to solve. (Allow students time to work.)

T: What did you think about to solve this problem?

S: I started by imagining the mat without the picture on top. I added the extra part of the mat ($1\frac{1}{2}$ inches) to the picture to find the length and width of the mat. Then, I multiplied and found the area of the mat. I subtracted the picture's area from the mat and got the answer. → I started to use improper fractions, but the numbers were really large, so I used the area model. → I used the area model for the mat's area because I saw the measurements were going to have fractions. Then, I just multiplied 8 × 10 to find the area of the picture. → After I figured out the area of the mat, I drew a tape diagram to show the part I knew and the part I needed to find. → I visualized 4 rectangles and then added their areas.





NOTES

Debrief Questions

- What are the strategies that we have used to find the area of a rectangle? Which one do you find the easiest? The most difficult? How do you decide which strategy you will use for a given problem? What kinds of things do you think about when deciding?
- When did you use the distributive property, and when did you multiply improper fractions? Why did you make those choices?
- What are some situations in real life where finding the area of something would be needed or useful?

Multiple Means of Engagement

Some students may need a quick refresher on changing mixed numbers to improper fractions or vice versa. Students should be reminded that a mixed number is an addition sentence. So, when converting to an improper fraction, the whole number can be expressed in the unit of the fractional part and then both like fractions added.

YOUR NOTES

Lesson 14

Solve real-world problems involving area of figures with fractional side lengths using visual models and/or equations.

Note: Today's lesson uses the Problem Set. Solutions for each problem are included below.

Note: The problems in today's lesson can be time intensive. It may be that only two or three problems can be solved in the time allowed. Students will approach representing these problems from many perspectives. Allow students the flexibility to use the approach that makes the most sense to them.

Materials: (S) Problem Set (see Appendix)

Suggested Delivery of Instruction for Solving Lesson 14's Word Problems

1. Model the problem.

Have two pairs of students who can successfully model the problem work at the board while the others work independently or in pairs at their seats. Review the following questions before beginning the first problem:

- Can you draw something? This may or may not be a tape diagram today. An area model may be more appropriate.
- What can you draw?
- What conclusions can you make from your drawing?

As students work, circulate. Reiterate the questions above. After two minutes, have the two pairs of students share only their labeled diagrams. For about one minute, have the demonstrating students receive and respond to feedback and questions from their peers.

2. Calculate to solve and write a statement.

Give everyone two minutes to finish working on that question, sharing his work and thinking with a peer. All students should write their equations and statements of the answer.

YOUR NOTES

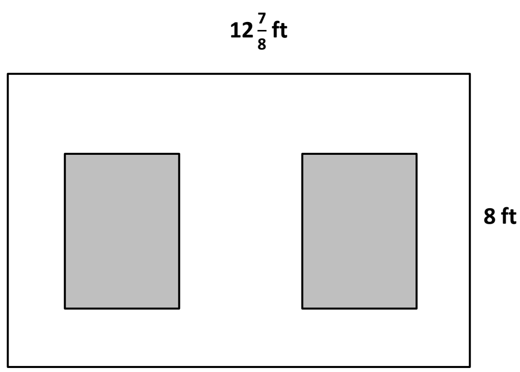
3. Assess the solution for reasonableness.

Give students one to two minutes to assess and explain the reasonableness of their solutions.

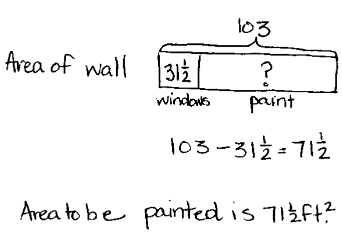
OPTIONAL FOR FLEX DAY: PROBLEM 1

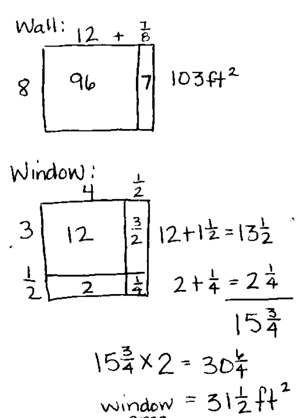
Problem 1

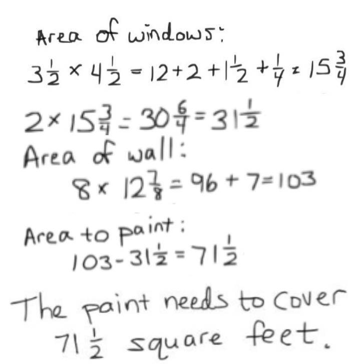
George decided to paint a wall with two windows. Both windows are $3\frac{1}{2}$-ft by $4\frac{1}{2}$-ft rectangles. Find the area the paint needs to cover.



Students must keep track of three different areas to solve Problem 1. Using a part–whole tape diagram to represent these areas may be helpful to some students, while others may find using the area model to be more helpful. Students have choices in the strategy for computing the areas as well. Some may choose to use the distributive property. Others may choose to multiply improper fractions. Once students have solved, ask them to justify their choice of strategy. Were they able to tell which strategy to use from the beginning? Did they change direction once they began? If so, why? Flexibility in thinking about these types of problems should be a focus.





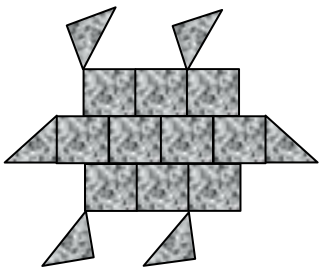


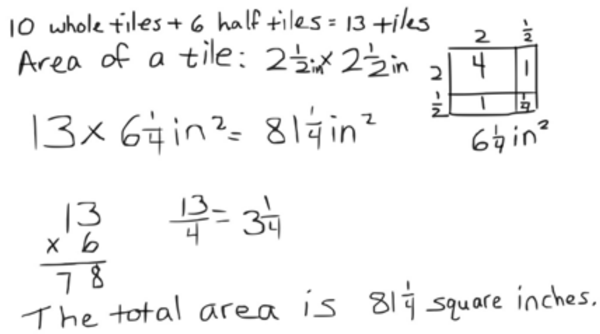
YOUR NOTES

Problem 2

Joe uses square tiles, some of which he cuts in half, to make the figure pictured below. If each square tile has a side length of $2\frac{1}{2}$ inches, what is the total area of the figure?

The presence of the triangles in the design may prove challenging for some students. Students who understand area as a procedure of multiplying sides—but do not understand the meaning of area—may need scaffolding to help them reason about mentally reassembling the 6 halves to find 3 whole tiles.

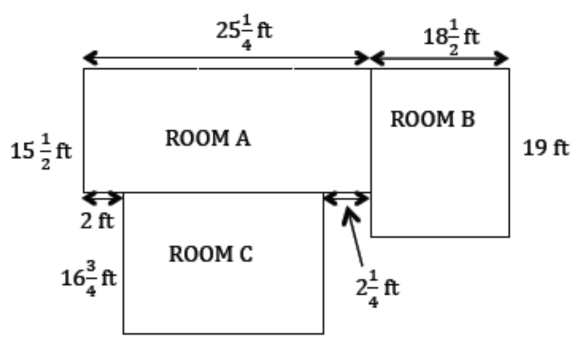




Problem 3

All-In-One Carpets is installing carpeting in three rooms. How many square feet of carpet are needed to carpet all three rooms?

While this problem is a fairly straightforward, additive area problem, an added complexity occurs in finding the dimensions of Room C. The complexity of this problem also lies in the need to keep three different areas organized before finding the total area. Again, once students have had an opportunity to work through the protocol, discuss the pros and cons of various approaches, including the reasoning for their choice of strategy.



YOUR NOTES

Room A: $25\frac{1}{4} \times 15\frac{1}{2} = 375 + \frac{25}{2} + \frac{15}{4} + \frac{1}{8}$

$= 375 + 12\frac{1}{2} + 3\frac{3}{4} + \frac{1}{8}$

$= 390 + \frac{11}{8}$

$= 391\frac{3}{8}$

$A_{Room A} = 391\frac{3}{8}\ ft^2$

$$\begin{array}{r} 25 \\ \times 15 \\ \hline 125 \\ +250 \\ \hline 375 \end{array}$$

Room B: $18\frac{1}{2} \times 19 = 342 + \frac{19}{2}$

$= 342 + 9\frac{1}{2}$

$= 351\frac{1}{2}$

$A_{Room B} = 351\frac{1}{2}\ ft^2$

$$\begin{array}{r} 19 \\ \times 18 \\ \hline 152 \\ +190 \\ \hline 342 \end{array}$$

Room C: $21 \times 16\frac{3}{4} = 336 + \frac{21 \times 3}{4}$

$= 336 + \frac{63}{4}$

$= 336 + 15\frac{3}{4}$

$= 351\frac{3}{4}$

$A_{Room C} = 351\frac{3}{4}\ ft^2$

$$\begin{array}{r} 21 \\ \times 16 \\ \hline 126 \\ +210 \\ \hline 336 \end{array}$$

$391\frac{3}{8} + 351\frac{1}{2} + 351\frac{3}{4}$

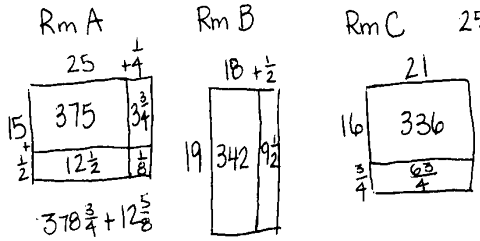
$= 1{,}093 + \frac{3}{8} + \frac{4}{8} + \frac{6}{8}$

$= 1{,}093 + \frac{13}{8}$

$= 1{,}094\frac{5}{8}$

$1{,}094\frac{5}{8}\ ft^2$ of carpet is needed.

Rm A Rm B Rm C $25\frac{1}{4} - 4\frac{1}{4} = 21$



Rm A:

$378\frac{3}{4} + 12\frac{5}{8}$

$= 378\frac{6}{8} + 12\frac{5}{8}$

$= 390 + \frac{11}{8}$

$= 391\frac{3}{8}$

Rm B:

$= 351\frac{1}{2}$

Rm C:

$= 336 + 15\frac{3}{4}$

$= 351\frac{3}{4}$

Total:

$$\begin{array}{r} 391\frac{3}{8} \\ 351\frac{4}{8} \\ +\ 351\frac{6}{8} \\ \hline 1093\frac{13}{8} \end{array}$$

$= 1{,}094\frac{5}{8}$

All-in-One needs $1{,}094\frac{5}{8}\ ft^2$ of carpet.

YOUR NOTES

Problem 4

Mr. Johnson needs to buy sod for his front lawn.

a. If the lawn measures $36\frac{2}{3}$ ft by $45\frac{1}{6}$ ft, how many square feet of sod will he need?

b. If sod is only sold in whole square feet, how much will Mr. Johnson have to pay?

Sod Prices

Area	Price per Square Foot
First 1,000 sq ft	\$0.27
Next 500 sq ft	\$0.22
Additional square feet	\$0.19

The dimensions of the yard are larger than any others in the Problem Set to encourage students to use the distributive property to find the total area. Because the total area (1,656 $\frac{1}{9}$ ft^2) is numerically closer to 1,656, students may be tempted to round down. Reasoning about the $\frac{1}{9}$ ft^2 area can provide an opportunity to discuss the pros and cons of sodding that last fraction of a square foot. In the final component of the protocol, ask the following or similar questions:

- Is it worth the extra money for such a small amount of area left to cover? While 19 cents is a small cost, what if the sod had been more expensive?
- What if the costs had been structured so that the last whole square foot of sod had lowered the price of the entire amount?
- What could Mr. Johnson do with the other 8 ninths?

a. $36\frac{2}{3} \times 45\frac{1}{6}$

$45 \times 36 = 1{,}620$ (270 + 1350)

$36 \times \frac{1}{6} = 6$

$45 \times \frac{2}{3} = 30$

$\frac{2}{3} \times \frac{1}{6} = \frac{1}{9}$

$1{,}620 + 30 + 6 + \frac{1}{9} = 1{,}656\frac{1}{9}$

He needs $1{,}656\frac{1}{9}$ ft^2 of sod.

b. 1657 whole s.f. :

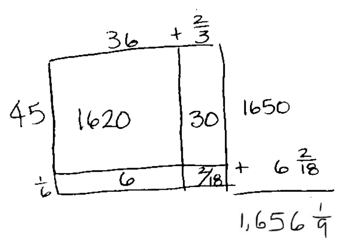
$1000 \times \$0.27 = \$270.00$

$500 \times \$0.22 = \$110.00$

$157 \times \$0.19 = \$29.83$

\$409.83

He will have to pay \$409.83.



Mr Johnson needs to buy 1657 sq. ft of sod and it will cost \$ 409.83.

Problem 5

Jennifer's class decides to make a quilt. Each of the 24 students will make a quilt square that is 8 inches on each side. When they sew the quilt together, every edge of each quilt square will lose $\frac{3}{4}$ of an inch.

a. Draw one way the squares could be arranged to make a rectangular quilt. Then, find the perimeter of your arrangement.

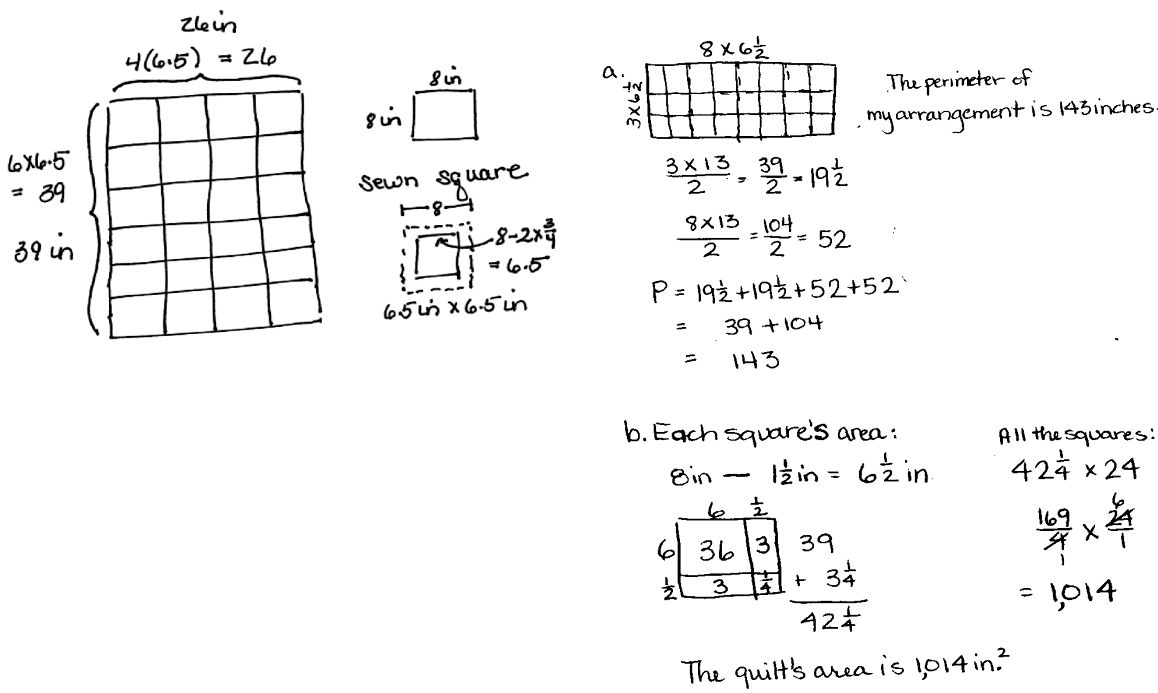
b. Find the area of the quilt.

YOUR NOTES

There are many ways to lay out the quilt squares. Allow students to draw their layouts, and then compare the perimeters. Ask the following questions:

- Does the difference in perimeter affect the area? Why or why not?
- Are there advantages to one arrangement of the blocks over another (e.g., lowering the cost for an edging by minimizing the perimeter or fitting the dimensions of the quilt to a specific wall or bed size)?

Problem 5 harkens back to Problem 2 but with an added layer of complexity. Students might be asked to compare and contrast the two problems. In this problem, students must account for the seam allowances on all four sides of the quilt squares before finding the area. Students find that each quilt block becomes $42\frac{1}{4}$ inches square after sewing and may simply multiply this area by 24.



NOTES

Debrief Questions

- Do these problems remind you of any others that we have seen in this mission? In what ways are they like other problems? In what ways are they different?
- What did you learn from looking at your classmates' drawings? Did that support your understanding of the problems in a deeper way? When you checked for reasonableness, what process was used?
- When finding the areas, which strategy did you use more often—distribution or improper fractions? Is there a pattern to when you used which? How did you decide? What advice would you give a student who was not sure what to do?
- Which problems did you find the most difficult? Which one was easiest for you? Why?

Multiple Means of Action and Expression

If students struggle with Problem 2, give them 13 square units, and allow them to make designs with the tiles and find the areas. They quickly see that the layout of the tiles does not change the area the tiles cover. They can then re-create the design in Problem 2, physically reassembling the half tiles as necessary to reason about the wholes.

Multiple Means of Engagement

Problem 5 may be extended for enrichment. Ask students to find the arrangement that gives the largest perimeter and then the smallest. The problem can also be changed to having seams only between squares so there are three different square areas to calculate. Another extension could be offered by asking students to find the area of the seams. (Find the unfinished area of the 24 squares, and subtract the finished area.)

Multiple Means of Engagement

Problem 3 might be extended by inviting students to research actual carpet prices from local ads or the Internet and calculate what such a project might cost in real life. Comparison between the costs of using different types of flooring (hardwood versus carpet, for example) may also be made.

YOUR NOTES

Lesson 15

Solve real-world problems involving area of figures with fractional side lengths using visual models and/or equations.

Note: Today's lesson uses the Problem Set. Solutions for each problem are included below.

Note: While there are only four problems, most are multi-step and require time to solve.

Materials: (S) Problem Set (see Appendix)

Suggested Delivery of Instruction for Solving Lesson 15's Word Problems

1. Model the problem.

Have two pairs of students who can successfully model the problem work at the board while the others work independently or in pairs at their seats. Review the following questions before beginning the first problem:

- Can you draw something? This might be a tape diagram or an area model.
- What can you draw?
- What conclusions can you make from your drawing?

As students work, circulate. Reiterate the questions above. After two minutes, have the two pairs of students share only their labeled diagrams. For about one minute, have the demonstrating students receive and respond to feedback and questions from their peers.

2. Calculate to solve and write a statement.

Give everyone two minutes to finish working on that question, sharing her work and thinking with a peer. All students should write their equations and statements of the answer.

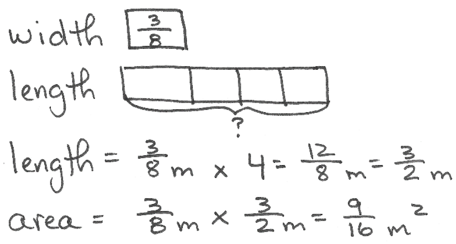
3. Assess the solution for reasonableness.

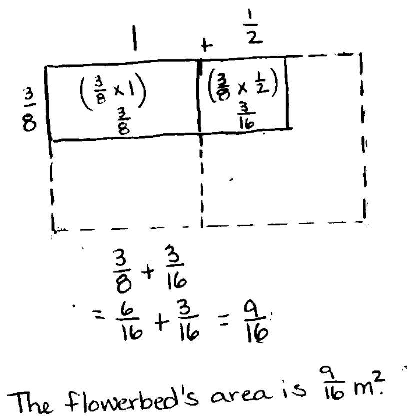
Give students one to two minutes to assess and explain the reasonableness of their solutions.

YOUR NOTES

Problem 1

The length of a flowerbed is 4 times as long as its width. If the width is $\frac{3}{8}$ meter, what is the area?





While this problem is quite simple to calculate, two complexities must be navigated. First, the length is not given. Second, the resulting area is less than a whole square meter. Once students have arrived at a solution, ask if their results make sense and why. If students need support, discuss what this might look like if the flowerbed was tiled with 1-meter square tiles. The length of the bed necessitates that 2 whole tiles be used. (How is it that the area is less than 1?) Students might draw or represent the problem with concrete materials to explain their thinking. The folding for these units may prove challenging but worthwhile because these types of problems are often done procedurally by students rather than with a deep understanding of what their answers represent. As in Lesson 14, continue to have students explain their choice of strategy in terms of efficiency. As students share their approaches with the class, encourage those who had difficulty to ask how the presenters got started with their drawings and calculations. Also, encourage students to explain any false starts they experienced when solving and how and why their thinking changed.

Problem 2

Mrs. Johnson grows herbs in square plots. Her basil plot measures $\frac{5}{8}$ yd. on each side.

a. Find the total area of the basil plot.
b. Mrs. Johnson puts a fence around the basil. If the fence is 2 ft from the edge of the garden on each side, what is the perimeter of the fence in feet?
c. What is the total area, in square feet, that the fence encloses?

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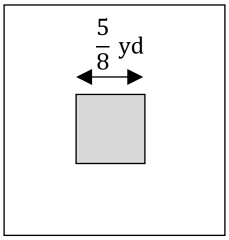
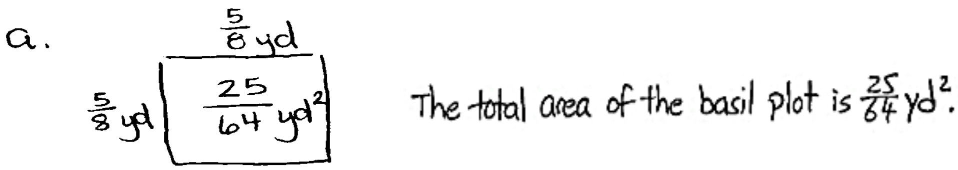
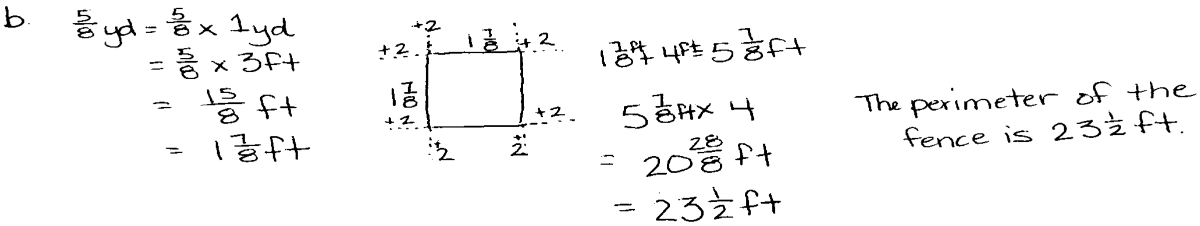
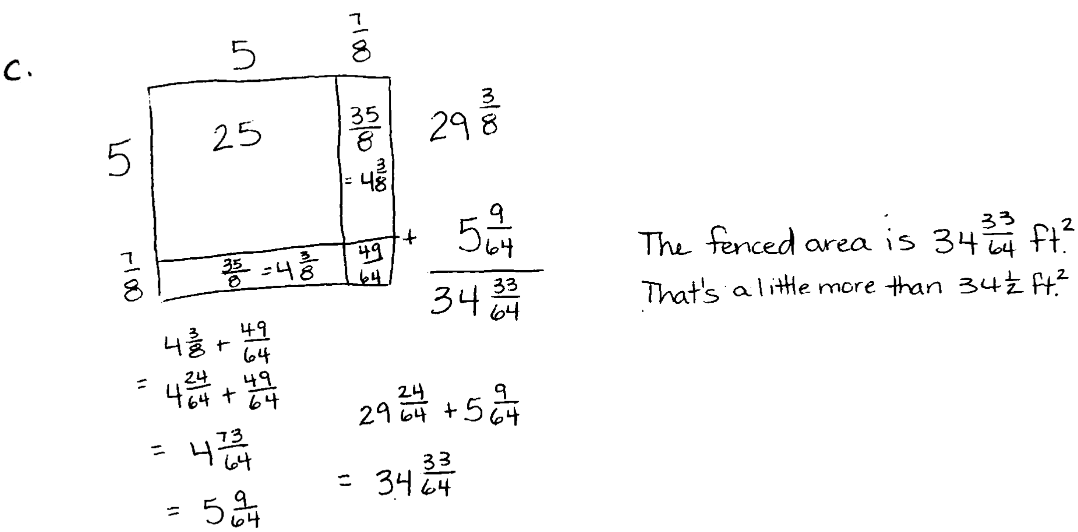








As in Problem 1, the fraction multiplication involved in completing Part (a) is not rigorous. However, this problem offers an opportunity to explore the relationships of square yards to square feet and the importance of understanding the actual size of such units. The expression of the area as $\frac{25}{64}$ yd^2 may be conceptually challenging for students. They might be encouraged to relate this to a benchmark of 1 half or 1 third square yard (which is 3 square feet). Students might be asked to show what a tiling of this garden plot would look like.

It may even be helpful to use yardsticks to show the actual size of the herb plot. The area expressed as a bit more than 5 square feet may be surprising to students. Help students make connections to the shading models of Mission 4 and how the

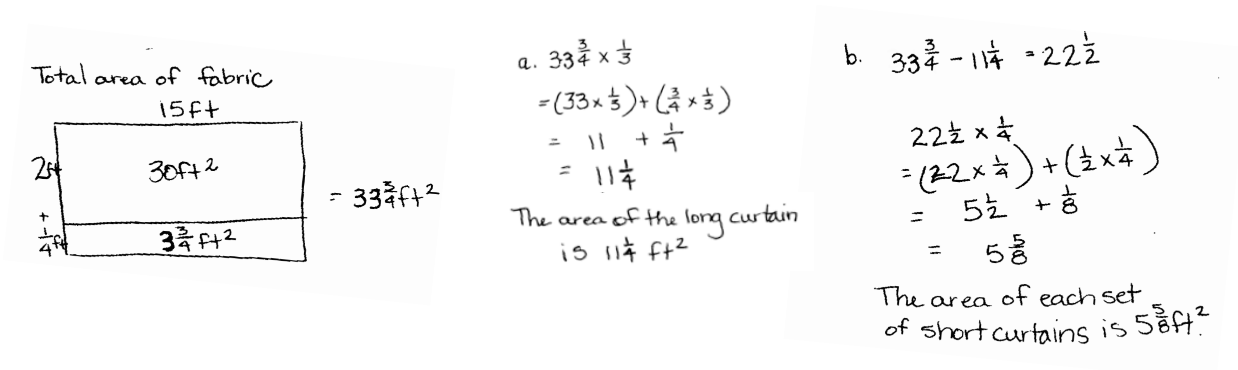
YOUR NOTES

representation of the area model for the basil plot compares and contrasts to the representation of fraction multiplication. Part (b) offers a bit of complexity in that the dimensions of the garden are given in yards, yet the distance from the garden to the fence is given in feet. Part (c) requires that students use the fence measurements to find the total area enclosed by the fence. Multiple methods may be used to accomplish this.

Problem 3

Janet bought 5 yards of fabric $2\frac{1}{4}$-feet wide to make curtains. She used $\frac{1}{3}$ of the fabric to make a long set of curtains and the rest to make 4 short sets.

a. Find the area of the fabric she used for the long set of curtains.
b. Find the area of the fabric she used for each of the short sets.



As in Problem 2, there are different units within a multi-step problem. After students have solved the problem, allow them to share whether they converted both measurements to feet or yards and the advantages and disadvantages of both. A discussion of the relationship of the square yards to the square feet may also be fruitful. Discuss the various strategies students may have used to find the fabric left for the shorter curtains.

Problem 4

Some wire is used to make 3 rectangles: A, B, and C. Rectangle B's dimensions are $\frac{3}{5}$ cm larger than Rectangle A's dimensions, and Rectangle C's dimensions are $\frac{3}{5}$ cm larger than Rectangle B's dimensions. Rectangle A is 2 cm by $3\frac{1}{5}$ cm.

a. What is the total area of all three rectangles?
b. If a 40-cm coil of wire was used to form the rectangles, how much wire is left?

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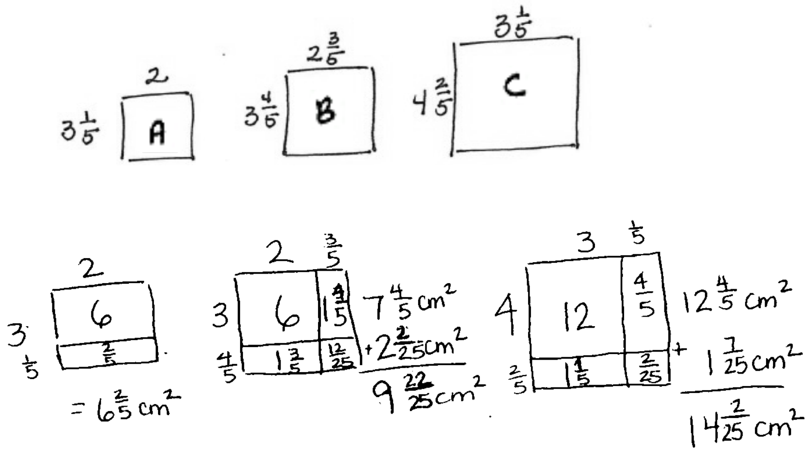
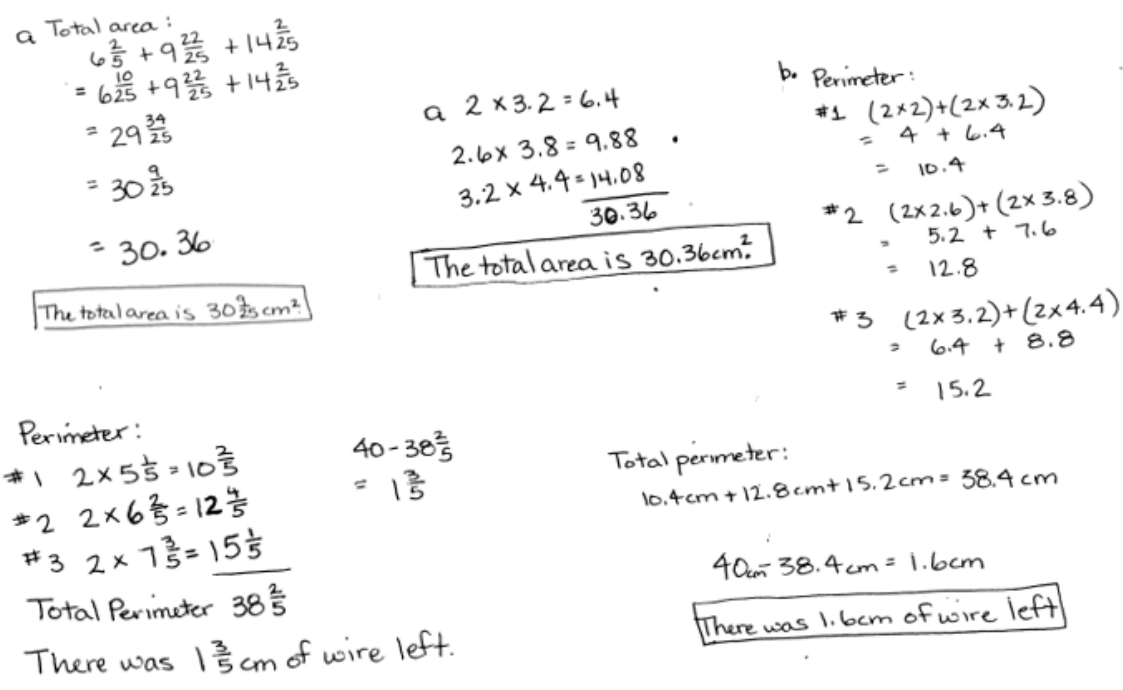


The complexity of this problem stems from students making sense of the way each rectangle increases in dimension. As always, encourage students to start by drawing each of the three rectangles. Since the denominators are easily expressed as hundredths, some students may use decimals to calculate these areas. Should this occur, help make the connection for other students to that learning from Mission 4.

For part (b), students must shift their thinking to the perimeter and use the outer dimensions of the rectangles to find the total amount of wire used. Again, students may use decimals for the calculations. Be sure to have students compare their decimal and fraction solutions with one another for equivalence and explain why they chose each type of fraction.



NOTES

Debrief Questions

- Compare the problems for which the distributive property seems most efficient and the problems for which multiplying improper fractions (or using decimals to multiply) seems so. What influences your choice of strategy?
- Sort problems from yesterday's and today's lessons (Lessons 14 and 15) from simple to complex. What do the problems have in common? Have students compare their sort to a partner's.

Multiple Means of Engagement

Problem 2 may be extended by having students convert the fence perimeter back into yards and the fenced area back into square yards. The understanding that 1 yd^2 = 9 ft^2 makes this an interesting challenge.

Multiple Means of Engagement

Encourage students to look for patterns in the growth of the perimeters. As a challenge, ask students to tell the perimeter of the fifth or tenth rectangle in the series.

YOUR NOTES

Topic D: Drawing, Analysis, and Classification of Two-Dimensional Shapes

In Topic D, students draw two-dimensional shapes to analyze their attributes and use those attributes to classify them.

Lesson 16

Draw trapezoids to clarify their attributes, and define trapezoids based on those attributes.

Note: The color hierarchy of quadrilaterals may be used to make color-coded cut-outs of each quadrilateral.

Materials: (T) Collection of polygons (Template 1), ruler, protractor, set square (or right angle template), quadrilateral hierarchy: color (Template 3) (S) Collection of polygons (Template 1, 1 per pair of students); ruler; protractor; set square (or right angle template); scissors; crayons, markers, or colored pencils; blank paper for drawing; quadrilateral hierarchy (Template 2)

Problem 1

a. Sort polygons by the number of sides.
b. Sort quadrilaterals into trapezoids and non-trapezoids.

T: Work with your partner to sort the collection of polygons by the number of sides they have.
S: (Sort.)
T: What are polygons with four sides called?
S: Quadrilaterals.
T: Which shapes are quadrilaterals?
S: Shapes A, B, D, F, G, H, J, K, and N.
T: What attribute do you need to consider to separate the quadrilaterals into two groups—trapezoids and non-trapezoids?
S: We look for sides that are parallel. → Trapezoids have at least one set of parallel sides, so quadrilaterals with parallel sides go in the trapezoid pile.
T: Separate the trapezoids in your collection of quadrilaterals from the non-trapezoids.

YOUR NOTES

S: (Sort. The trapezoid set includes Shapes A, B, D, J, and K.)

T: Talk to your partner. How are the shapes in the trapezoid group alike? How are they different?

S: They all have four straight sides, but they don't all look the same. → They are all quadrilaterals, but they have different side lengths and angle measures. → Some of the trapezoids are rhombuses, rectangles, or squares. → They all have at least one pair of sides that are parallel.

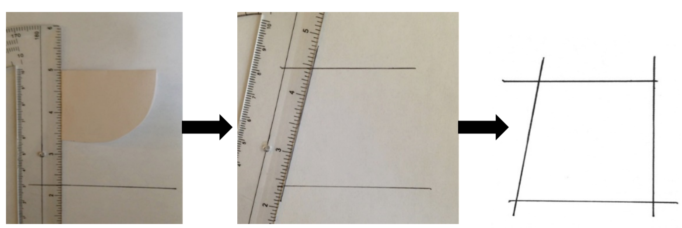
Problem 2

a. Draw a trapezoid according to the definition of a trapezoid.

b. Measure and label its angles to explore their relationships.

T: Look at the sorted shapes. I am going to ask you to draw a trapezoid in a minute. What attributes do you need to include? Turn and talk.

S: We have to draw four straight sides. → Two of the sides need to be parallel to each other. → We could draw any of the shapes in our trapezoid pile. → If we have one set of parallel sides, it will be a trapezoid.



T: Use your ruler and set square (or right angle template) to draw a pair of parallel lines on your blank paper positioned at any angle on the sheet.

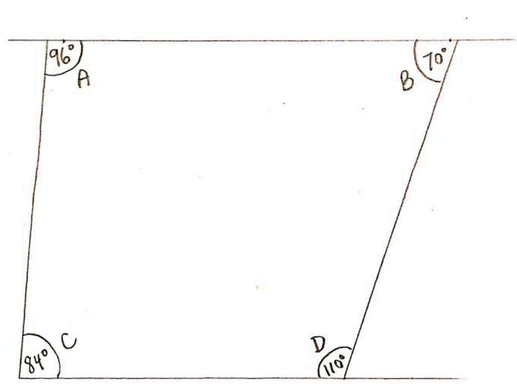
S: (Draw.)

T: Finish your trapezoid by drawing a third and fourth segment that cross the parallel pair of lines. Make sure they do not cross each other.

S: (Draw.)

T: Compare your trapezoid with your partner's. What is alike? What is different?

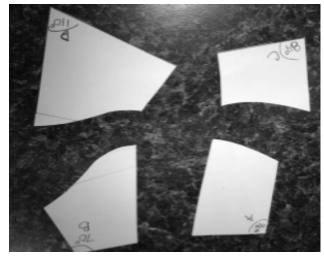
S: My horizontal parallel sides are closer together than my partner's. → My trapezoid is a rectangle, but my partner's isn't. → My trapezoid is taller than my partner's. → I have right angles in mine, but my partner does not. → My trapezoid is a square, but my partner's is not.



T: Label the angles of one of the trapezoids as A, B, C, and D on the inside of the shape. Label the top left angle as angle A. Label the top right angle as angle B. On the

YOUR NOTES

bottom, label the left angle as angle C, and finally, label the last angle on the bottom right as angle D. Now, measure these four interior angles of your trapezoid with your protractor, and write the measurements inside each angle.



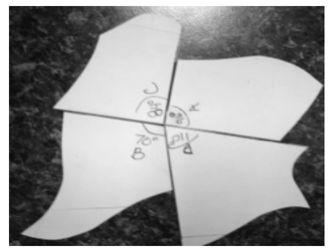
S: (Measure.)

T: Cut out your trapezoid.

S: (Cut.)

T: Cut your trapezoid into two parts by cutting between the parallel sides with a wavy cut (as shown to the right).

S: (Cut.)



T: Place $\angle A$ alongside $\angle C$. (See the image.) What do you notice in your trapezoid and in your partner's?

S: The angles line up. → The two angles make a straight line. → If I add the angles together, it is 180 degrees. → It is a straight line, but my angles only add up to something close to 180 degrees. → My partner's trapezoid did the same thing.

T: I heard you say that the angles make a straight line. What is the measure of a straight angle?

S: 180 degrees.

T: I also heard a few of you say that your angles did not add up to exactly 180 degrees. How do you explain that?

S: It sure looked like a straight line, so maybe we read our protractor a little bit wrong. → I might not have lined up the protractor exactly with the line I was using to measure.

T: Place $\angle B$ alongside $\angle D$. (See the image.) What do you notice?

S: It's the same as before. The angles make a straight line. → These angles add up to 180 degrees, too.

T: How many pairs of angles add up to 180 degrees?

S: Two pairs.

T: Cut each part of your trapezoid into two pieces using a wavy cut.

S: (Cut.)

T: Place all four of your angles together at a point. (Demonstrate. See the image.) What do you notice about the angles?

S: They all fit together like a puzzle. → The angles go all the way around. → Angles A and C made a straight line, and angles B and D made a straight line. I could put the straight lines together. The two straight angles make 360 degrees.

T: How does this compare to your partner's trapezoid? Turn and talk.

S: It's the same in my partner's trapezoid. → The angles in my partner's trapezoid weren't the same size as mine, but they still all fit together all the way around.

NOTES

Debrief Questions

- (Consolidate the lists of attributes students generated for trapezoids.) Where do these pairs seem to occur consistently? Is this true for all quadrilaterals? Just trapezoids?
- (Use the trapezoids that students produce in Problem 1 to articulate the formal definition of trapezoids. Post these definitions in the classroom for reference as Topic D proceeds.)

  A trapezoid:

  Is a quadrilateral in which at least one pair of opposite sides is parallel.
- (Begin the construction of the hierarchy diagram; see Lesson 16 Template 2 in the Appendix. Students might draw or glue examples of trapezoids and quadrilaterals and/or list attributes within the diagram.) Explain how you decided to place each of the figures on the hierarchy. What are questions you asked yourself as you classified them?
- Respond to the following statements with *true* or *false*. Explain your reasoning.
  1. All trapezoids are quadrilaterals. (True. All trapezoids have 4 straight sides.)
  2. All quadrilaterals are trapezoids. (False. A trapezoid must have at least one pair of opposite, parallel sides, but quadrilaterals need only 4 sides. No side needs to be parallel to another.)

Multiple Means of Engagement

Problem 1 in the lesson provides an opportunity for a quick formative assessment. If students have difficulty sorting and articulating attributes, consider a review of concepts from Grade 4 Mission 4.

Drawing Figures

If students need specific scaffolding for drawing figures, please see Grade 4 Mission 4.

To draw parallel lines with a set square:

1. Draw a line.
2. Line up one side of the set square on the line.
3. Line up a ruler with the perpendicular side of the set square.
4. Slide the set square along the ruler until the desired place for a second line is reached.
5. Draw along the side of the set square to mark the second, parallel line.
6. Remove the set square, and extend the second line with the ruler, if necessary.

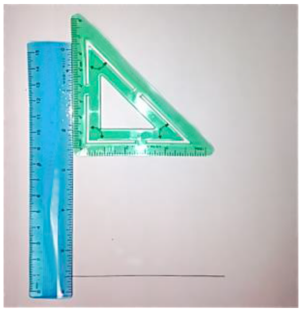
YOUR NOTES

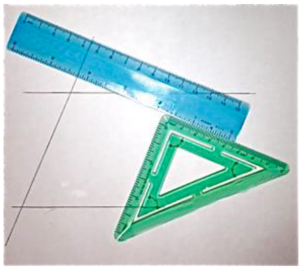
Lesson 17

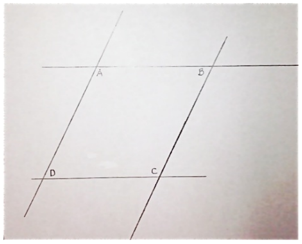
Draw parallelograms to clarify their attributes, and define parallelograms based on those attributes.

Materials: (T) Ruler, protractor, set square (or right angle template), quadrilateral hierarchy with parallelogram: color (Template 2) (S) Ruler; protractor; set square (or right angle template); scissors; crayons, markers, or colored pencils; blank paper for drawing; quadrilateral hierarchy with parallelogram (Template 1)

Problem 1







a. Draw a parallelogram, and articulate the definition.
b. Measure and label its angles to explore their relationships.
c. Measure to explore diagonals of parallelograms.

T: Use your ruler and set square to draw a pair of parallel lines on your blank paper positioned at any angle on the sheet.
S: (Draw.)
T: Because we are about to draw a quadrilateral beginning with one pair of parallel sides, what name can we give every figure we will draw today?
S: Trapezoid.
T: If I want to draw a trapezoid that can also be called a parallelogram, what will I need to draw next?
S: Parallelograms have two sets of parallel sides, so you have to draw another pair of parallel lines. → Draw another pair of parallel lines that cross the first ones.
T: Use your tools to draw a second pair of parallel lines that intersect your first pair.
S: (Draw.)
T: Measure the sides of your parallelogram, and compare your parallelogram with your partner's. What is alike? What is different?
S: The opposite sides in my parallelogram are the same lengths, and my partner's are, too. → My parallel sides are closer together than my partner's. → My parallelogram is a rectangle, but my partner's isn't. → My parallelogram has four equal sides, and my partner's has two different pairs of equal sides. → I have right angles in mine, but my partner does not. → I drew a square, and my partner drew a rectangle.

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T: Label the angles of your parallelogram inside the shape. Label the upper angle on the left as angle A and the upper angle on the right as angle B. Continue clockwise by labeling the lower right angle as angle C and the angle directly below A as angle D. Then, measure the angles of your parallelogram with your protractor, and write the measurements inside each angle.

S: (Measure.)

T: Cut out your parallelogram.

S: (Cut.)

T: Make a copy of your parallelogram on another blank sheet by tracing it and labeling the vertices with the same letters.

S: (Trace and label.)

T: Cut your first parallelogram into four parts by cutting between each set of parallel sides with a wavy cut.

S: (Cut.)

T: Put angle A on top of angle C, and put angle B on top of angle D. What do you notice about your parallelogram's angles and about your partner's? Turn and talk.

S: Angles A and C are the same size, and so are angles B and D. → Our parallelograms don't look anything alike, but the angles opposite each other in each of our parallelograms are equal.

T: Place your angles alongside each other, and find as many combinations that form straight lines as you can.

S: (Work.)

T: Compare your findings with your partner's. What do you notice? How many pairs did you find?

S: We both found four pairs that make straight lines. → Yesterday, some of us only had two pairs of angles that made straight lines. Today, all of us have four pairs.

T: So, thinking about what we drew and what we have discovered about these angles, when can a trapezoid also be called a parallelogram? Turn and talk.

S: When a trapezoid has more than one pair of parallel sides, it can be called a parallelogram. → Trapezoids have at least two pairs of angles that add up to 180 degrees. When they have more than that, they can also be called a parallelogram.

T: Place all four of your angles together at a point. What do you notice about the angles in your parallelogram and in your partner's?

S: (Work.) All four of my angles add up to 360 degrees. → My parallelogram was different than my neighbors, but the angles go all the way around again, like they did with our trapezoids.

T: Use your ruler to draw the diagonals on the copy you made of your parallelogram.

S: (Draw.)

T: Measure each diagonal, and record the measurements on your paper. Are these segments equal to each other?

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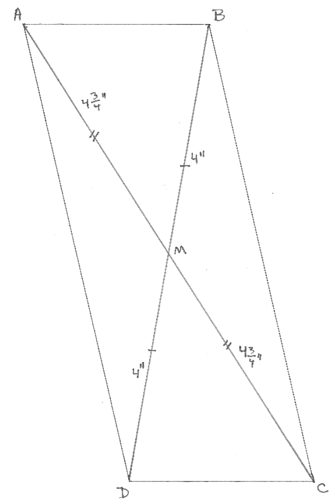


S: I drew a long skinny parallelogram, and my diagonals aren't the same length. But my partner drew a square, and his are the same length. → Some people have equal diagonals, and some people don't.

T: I hear you saying that the diagonals of a parallelogram may or may not be equal to each other. Label the point where your diagonals intersect as point M.

S: (Draw and label.)

T: Measure from each corner of your parallelogram to point M. Record all of the measurements on the figure. Compare your measurements to those of your partner.

S: (Measure and compare.)

T: What do you notice about the diagonals of your parallelogram now?

S: The length from opposite corners to point M on the same diagonal is equal. → The diagonals cut each other into two equal parts. → One diagonal crosses the other at its midpoint. → M is the midpoint of both diagonals. → Even though our parallelograms look really different, our diagonals still cross at their midpoints.

T: The diagonals of a parallelogram bisect each other. Say, "bisect."

S: Bisect.

T: Let's break down this word into parts. Think about the first part, bi-. How many wheels are on a bicycle?

S: Two.

T: What does the word section mean?

S: Parts of something.

T: Sect also means to cut. Turn and talk to your partner about why bisect is a good name for what you see in all the parallelograms' diagonals.

S: Bi- means two. These segments are cut in two equal parts. → Bi- means two, and sect means cut, so bisect means to cut in two equal parts.

NOTES

Debrief Questions

- (Consolidate the lists of attributes students generated for parallelograms.) What attributes do all parallelograms share? Where do these pairs seem to occur consistently? Is this true for all quadrilaterals? Trapezoids? Parallelograms?
- (Use the parallelograms that students produce in Problem 1 to articulate the formal definition of a parallelogram. Continue posting these definitions in the classroom for reference as Topic D proceeds.)
- When can a quadrilateral also be called a parallelogram? When can a trapezoid also be called a parallelogram?

  Respond to the following statements with *true* or *false*. Explain your reasoning.

  1. All parallelograms are trapezoids. (True. The defining attribute of a trapezoid is that it has at least one set of parallel sides. Since parallelograms have two sets of parallel sides, they also fit the definition of a trapezoid.)
  2. All trapezoids are parallelograms. (False. If the trapezoid has only one set of parallel sides, it is not also a parallelogram.)
  3. All parallelograms are quadrilaterals. (True. All parallelograms have four straight sides.)
  4. All quadrilaterals are parallelograms. (False. There are many four-sided shapes that do not have two pairs of parallel sides.)

  A parallelogram:

  Is a quadrilateral in which both pairs of opposite sides are parallel.

  A quadrilateral:

  Consists of four different points A, B, C, D in the plane and four segments, $\overline{AB}, \overline{BC}, \overline{CD}, \overline{DA}$.

  Is arranged so that the segments intersect only at their endpoints.

  Has no two adjacent segments that are collinear.
- (Continue the construction of the hierarchy diagram from Lesson 16. Students might draw or glue examples of parallelograms and/or list attributes within the diagram. Encourage students to explain their placements of the figures in the hierarchy.)

Multiple Means of Engagement

If students need specific scaffolding for drawing figures, please see Grade 4 Mission 4.

Multiple Means of Engagement

The discussion of parallelograms as special trapezoids is based on the inclusive definition of a trapezoid as a quadrilateral with *at least* one set of parallel sides. That is, trapezoids may have more than one set of parallel sides.

Also note that the dialogue as written here assumes recall of Grade 4 geometric concepts. Additionally, scaffolded questions may be necessary for students to verbalize the conditions necessary to classify a trapezoid as a parallelogram.

YOUR NOTES

Lesson 18

Draw rectangles and rhombuses to clarify their attributes, and define rectangles and rhombuses based on those attributes.

Materials: (T) Quadrilateral hierarchy with square: color (Template 2) (S) Ruler, set square or square template, protractor, scissors, quadrilateral hierarchy with square (Template 1)

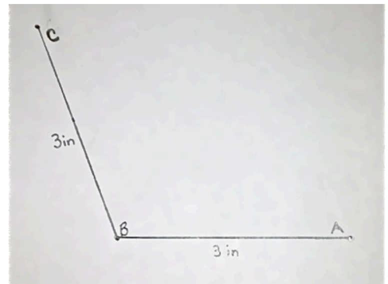
Problem 1

a. Draw a rhombus, and articulate the definition.
b. Measure and label its angles to explore their relationships.
c. Measure to explore diagonals of rhombuses.

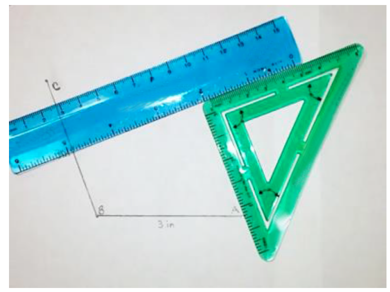
T: Give the least specific name for all the shapes we have drawn so far.
S: Quadrilaterals.
T: Tell your partner a more specific name for a shape we have drawn, and explain what property it has that gives it that name.
S: Trapezoids because we have drawn shapes with at least one pair of parallel sides. → Some of the quadrilaterals could be called trapezoids and parallelograms. Parallelograms have two pairs of parallel sides.
T: How did we start drawing the trapezoids and parallelograms?
S: By drawing a pair of parallel sides.
T: If we wanted to draw a parallelogram that is also a rhombus, what would we need to think about?
S: It would need to have four sides the same length. → It would need another pair of parallel sides, but we would need to measure to be sure we drew all the sides the same length.
T: Draw an angle with sides that are equal length. Then, label the vertex as B and the endpoints of the sides as A and C.
S: (Draw an angle.)
T: Draw a line parallel to one of the sides through the endpoint of the other side.
S: (Draw a parallel line.)
T: Now, do the same for the second side.
S: (Draw a second parallel line.)
T: Label the last angle as D.
S: (Label the angle.)

YOUR NOTES

T: Measure the sides, and compare your figure with your partner's. What is the most specific name for this shape? How do you know?



S: My sides were two inches long. My partner's were three inches long, but they both have two sets of parallel sides, and the sides are all the same length. So, we both drew a rhombus. → It's a parallelogram with four equal sides. → Mine is a parallelogram with equal sides, but my partner's is a square. We both drew a rhombus with four equal sides, even though I started with an acute angle and he started with a right angle.

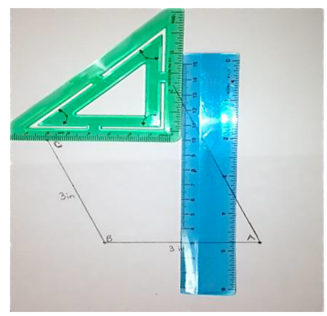


T: Measure the angles, and mark them inside the rhombus.

S: (Measure and mark the angles.)

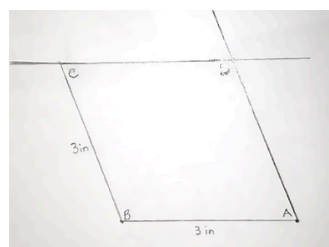
T: What do you notice? Turn and talk.

S: The angles that are beside each other add up to a straight angle. → There are two pairs of angles. Each pair adds up to 180°. → Angles between parallel lines equal 180°. → The opposite angles are the same size.



T: Use your ruler to draw the diagonals of your rhombus. Then, measure them and the distance from each corner to the point where they intersect. Tell your partner what you notice.

S: These diagonals are equal. → The diagonals bisect each other. → The point where they cross is the midpoint of both diagonals.



T: Now, measure the angles formed by the diagonals. What is the measure?

S: They are right angles. → The angles are all 90°.

T: What is the name for lines that intersect at a right angle?

S: Perpendicular lines.

T: Because they bisect each other at a 90° angle, we call these diagonals **perpendicular bisectors**.

T: From our drawing, what attribute needs to be present to call this parallelogram a rhombus?

S: All four sides must be equal.

T: What else did we discover about the diagonals of a rhombus?

S: The diagonals are perpendicular bisectors.

Problem 2

YOUR NOTES

a. Draw a rectangle according to the definition of a rectangle.
b. Measure and label its angles to explore their relationships.
c. Measure to explore the diagonals of rectangles.

T: If I want to draw a parallelogram that is also a rectangle, what must I include in my drawing?
S: Rectangles are parallelograms, so they need two sets of parallel sides. → Rectangles have right angles and opposite sides that are parallel and equal.
T: Use your ruler and set square to draw a rectangle.
S: (Draw a rectangle.)
T: Cut out your rectangle, and by folding, confirm that the angles are all 90° and the opposite sides are the same lengths.
S: (Cut and fold the rectangle.)
T: Now, measure the diagonals, the segments of the diagonals, and the angles around the intersection point. Record your measurements on the figure.
S: (Measure and record on the figure.)
T: What do you notice? Turn and talk.
S: The diagonals are equal lengths. → The segments of the diagonals are equal. → The angles between the parallel lines equal 180°. → The diagonals are equal and bisect each other.
T: Are the diagonals perpendicular bisectors? How do you know?
S: They are not perpendicular bisectors because they don't form right angles.
T: What properties must be present for a parallelogram to also be a rectangle?
S: The sides across from each other have to be the same length. → All angles are 90°. → Diagonals bisect each other.

NOTES

Debrief Questions

- (Allow students to share all the different rhombuses and rectangles.) What attributes do all rhombuses share? What attributes appear on the rhombus list that were not on the list for parallelograms? What attributes do all rectangles share? Is this true for all quadrilaterals? Rhombuses? Rectangles? (Use the rhombuses and rectangles produced in Problem 1 to articulate the formal definitions. Continue posting definitions for comparisons.)
- When can a quadrilateral also be called a rhombus? When can a quadrilateral also be called a rectangle?

  A rhombus:
  Is a quadrilateral with all sides of equal length.

  A rectangle:
  Is a quadrilateral with four right angles.
- Respond to the following statements with true or false. Explain your reasoning.
  1. All parallelograms are rhombuses. (False. Parallelograms have pairs of opposite sides that are the same length, but both pairs do not necessarily have the same length.)
  2. All rhombuses are parallelograms. (True. Rhombuses have all sides that are equal in length, and the opposite sides are also parallel.)
  3. All parallelograms are rectangles. (False. It is possible to have a quadrilateral with two pairs of parallel sides that do not have four right angles.)
  4. All rectangles are parallelograms. (True. If a quadrilateral has four right angles, it must have two pairs of parallel sides.)
- Continue the construction of the hierarchy diagram from Lessons 16 and 17. Students might draw or glue examples of rhombuses and rectangles and list attributes within the diagram. Encourage them to explain their placement of the figures in the hierarchy.

Multiple Means of Engagement

If students are confused about the segments of a quadrilateral lying in the same plane or intersecting only at their endpoints, use the straws from Lesson 16 to demonstrate counter examples.

YOUR NOTES

Lesson 19

Draw kites and squares to clarify their attributes, and define kites and squares based on those attributes.

Materials: (T) Quadrilateral hierarchy with kite: color (Template 2) (S) Ruler, set square or square template, protractor, scissors, quadrilateral hierarchy with kite (Template 1)

Problem 1

a. Draw a square, and articulate the definition.
b. Measure and label its angles to explore their relationships.
c. Measure to explore diagonals of squares.

T: What shapes have we drawn so far?

S: Quadrilaterals. → Rhombuses and rectangles. → Trapezoids and parallelograms, but in a rhombus all sides are the same length.

T: Can a rectangle ever be a rhombus? Can a rhombus ever be a rectangle? Turn and talk.

S: Well, a rectangle and a rhombus are both parallelograms. A rectangle must have right angles, but a rhombus can have angles that are not right angles. → Rhombuses must have four equal sides, but rectangles might not. → A square is a rhombus and a rectangle at the same time.

T: Let's see if we can answer this question by drawing.

T: Draw a segment 3 inches long on your blank paper, and label the endpoints A and B.

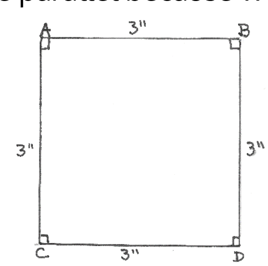
S: (Draw a segment.)

T: (Demonstrate.) Now, using your set square, draw three-inch segments from both point A and point B at a 90° angle to $\overline{AB}$. Draw both line segments toward the bottom of your paper.

S: (Draw additional line segments.)

T: Label the endpoints as C and D. Are $\overline{AC}$ and $\overline{BD}$ parallel? How do you know?

S: I checked with my set square. They are parallel. → They must be parallel because we drew them both as right angles to the same segment.



T: Use your straightedge to connect points C and D.

T: Measure segment CD. What is its length?

S: $\overline{CD}$ is also 3 inches long.

T: What have we drawn? How do you know?

S: A square. It has four right angles and four equal sides.

YOUR NOTES

T: Based on the properties of parallel sides, tell your partner another name for this shape, and justify your choice.

S: It is a trapezoid. It has a pair of parallel sides. → I can call this a parallelogram because there are two sets of parallel sides.

T: Use your protractor to measure angles C and D. What are their measures?

S: 90°. → All of the angles are 90°.

T: Since this is a parallelogram with four right angles and two sets of opposite equal sides, what can we call it?

S: A rectangle.

T: Since this is a parallelogram with four equal sides, what can we call it?

S: A rhombus.

T: Let's return to our question, can a rhombus ever be a rectangle? Can a rectangle ever be a rhombus? Why or why not?

S: Yes. A square is a rhombus and a rectangle at the same time. → A rectangle can be a rhombus if it is a square. → A rhombus can be a rectangle if it is a square.

T: Using what you just drew, list the attributes of a square with your partner.

S: A square has four sides that are equal and four right angles. → A square has opposite sides that are parallel, four right angles, and sides that are all equal in length. → A square is a rectangle with four sides that are equal length. → A square is a rhombus with four right angles.

T: Draw the diagonals of the square. Before we measure them, predict whether the diagonals will bisect each other, and justify your predictions using properties. Turn and talk.

S: The parallelograms we drew had bisecting diagonals, and this is definitely a parallelogram. I think the diagonals will bisect. → I think they will bisect each other because a square is a rectangle, and all the rectangles' diagonals we measured bisected each other. → We drew rhombuses yesterday, and all those diagonals bisected each other. A square is a rhombus, so that should be true in a square, too.

T: Measure the lengths of the diagonals. Then, to test your prediction, measure the distance from each corner to the point where they intersect.

S: (Draw and measure.)

T: What did you find?

S: The diagonals do bisect each other.

T: Now, use the protractor to measure the angles where the diagonals intersect.

S: (Measure the intersecting angles.)

T: What did you find?

S: They intersect at right angles. → All the angles are 90°. → The diagonals are perpendicular to each other.

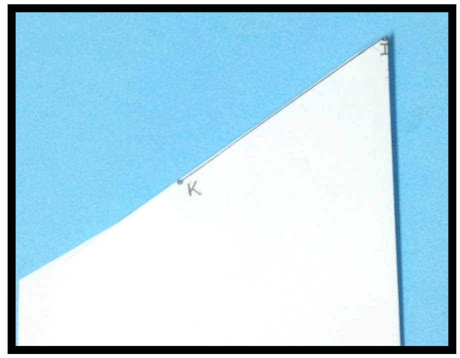
T: When the diagonals of a quadrilateral bisect each other at a 90° angle, we say the diagonals are perpendicular bisectors.

YOUR NOTES

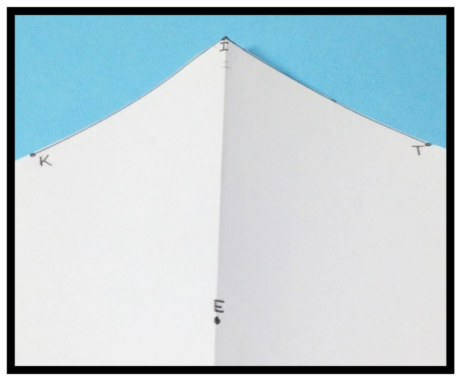
Problem 2

a. Draw a kite, and articulate the definition.
b. Measure and label its sides and angles to explore their relationships.
c. Measure to explore diagonals of kites.

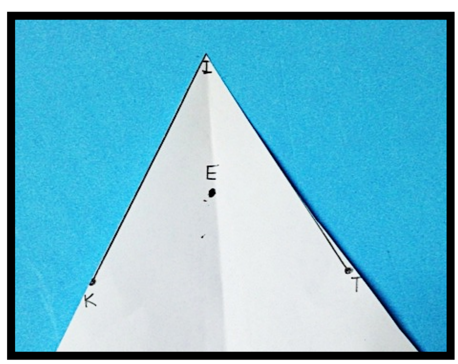
T: We have one more quadrilateral to explore. Let's see if you can guess the figure if I give you some real-world clues. It works best outside on windy days, and it is flown with a string. (Give clues until the figure is named.)



S: A kite.
T: Sketch a kite.
S: (Sketch.)
T: Compare your kite to your neighbor's. How are they alike? How are they different? Turn and talk.
S: Mine is narrow, and my partner's is wider. → Mine is taller, and my partner's is shorter. → They all have four sides.

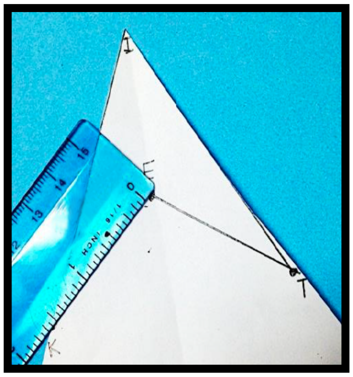


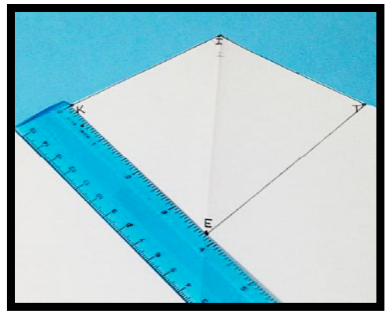
T: Let's draw a kite using our tools. Draw an angle of any measure with two sides that are the same length but at least two inches long. Mark the vertex as I and the endpoints of the segments as K and T.
S: (Draw a kite.)
T: Use your scissors to cut along the rays of your angle.
S: (Cut along the rays.)
T: Fold your angle in half matching points K and T. (Refer to the image.) Open it, and mark a point on the fold, and label it E.



S: (Fold and label.)
T: Use your ruler to connect your point to the ends of the other segments. Then, cut out your kite.
S: (Cut out the kite.)
T: Measure the two sides that you just drew. What do you notice about the sides? How are they different from parallelograms?
S: There are two sets of sides that are equal to each other, but they are next to each other, not across from each other. → Opposite sides are not equal on mine, but adjacent sides are. → None of these sides are parallel to each other.

YOUR NOTES





T: Use your protractor to measure the angles of your kite, and record the measurements on your figure.

S: (Measure and record the angles of the kite.)

T: What do you notice? Turn and talk. (Allow students time to share with a partner.)

S: There are always at least two angles of equal measure. They are across from each other where the unequal sides meet. → My kite is also a square. It has two pairs of equal angles.

T: Now, draw the diagonals of the kite. Measure the length of the diagonals, the segments of the diagonals, and the angles where the diagonals intersect.

S: (Draw and measure the diagonals, segments, and angles.)

T: What can you say about the diagonals of a kite? Turn and talk.

S: My diagonals cross outside my kite, but they are still perpendicular. → The diagonals are not the same length. → The diagonals meet at 90° angles; they are perpendicular. → One diagonal bisects the other, but they are not both bisected.

T: What are the attributes of a kite? Tell your partner.

S: A kite is a quadrilateral with equal adjacent sides. → It is a quadrilateral with two pairs of adjacent sides that have equal lengths.

T: A kite is a quadrilateral that has adjacent sides, or sides next to each other, that are equal. Can a kite ever be a parallelogram? Can a parallelogram ever be a kite? Why or why not? Turn and talk.

S: Yes, a parallelogram can be a kite. A square and a rhombus both have all equal sides, so that fits the definition of a kite. → Squares and rhombuses have sides next to each other that are equal. They are the only parallelograms that could also be called kites. → Any quadrilateral with all sides equal would have adjacent sides equal, so a rhombus and a square are kites.

NOTES

Debrief Questions

- (Allow students to share the myriad of squares and kites.) Compare and contrast these quadrilaterals.
- (Use the figures produced in Problem 1 to articulate the formal definitions of both squares and kites. Continue to post the definitions.)

  A square:

  Is a rhombus with four right angles.

  Is a rectangle with four equal sides.

  A kite:

  Is a quadrilateral in which two consecutive sides have equal length.

  Has two remaining sides of equal length.
- (Consolidate the lists of attributes students generated for squares and kites.) What attributes do all squares share? What attributes do all kites share? When is a quadrilateral a kite but not a square or rhombus?
- When can a quadrilateral also be called a square?
- Respond to the following statements with *true* or *false*. Explain your reasoning.
  1. All squares are quadrilaterals. (True. All squares have four sides.)
  2. All quadrilaterals are squares. (False. Not all quadrilaterals have equal side lengths.)
  3. All rhombuses are squares. (False. All rhombuses must have four equal sides, but the angles do not have to be 90°.)
  4. All squares are rhombuses. (True. The defining attribute of a rhombus is that its four sides must be equal in length. Squares also have four sides of equal length.)
- (Finish the construction of the hierarchy diagram. (See the Lesson 19 Templates 1 and 2 in the Appendix.) Students might draw or glue examples of squares and kites or list attributes within the diagram. Encourage students to explain their placement of the figures in the hierarchy.)

Multiple Means of Representation

English language learners and others may feel overwhelmed distinguishing terms in this lesson. To support understanding, point to a picture or make gestures to clarify the meaning of *parallel*, *rhombus*, *attribute*, etc., each time they are mentioned. Building additional checks for understanding into instruction may also prove helpful, as might recording student observations of shape attributes and definitions in a list, table, or graphic organizer.

Kites

If no student produces a concave kite (an arrowhead) through the process of drawing in the lesson, draw one for students to consider. It is important to note that although the diagonals do not intersect within the kite, the same relationships hold true. The lines containing the diagonals intersect at a right angle, and only one bisects the other. Students who produce such a kite may need help drawing the diagonals.

YOUR NOTES

Lesson 20

Classify two-dimensional figures in a hierarchy based on properties.

Materials: (T) Quadrilateral hierarchy with kite: color (Lesson 19 Template 2), image of a trapezoid that is not a parallelogram (S) Personal white board, shape name cards (Template 1, 1 per pair of students), shapes for sorting (Template 2, 1 per pair of students), protractor, ruler, set square, quadrilateral hierarchy with kite (Lesson 19 Template 1, 1 per pair of students), scissors, glue

Part 1

Justify responses to true or false statements about quadrilaterals based on properties.

a. Trapezoids are always quadrilaterals.
b. Quadrilaterals are always trapezoids.

T: (Project Sentence (a) on the board.) Talk to your partner about whether the statement is true or false. Justify your answer using properties of the shapes.

S: This is true because all trapezoids have the properties of quadrilaterals. They just have an extra property; they have at least one set of parallel sides. → Look at this trapezoid I drew. It has four segments in the same plane that only intersect at their endpoints. You cannot draw a trapezoid without these properties of quadrilaterals. It is true that trapezoids are always quadrilaterals.

T: (Project Sentence (b) on the board.) What about this statement? Trapezoids are always quadrilaterals. Are quadrilaterals always trapezoids? Why or why not? Turn and talk.

S: This is not true. There are lots of quadrilaterals that do not have any parallel sides. → If a quadrilateral does not have parallel sides, it cannot be a trapezoid. This statement is false.

T: (Write on the board: ____________ are always ____________. Give pairs of students one copy of the shape name cards.) Write this sentence frame on your personal white boards, and turn all your cards facedown on your table.

S: (Write the sentence frame.)

T: Each partner should choose a shape name card and place it in one of the blanks in the sentence frame. Work together to decide whether your statement is true or false, and use the properties of the figures to justify your answer. Then, switch the cards in the frame, and repeat the sequence. Finally, put the cards back on the table facedown. (Allow students time to work.)

Part 2

YOUR NOTES

Classify two-dimensional figures in a hierarchy using tools to confirm properties.

T: (Project an image of a trapezoid that is not a parallelogram and the quadrilateral hierarchy with kite: color (Lesson 19 - Template 2) for students to see.) What does this shape look like?

S: Quadrilateral. → A trapezoid.

T: How could I use my tools to be sure of these classifications? What properties would I need to confirm in order to classify this shape as a trapezoid? Turn and talk.

S: It is two-dimensional, and it has four sides, so we know it is a quadrilateral. → I can see it is a quadrilateral, but to be sure it is a trapezoid, I could use my set square to check if it has at least one pair of parallel sides.

T: I will confirm for you that this figure does have four segments in the same plane, and they only intersect at their endpoints. None of the endpoints are collinear, and it has one pair of parallel sides. With that information, could I place this figure inside the quadrilateral on the hierarchy diagram? Why or why not?

S: Yes. It is a quadrilateral.

T: (Place the figure on the diagram inside the quadrilateral only.) Could I place it inside the trapezoid on the hierarchy diagram? Why or why not?

S: Yes, it can go there because it has one set of parallel sides.

T: Can I place it inside the parallelogram on the hierarchy? Why or why not?

S: No. It does not have two sets of parallel sides, so it cannot go inside the parallelogram.

T: This figure is inside the quadrilateral ring and the trapezoid ring. What does that mean for its properties?

S: It has all the properties of a quadrilateral and all the properties of a trapezoid.

T: (Give a copy of quadrilateral hierarchy with kite (Lesson 19 – Template 1) and one shapes for sorting sheet to each pair of students.) Work with your partner to classify the shapes on your sheet. Use your tools to confirm their properties. Then, cut out the shapes, and glue them on the hierarchy diagram. Be prepared to defend their placements.

S: (Work.)

Circulate and ask questions of students as they confirm properties and sort. Encourage students to verbalize that attributes belonging to a category of figures also belong to all subcategories of the figure. The following sentence frame might be used: Because a ______________ is a ______________, it must have ______________. (For example: Because a rhombus is a trapezoid, it must have at least one set of parallel sides.)

NOTES

Debrief Questions

- (Review the formal definitions of all the quadrilaterals from the topic. Compare them with a view toward noticing the hierarchical nature. For example, a rhombus is a parallelogram with four equal sides. Point out that because a rhombus *is* a parallelogram, it has all the attributes of a parallelogram *and* four equal sides.)
- As the most specific quadrilateral that we have explored, can a square be correctly classified as any of the quadrilaterals on the hierarchy? (Making a list of all of a square's attributes using sides, angles, and diagonals, can help drive home the understanding of the hierarchy.)

Multiple Means of Representation

Depending on the English proficiency level of English language learners, it might be helpful to demonstrate how to justify responses to true or false statements, give extra response time, or provide sentence frames or starters, such as the following:

- The statement is true or false because ...
- I disagree because ...

YOUR NOTES

Lesson 21

Draw and identify varied two-dimensional figures from given attributes.

Note: Today's lesson uses the Problem Set. Solutions for each problem are included below.

Note: Today's lesson asks students to apply the nested relationships among quadrilaterals that have been explored throughout this topic. It should be conducted following a protocol similar to that of a problem-solving lesson involving word problems. Allow students to wrestle with the drawing tasks and then share the work during the Debrief Questions. Allow students to redraw, as necessary, after the Debrief Questions discussion. Task cards (24 per set) should be copied in sufficient quantity that pairs of students can share six cards.

Materials: (S) Task cards, 6 for each pair of students (Templates 1–4), ruler, set square, protractor, Problem Set (or blank paper)

T: (Project on the board: Draw a quadrilateral that has two pairs of equal sides. Tell as many names as you can for this shape. Circle the most specific name.) What shape could you draw to satisfy the attributes of this task? Turn and talk. Then, draw your shape.

S: I could draw a parallelogram. It has two pairs of equal sides. → A rectangle would work because it has two pairs of equal sides. → It says "two pairs of equal sides." I would draw a square. It has two sets of equal sides. The two sets also happen to be equal to each other. → A rhombus would work, too, because it is like a square. It has two sets of equal sides. → I could draw a kite. It has two pairs of equal sides. The sides that are equal are just next to each other rather than across from each other.

T: Compare your shape with your neighbor's. Did we all draw the same shape? Is there only one shape that would be correct for this task?

S: (Share work with partner.)

T: This is the shape I drew. (Project a rectangle.) Name this shape.

S: Rectangle. → Parallelogram. → Quadrilateral. → Polygon. → Trapezoid.

T: (Record student responses.) Which of the names we listed is the most specific?

S: Rectangle.

T: (Circle *rectangle* on the board.) Is there a quadrilateral that we should not construct for this task? Why not?

S: A trapezoid that is not a parallelogram because it would not have two pairs of equal sides. → An isosceles trapezoid would not work for this task because there would only be one set of equal sides.

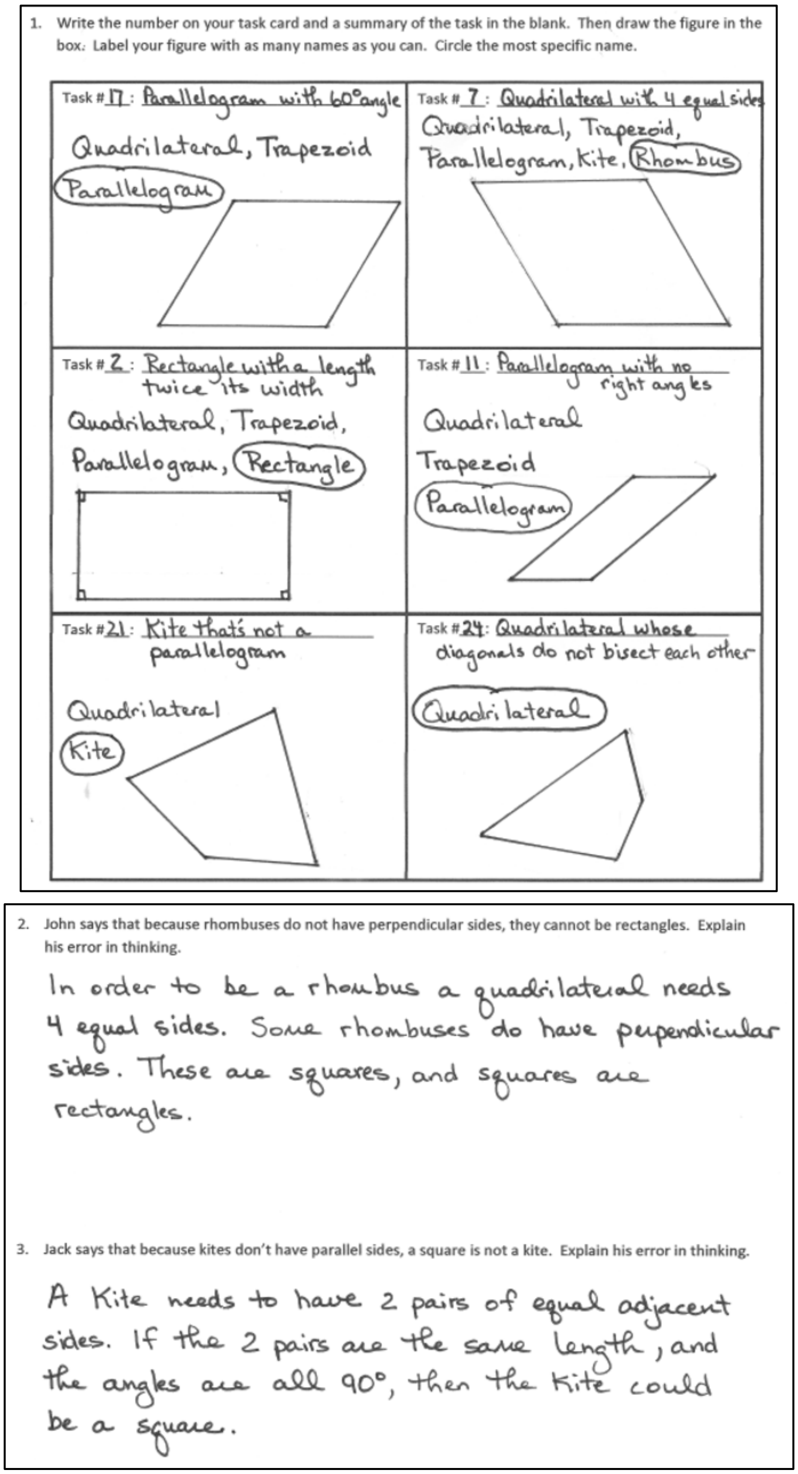
T: Pull six task cards from the envelope on your table. Record the number of the task and

YOUR NOTES

a brief summary of the task in the boxes on your Problem Set. Follow the directions on the cards to draw the shapes in the boxes.

S: (Work.)

The Problem Set serves as a recording sheet for the drawings in the lesson. Time should be given for students to share their approaches to constructing the figures on the task cards.



1. Write the number on your task card and a summary of the task in the blank. Then draw the figure in the box. Label your figure with as many names as you can. Circle the most specific name.

Task # 17: Parallelogram with 60° angle
Quadrilateral, Trapezoid, (Parallelogram)

Task # 7: Quadrilateral with 4 equal sides
Quadrilateral, Trapezoid, Parallelogram, Kite, (Rhombus)

Task # 2: Rectangle with a length twice its width
Quadrilateral, Trapezoid, Parallelogram, (Rectangle)

Task # 11: Parallelogram with no right angles
Quadrilateral, Trapezoid, (Parallelogram)

Task # 21: Kite that's not a parallelogram
Quadrilateral, (Kite)

Task # 24: Quadrilateral whose diagonals do not bisect each other
(Quadrilateral)

2. John says that because rhombuses do not have perpendicular sides, they cannot be rectangles. Explain his error in thinking.

In order to be a rhombus a quadrilateral needs 4 equal sides. Some rhombuses do have perpendicular sides. These are squares, and squares are rectangles.

3. Jack says that because kites don't have parallel sides, a square is not a kite. Explain his error in thinking.

A Kite needs to have 2 pairs of equal adjacent sides. If the 2 pairs are the same length, and the angles are all 90°, then the kite could be a square.

NOTES

Debrief Questions

- Find someone who completed two of the same tasks you did. Compare the shapes that you drew. Must they be the same shape to correctly follow the directions on the card? Why or why not?
- Which tasks produced quadrilaterals with the same specific name on everyone's Problem Set? Which tasks produced the most varied quadrilaterals?
- Choose three of your quadrilaterals, and paste them in the correct part of the hierarchy diagram. Explain why they belong there.
- Explain to your partner how you corrected John's error in Problem 2.
- What part of a kite's definition did Jack not understand in Problem 3? How did you correct his thinking?
- How do all the shapes that were drawn today fit the definition of a quadrilateral?

Multiple Means of Engagement

The task cards for today's lesson are numbered from simplest to most complex. Differentiate instruction by assigning tasks based on student need.

Multiple Means of Engagement

The relationships between sides and angles in quadrilaterals can serve as an interesting extension. Students can explore the effects of changing side lengths on angle size, and vice versa, with online tools like Interactive Quadrilaterals at:

http://www.mathisfun.com/geometry/quadrilaterals-interactive.html

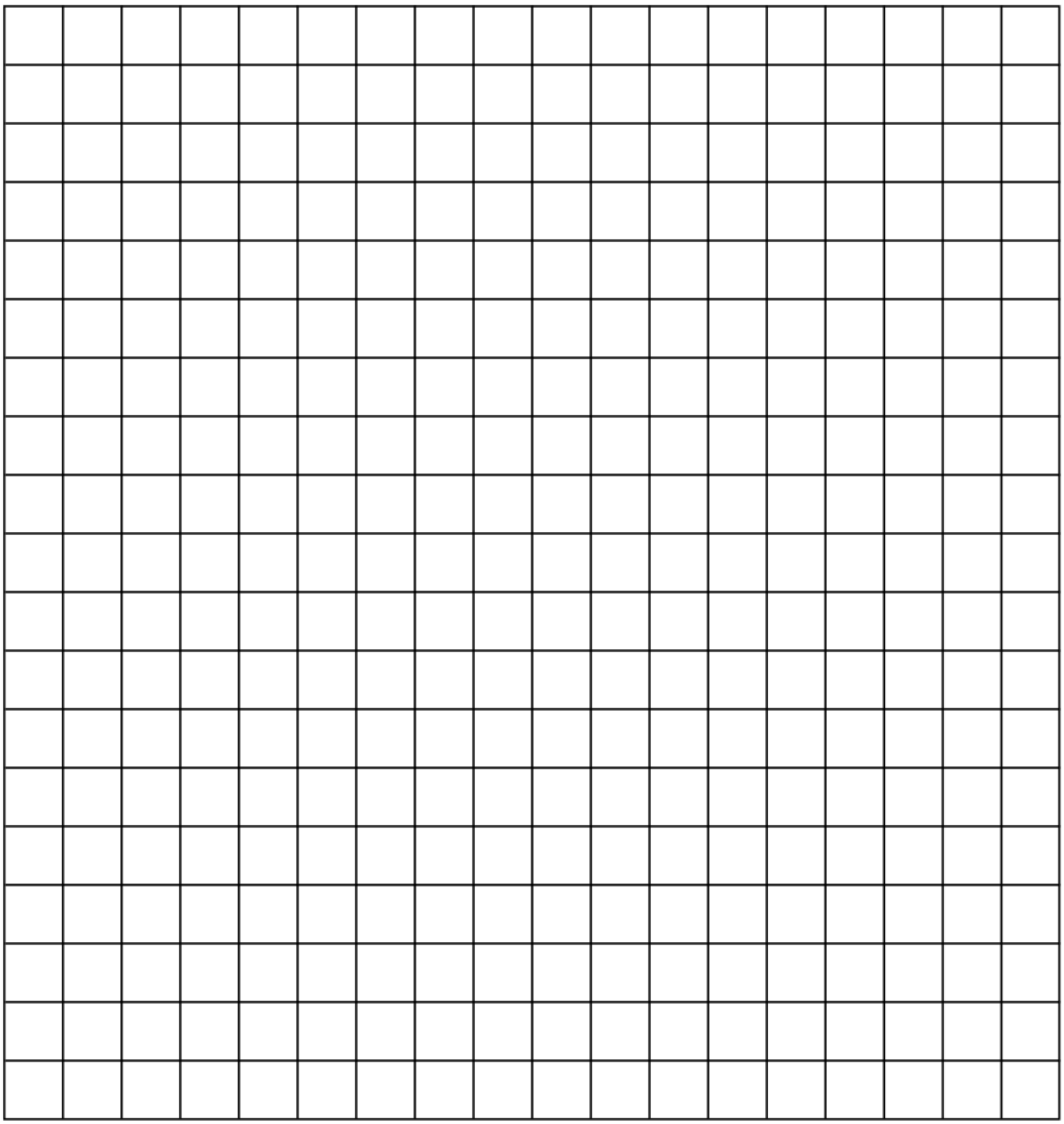
Appendix

Note: Student sheets should be printed at 100% scale to preserve size of figure for accurate measurements. Adjust printer or copier settings to "Actual Size" and set page scaling to "None."

Topic A: Concepts of Volume

Lesson 1

Centimeter Grid Paper (Template)



Isometric Dot Paper (Template)

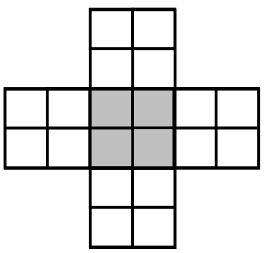
Lesson 2

Problem Set

Name ______________________________ Date ________________

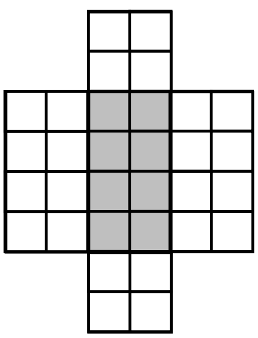
1. Shade the following figures on centimeter grid paper. Cut and fold each to make 3 open boxes, taping them so they hold their shapes. Pack each box with cubes. Write how many cubes fill each box.

a.



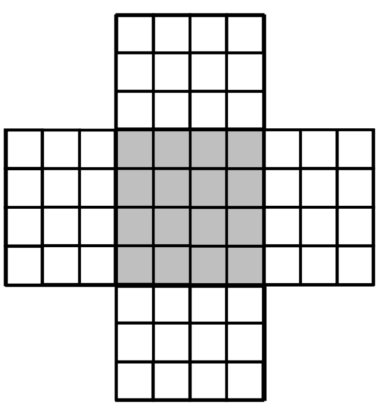
Number of cubes: ____________________

b.



Number of cubes: ____________________

c.



Number of cubes: ____________________

2. Predict how many centimeter cubes will fit in each box, and briefly explain your predictions. Use cubes to find the actual volume. (The figures are not drawn to scale.)

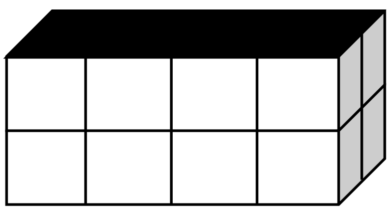
a. 

Prediction:

____

Actual:

____

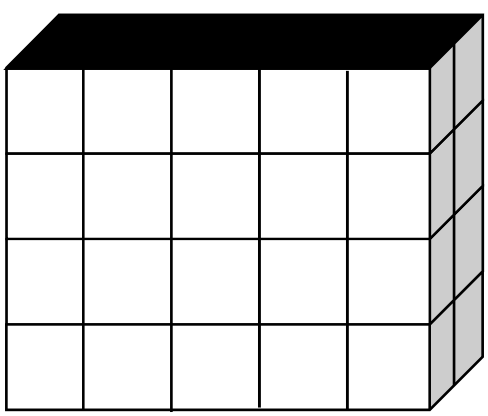
b. 

Prediction:

____

Actual:

____

c. 

Prediction:

____

Actual:

____

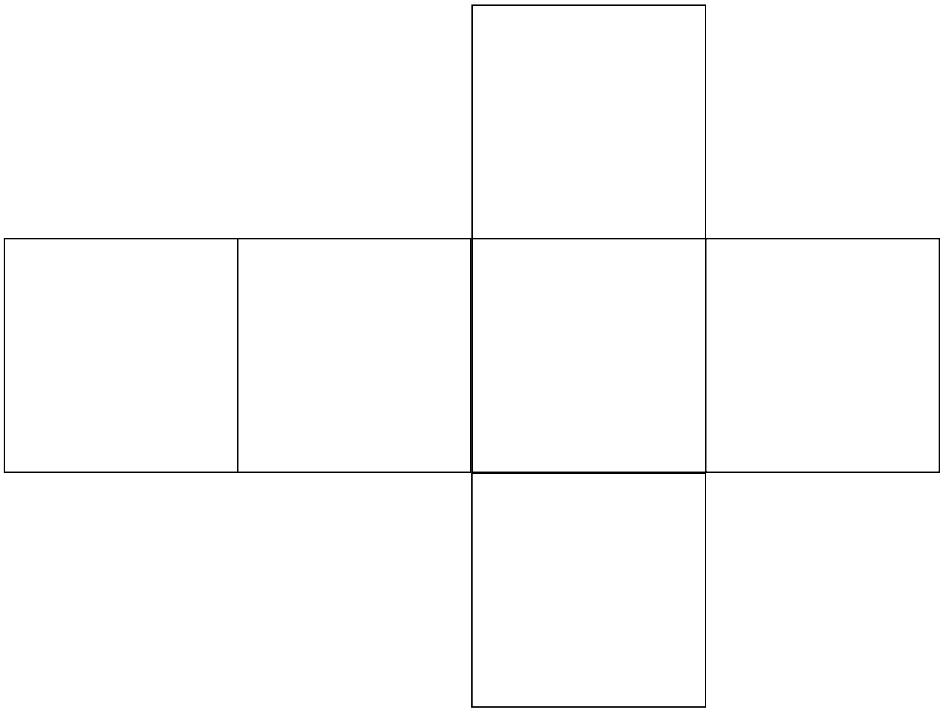
3. Cut out the net in the template, and fold it into a cube. Predict the number of 1-centimeter cubes that would be required to fill it.

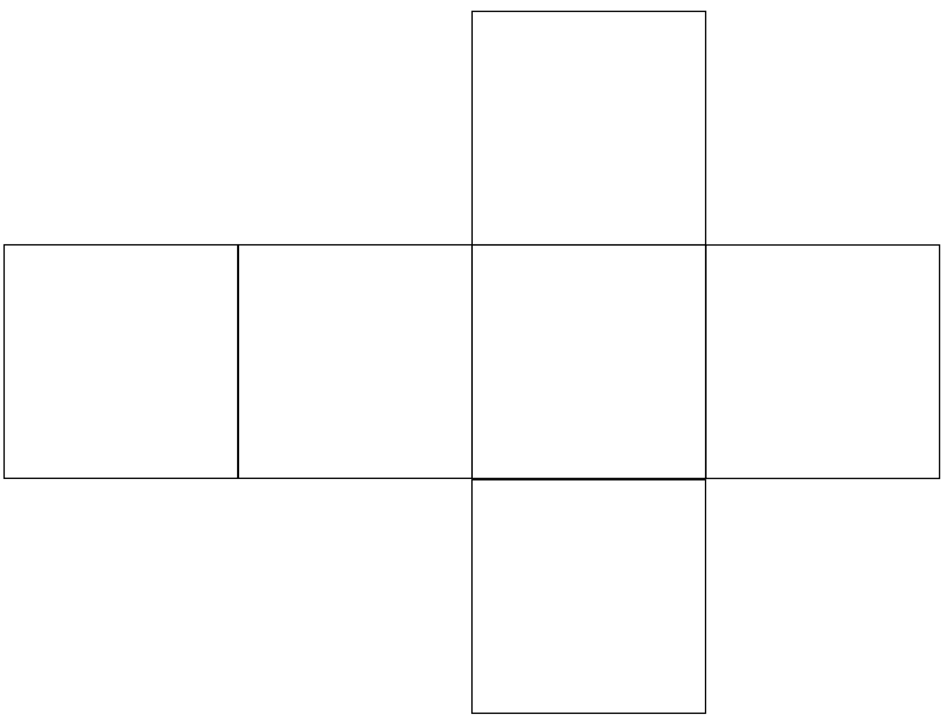
   a. Prediction: ______________________

   b. Explain your thought process as you made your prediction.

   c. How many 1-centimeter cubes are used to fill the figure? Was your prediction accurate?

Net (Template)

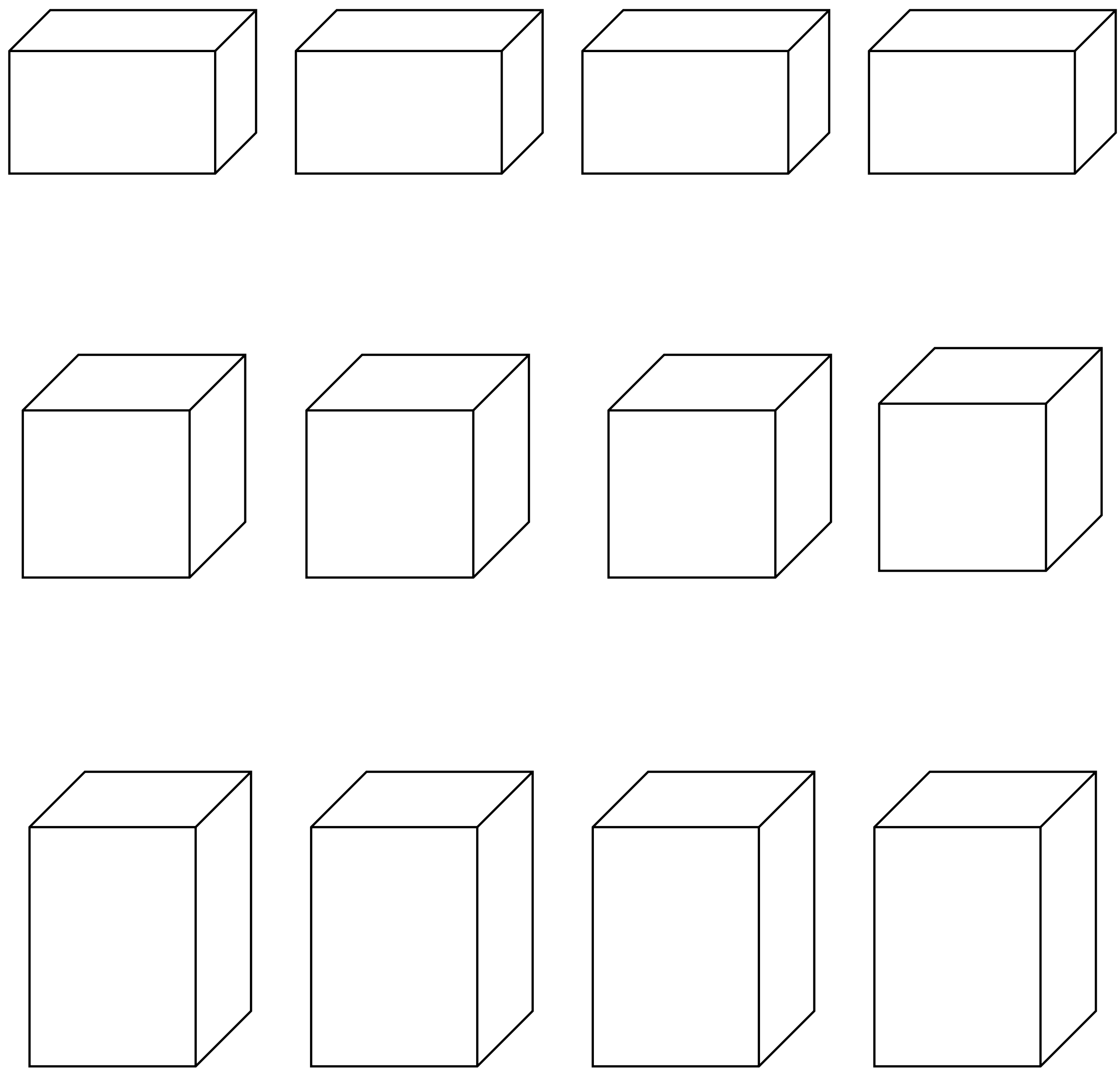




Lesson 3

Rectangular Prism Recording Sheet (Template)

Name ______________________________ Date ______________



Topic B: Volume and the Operations of Multiplication and Addition

Lesson 5

Problem Set

Name ______________________________ Date ______________

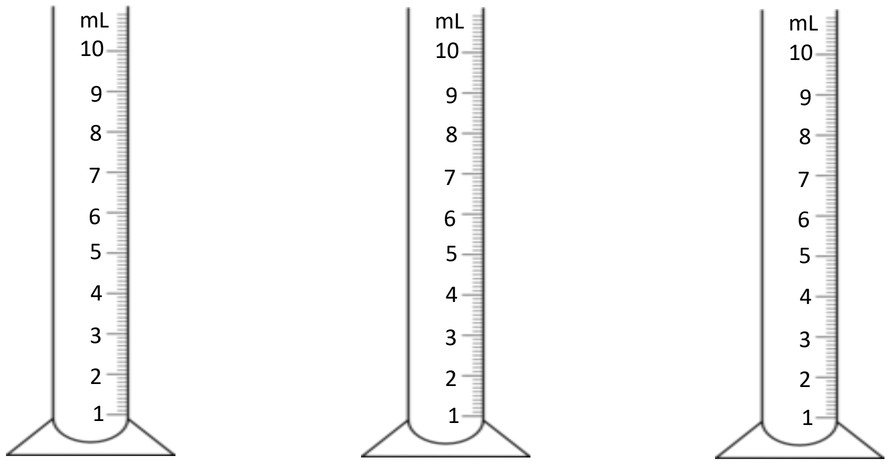
1. Determine the volume of two boxes on the table using cubes, and then confirm by measuring and multiplying.

Box Number	Number of Cubes Packed	Measurements			Volume
		Length	Width	Height	

2. Using the same boxes from Problem 1, record the amount of liquid that your box can hold.

Box Number	Liquid the Box Can Hold
	mL
	mL

3. Shade to show the water in the beaker.



At first:	After 1 mL water added:	After 1 cm cube added:
___________ mL	___________ mL	___________ mL

Lesson 7

Problem Set

Name ______________________________ Date ____________

Geoffrey builds rectangular planters.

1. Geoffrey's first planter is 8 feet long and 2 feet wide. The container is filled with soil to a height of 3 feet in the planter. What is the volume of soil in the planter? Explain your work using a diagram.

2. Geoffrey wants to grow some tomatoes in four large planters. He wants each planter to have a volume of 320 cubic feet, but he wants them all to be different. Show four different ways Geoffrey can make these planters, and draw diagrams with the planters' measurements on them.

Planter A	Planter B
Planter C	Planter D

3. Geoffrey wants to make one planter that extends from the ground to just below his back window. The window starts 3 feet off the ground. If he wants the planter to hold 36 cubic feet of soil, name one way he could build the planter so it is not taller than 3 feet. Explain how you know.

4. After all of this gardening work, Geoffrey decides he needs a new shed to replace the old one. His current shed is a rectangular prism that measures 6 feet long by 5 feet wide by 8 feet high. He realizes he needs a shed with 480 cubic feet of storage.

   a. Will he achieve his goal if he doubles each dimension? Why or why not?

   b. If he wants to keep the height the same, what could the other dimensions be for him to get the volume he wants?

   c. If he uses the dimensions in part (b), what could be the area of the new shed's floor?

Lesson 8

Problem Set

Name ______________________________ Date ____________

Using the box patterns, construct a sculpture containing at least 5, but not more than 7, rectangular prisms that meets the following requirements in the table below.

1.	My sculpture has 5 to 7 rectangular prisms. Number of prisms: __________	
2.	Each prism is labeled with a letter, dimensions, and volume.	
	Prism A ________ by ________ by ________ Volume = ________ Prism B ________ by ________ by ________ Volume = ________ Prism C ________ by ________ by ________ Volume = ________ Prism D ________ by ________ by ________ Volume = ________ Prism E ________ by ________ by ________ Volume = ________ Prism ___ ________ by ________ by ________ Volume = ________ Prism ___ ________ by ________ by ________ Volume = ________	
3.	Prism D has $\frac{1}{2}$ the volume of Prism _____.	Prism D Volume = ________ Prism _____ Volume = ________
4.	Prism E has $\frac{1}{3}$ the volume of Prism _____.	Prism E Volume = ________ Prism _____ Volume = ________
5.	The total volume of all the prisms is 1,000 cubic centimeters or less.	Total volume: ______________ Show calculations:

Project Requirements (Template 1)

Project Requirements

1. Each project must include 5 to 7 rectangular prisms.
2. All prisms must be labeled with a letter (beginning with A), dimensions, and volume.
3. Prism D must be $\frac{1}{2}$ the volume of another prism.
4. Prism E must be $\frac{1}{3}$ the volume of another prism.
5. The total volume of all of the prisms must be 1,000 cubic centimeters or less.

Project Requirements

1. Each project must include 5 to 7 rectangular prisms.
2. All prisms must be labeled with a letter (beginning with A), dimensions, and volume.
3. Prism D must be $\frac{1}{2}$ the volume of another prism.
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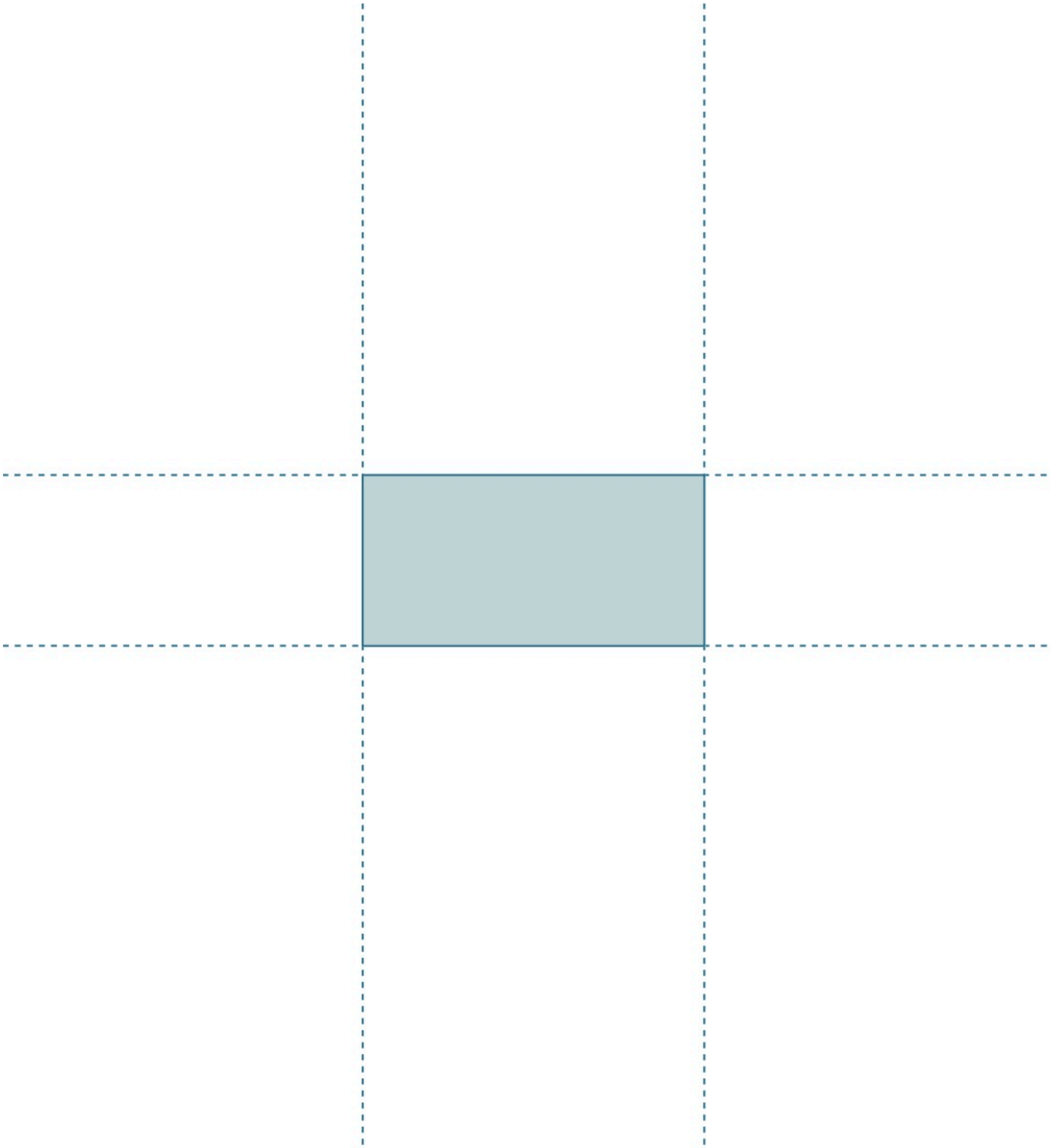
Project Requirements

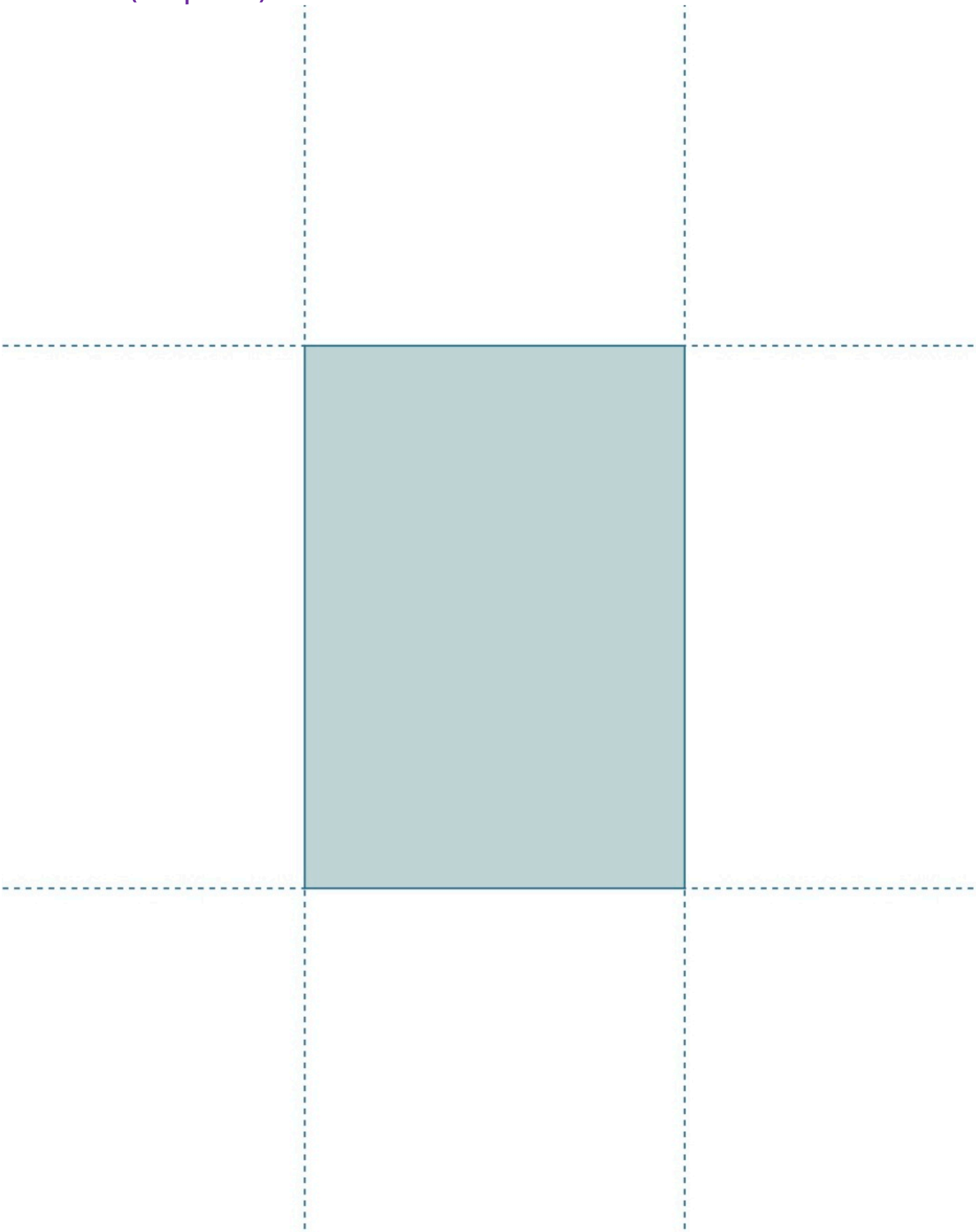
1. Each project must include 5 to 7 rectangular prisms.
2. All prisms must be labeled with a letter (beginning with A), dimensions, and volume.
3. Prism D must be $\frac{1}{2}$ the volume of another prism.
4. Prism E must be $\frac{1}{3}$ the volume of another prism.
5. The total volume of all of the prisms must be 1,000 cubic centimeters or less.

Box Pattern a (Template 2)

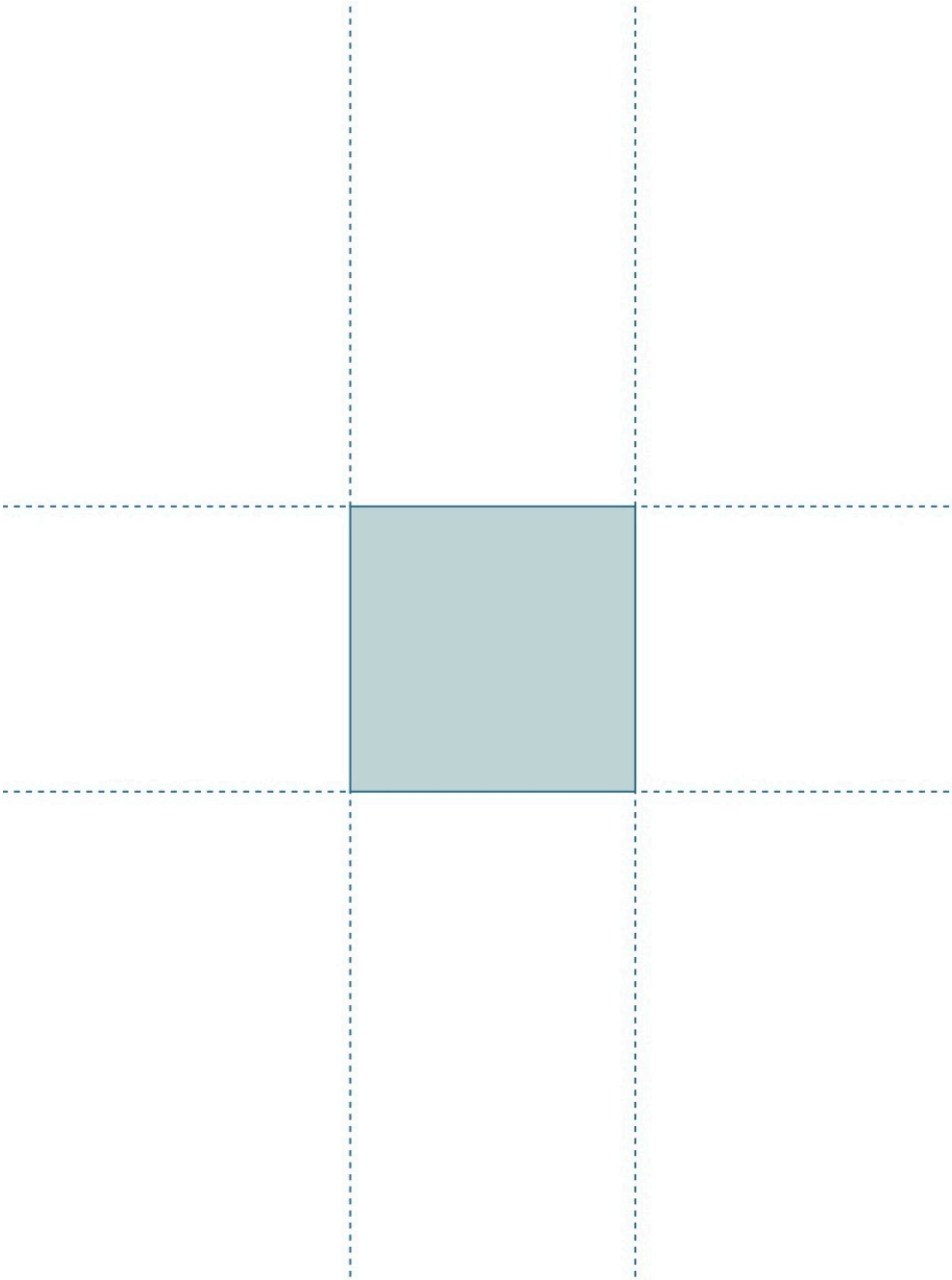
Note: Be sure to set printer to *actual size* before printing.



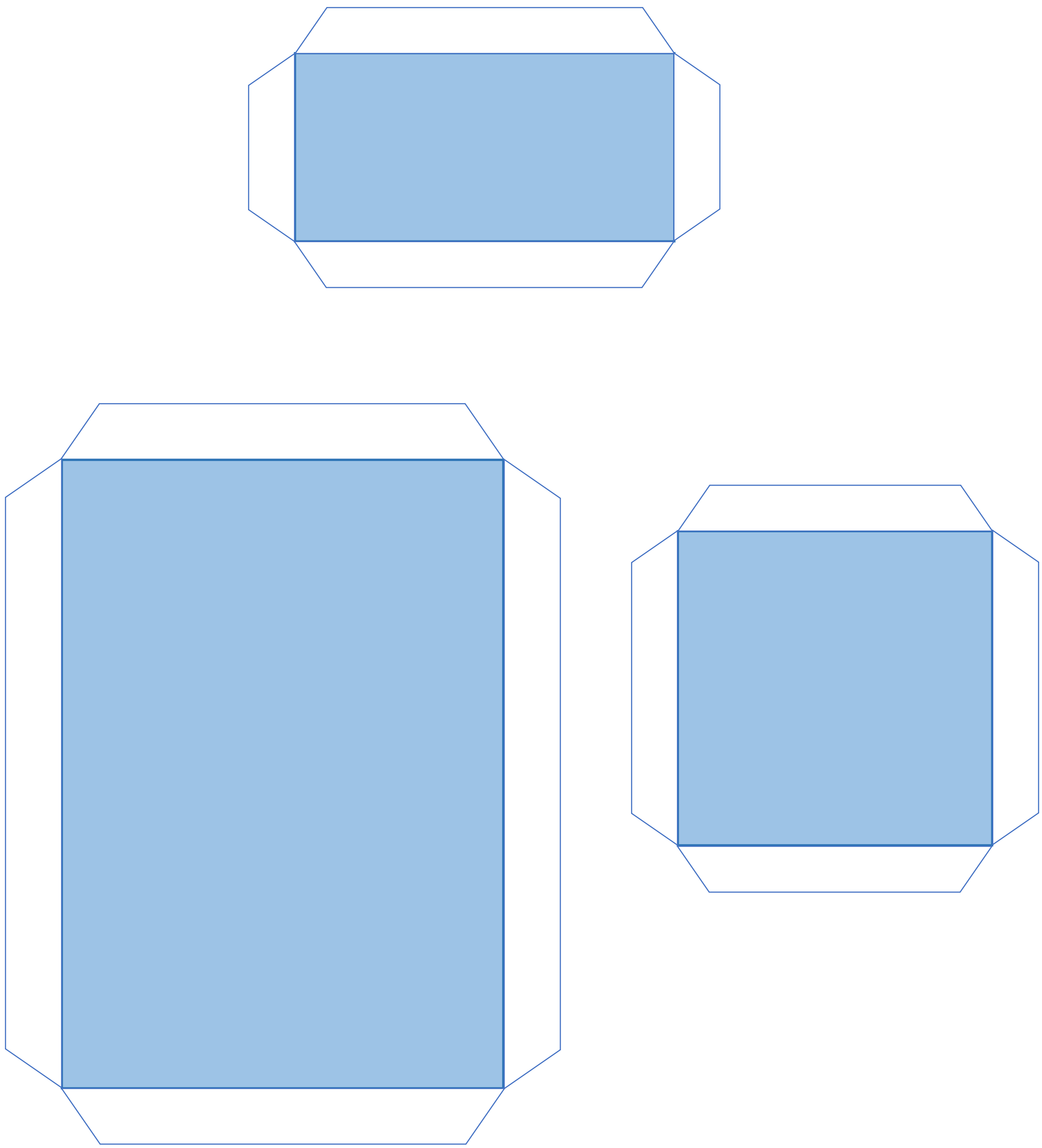
Box Pattern b (Template 3)



Box Pattern c (Template 4)



Lid Patterns (Template 5)



Evaluation Rubric (Template 6)

CATEGORY	4	3	2	1	Subtotal
Completeness of Personal Project and Classmate Evaluation	All components of the project are present and correct, and a detailed evaluation of a classmate's project has been completed.	Project is missing 1 component, and a detailed evaluation of a classmate's project has been completed.	Project is missing 2 components, and an evaluation of a classmate's project has been completed.	Project is missing 3 or more components, and an evaluation of a classmate's project has been completed.	(× 4) ________/16
Accuracy of Calculations	Volume calculations for all prisms are correct.	Volume calculations include 1 error.	Volume calculations include 2–3 errors.	Volume calculations include 4 or more errors.	(× 5) ________/20
Neatness and Use of Color	All elements of the project are carefully and colorfully constructed.	Some elements of the project are carefully and colorfully constructed.	Project lacks color or is not carefully constructed.	Project lacks color and is not carefully constructed.	(× 2) ________/4
					TOTAL: ________/40

Lesson 9

Problem Set

Name ______________________________ Date ______________

I reviewed project number ____________.

Use the rubric below to evaluate your friend's project. Ask questions and measure the parts to determine whether your friend has all the required elements. Respond to the prompt in italics in the third column. The final column can be used to write something you find interesting about that element if you like.

Space is provided beneath the rubric for your calculations.

	Requirement	Element Present? (✓)	Specifics of Element	Notes
1.	The sculpture has 5 to 7 prisms.		*# of prisms:*	
2.	All prisms are labeled with a letter.		*Write letters used:*	
3.	All prisms have correct dimensions with units written on the top.		*List any prisms with incorrect dimensions or units:*	
4.	All prisms have correct volume with units written on the top.		*List any prism with incorrect dimensions or units:*	
5.	Prism D has $\frac{1}{2}$ the volume of another prism.		*Record on next page:*	
6.	Prism E has $\frac{1}{3}$ the volume of another prism.		*Record on next page:*	
7.	The total volume of all the parts together is 1,000 cubic units or less.		*Total volume:*	

Calculations:

8. Measure the dimensions of each prism. Calculate the volume of each prism and the total volume. Record that information in the table below. If your measurements or volume differ from those listed on the project, put a star by the prism label in the table below, and record on the rubric.

Prism	Dimensions	Volume
A	_________ by _________ by _________	
B	_________ by _________ by _________	
C	_________ by _________ by _________	
D	_________ by _________ by _________	
E	_________ by _________ by _________	
	_________ by _________ by _________	
	_________ by _________ by _________	

9. Prism D's volume is $\frac{1}{2}$ that of Prism _____________.

   Show calculations below.

10. Prism E's volume is $\frac{1}{3}$ that of Prism _____________.

    Show calculations below.

11. Total volume of sculpture: _____________.

    Show calculations below.

Topic C: Area of Rectangular Figures with Fractional Side Lengths

Lesson 10

Problem Set

Name ______________________________ Date ____________

Sketch the rectangles and your tiling. Write the dimensions and the units you counted in the blanks. Then, use multiplication to confirm the area. Show your work. We will do Rectangles A and B together.

1. **Rectangle A:**

Rectangle A is

________ units long ________ units wide

Area = ________ units²

2. **Rectangle B:**

Rectangle B is

________ units long ________ units wide

Area = ________ units²

3. **Rectangle C:**

Rectangle C is

________ units long ________ units wide

Area = ________ units²

4. **Rectangle D:**

5. **Rectangle E:**

Rectangle D is

_________ units long _________ units wide

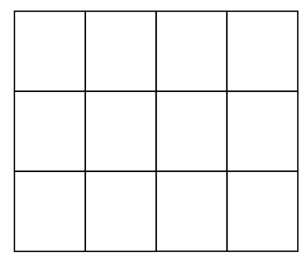
Area = _________ units^2

Rectangle E is

_________ units long _________ units wide

Area = _________ units^2

6. The rectangle to the right is composed of squares that measure $2\frac{1}{4}$ inches on each side. What is its area in square inches? Explain your thinking using pictures and numbers.



7. A rectangle has a perimeter of $35\frac{1}{2}$ feet. If the length is 12 feet, what is the area of the rectangle?

Lesson 11

Problem Set

Name ________________________________ Date ____________

Draw the rectangle and your tiling. Write the dimensions and the units you counted in the blanks. Then, use multiplication to confirm the area. Show your work.

1. **Rectangle A:**

Rectangle A is

________ units long ________ units wide

Area = ________ units²

2. **Rectangle B:**

Rectangle B is

________ units long ________ units wide

Area = ________ units²

3. **Rectangle C:**

Rectangle C is

________ units long ________ units wide

Area = ________ units²

4. **Rectangle D:**

Rectangle D is

________ units long ________ units wide

Area = ________ units²

5. Colleen and Caroline each built a rectangle out of square tiles placed in 3 rows of 5. Colleen used tiles that measured $1\frac{2}{3}$ cm in length. Caroline used tiles that measured $3\frac{1}{3}$ cm in length.

   a. Draw the girls' rectangles, and label the lengths and widths of each.

   b. What are the areas of the rectangles in square centimeters?

   c. Compare the areas of the rectangles.

6. A square has a perimeter of 51 inches. What is the area of the square?

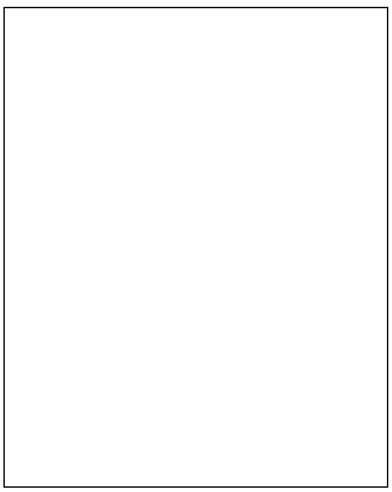
Lesson 12

Problem Set

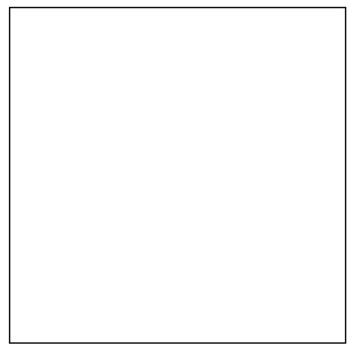
Name ______________________________ Date ______________

1. Measure each rectangle to the nearest $\frac{1}{4}$ inch with your ruler, and label the dimensions. Use the area model to find each area.

a.

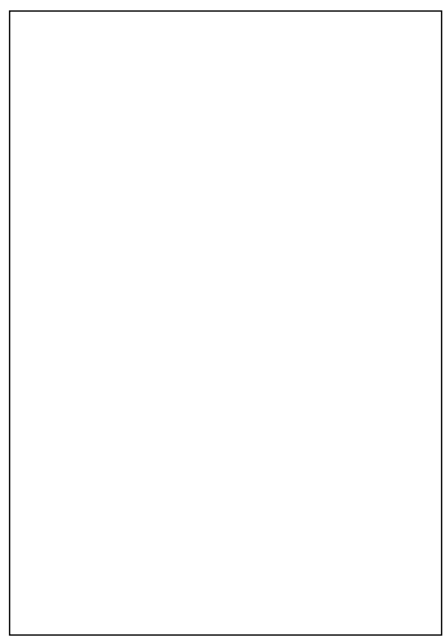


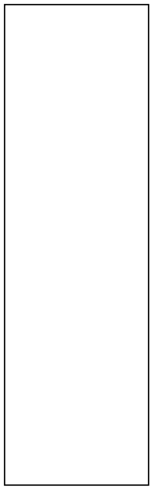
b.



c.

d.



e. 

f. 

2. Find the area of rectangles with the following dimensions. Explain your thinking using the area model.

a. $1 \text{ ft} \times 1\frac{1}{2} \text{ ft}$

b. $1\frac{1}{2} \text{ yd} \times 1\frac{1}{2} \text{ yd}$

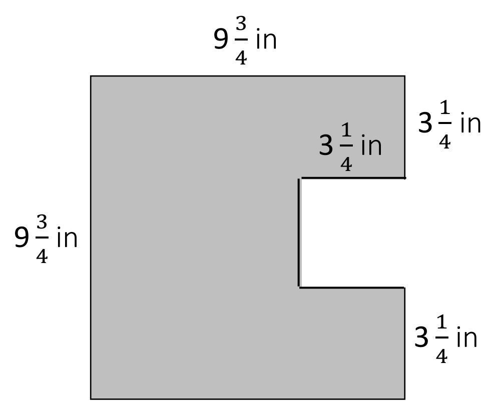
c. $2\frac{1}{2} \text{ yd} \times 1\frac{3}{16} \text{ yd}$

3. Hanley is putting carpet in her house. She wants to carpet her living room, which measures 15 ft × $12\frac{1}{3}$ ft. She also wants to carpet her dining room, which is $10\frac{1}{4}$ ft × $10\frac{1}{3}$ ft. How many square feet of carpet will she need to cover both rooms?

4. Fred cut a $9\frac{3}{4}$-inch square of construction paper for an art project. He cut a square from the edge of the big rectangle whose sides measured $3\frac{1}{4}$ inches. (See the picture below.)

   a. What is the area of the smaller square that Fred cut out?

   b. What is the area of the remaining paper?

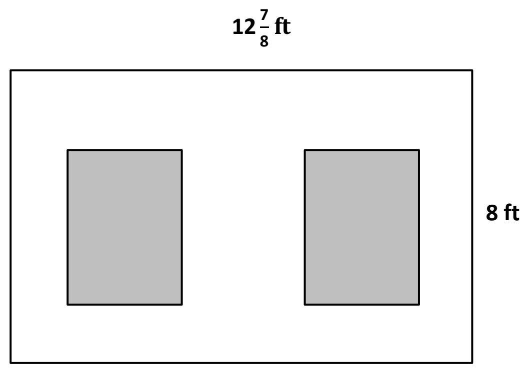


Lesson 14

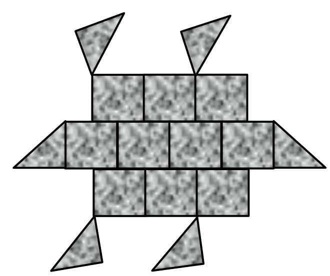
Problem Set

Name ______________________________ Date ____________

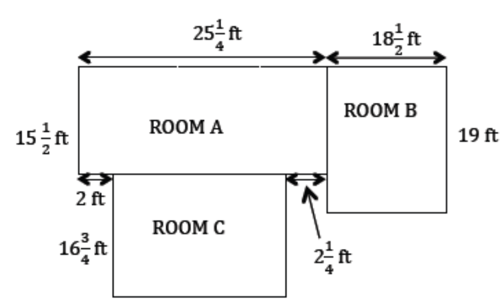
1. George decided to paint a wall with two windows. Both windows are $3\frac{1}{2}$-ft by $4\frac{1}{2}$-ft rectangles. Find the area the paint needs to cover.



2. Joe uses square tiles, some of which he cuts in half, to make the figure below. If each square tile has a side length of $2\frac{1}{2}$ inches, what is the total area of the figure?



3. All-In-One Carpets is installing carpeting in three rooms. How many square feet of carpet are needed to carpet all three rooms?



4. Mr. Johnson needs to buy sod for his front lawn.
   a. If the lawn measures $36\frac{2}{3}$ ft by $45\frac{1}{6}$ ft, how many square feet of sod will he need?

   b. If sod is only sold in whole square feet, how much will Mr. Johnson have to pay?

Sod Prices

Area	**Price per Square Foot**
First 1,000 sq ft	\$0.27
Next 500 sq ft	\$0.22
Additional square feet	\$0.19

5. Jennifer's class decides to make a quilt. Each of the 24 students will make a quilt square that is 8 inches on each side. When they sew the quilt together, every edge of each quilt square will lose $\frac{3}{4}$ of an inch.
   a. Draw one way the squares could be arranged to make a rectangular quilt. Then, find the perimeter of your arrangement.

   b. Find the area of the quilt.

Lesson 15

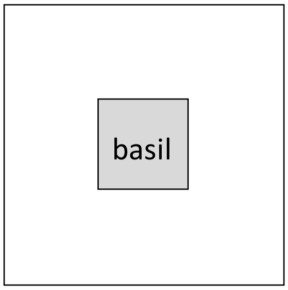
Problem Set

Name ______________________________ Date ______________

1. The length of a flowerbed is 4 times as long as its width. If the width is $\frac{3}{8}$ meter, what is the area?

2. Mrs. Johnson grows herbs in square plots. Her basil plot measures $\frac{5}{8}$ yd on each side.

   a. Find the total area of the basil plot.



   b. Mrs. Johnson puts a fence around the basil. If the fence is 2 ft from the edge of the garden on each side, what is the perimeter of the fence in feet?

c. What is the total area, in square feet, that the fence encloses?

3. Janet bought 5 yards of fabric $2\frac{1}{4}$-feet wide to make curtains. She used $\frac{1}{3}$ of the fabric to make a long set of curtains and the rest to make 4 short sets.

   a. Find the area of the fabric she used for the long set of curtains.

   b. Find the area of the fabric she used for each of the short sets.

4. Some wire is used to make 3 rectangles: A, B, and C. Rectangle B's dimensions are $\frac{3}{5}$ cm larger than Rectangle A's dimensions, and Rectangle C's dimensions are $\frac{3}{5}$ cm larger than Rectangle B's dimensions. Rectangle A is 2 cm by $3\frac{1}{5}$ cm.

   a. What is the total area of all three rectangles?

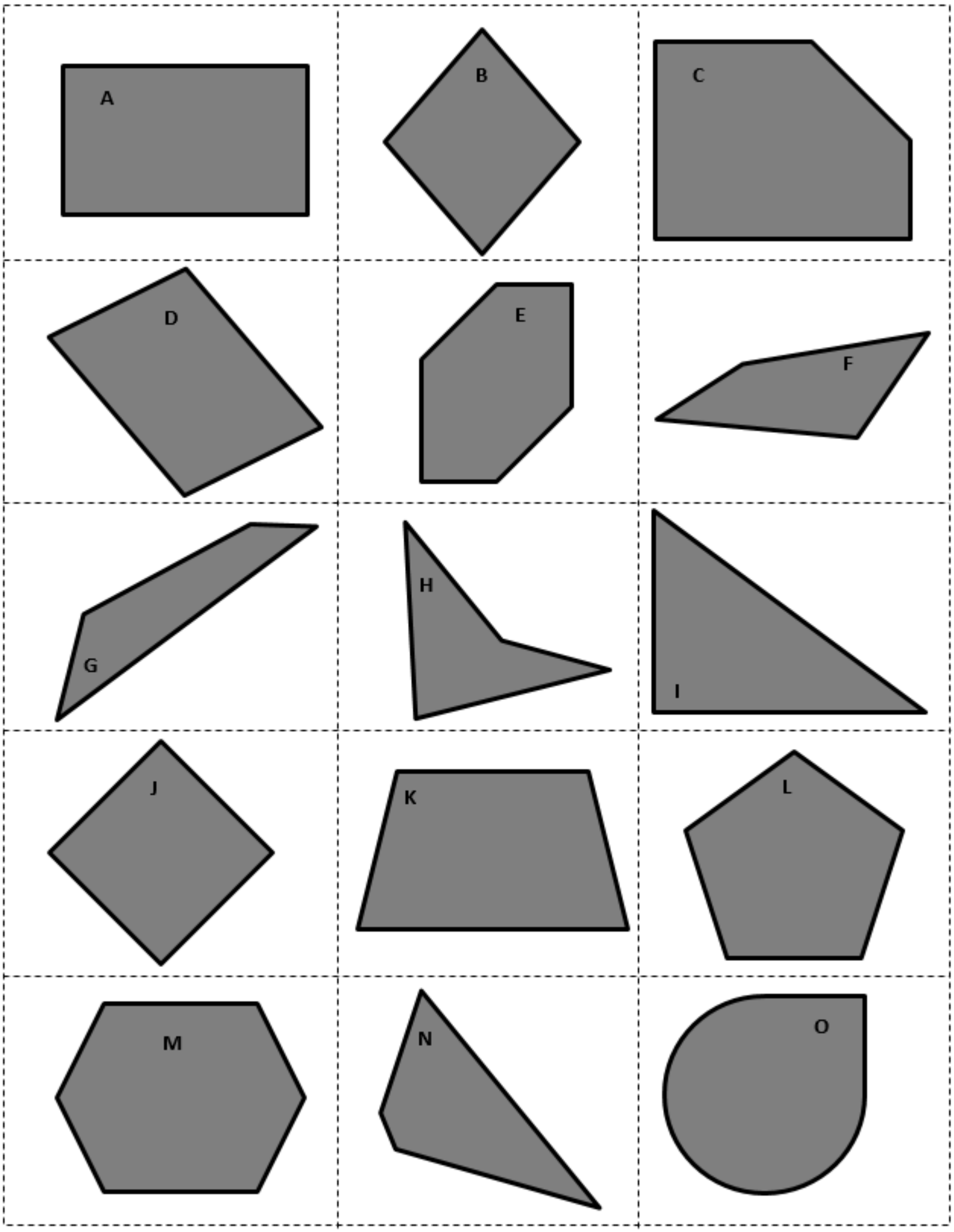
   b. If a 40-cm coil of wire was used to form the rectangles, how much wire is left?

Topic D: Drawing, Analysis, and Classification of Two-Dimensional Shapes

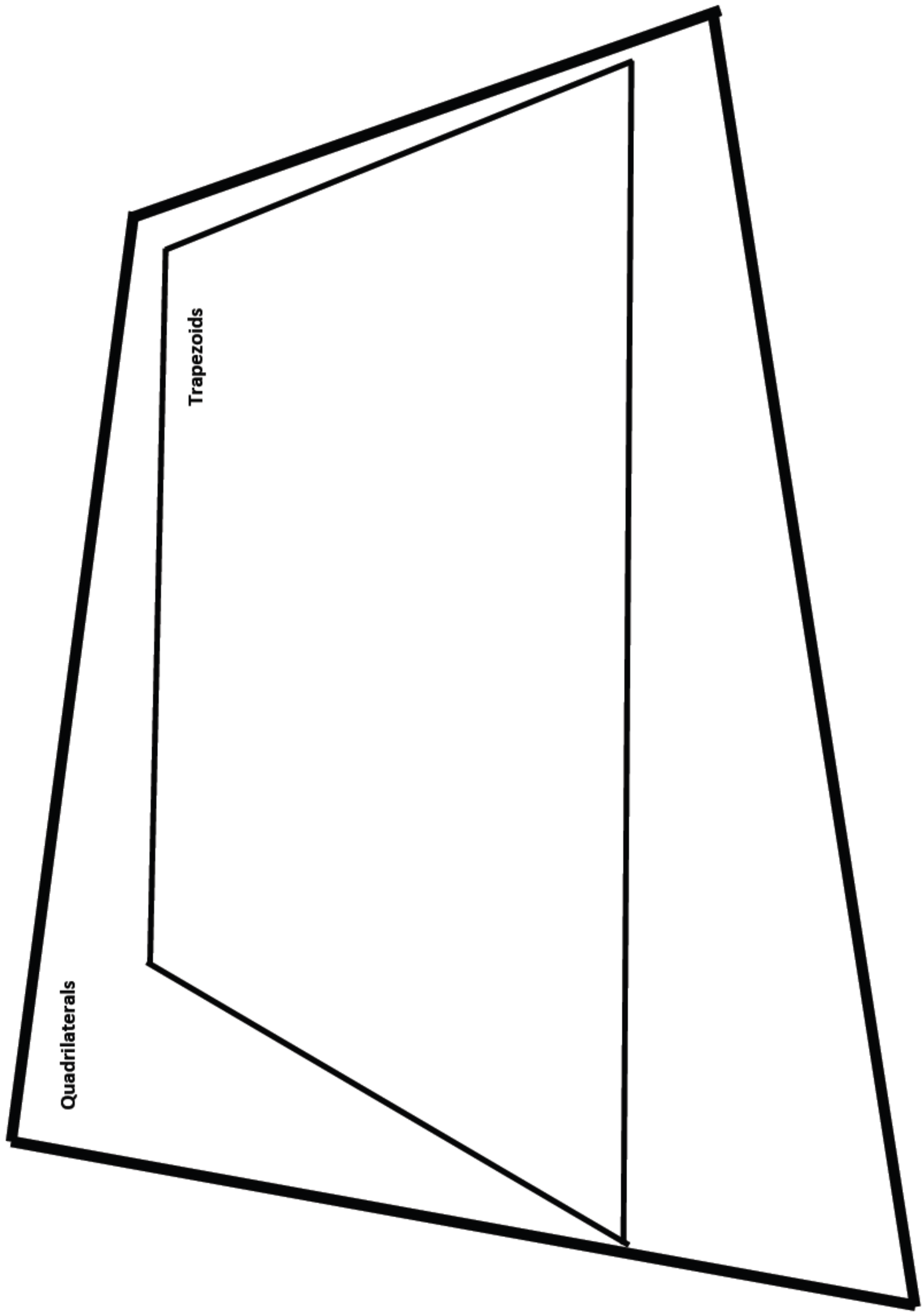
Lesson 16

Collection of Polygons (Template 1)

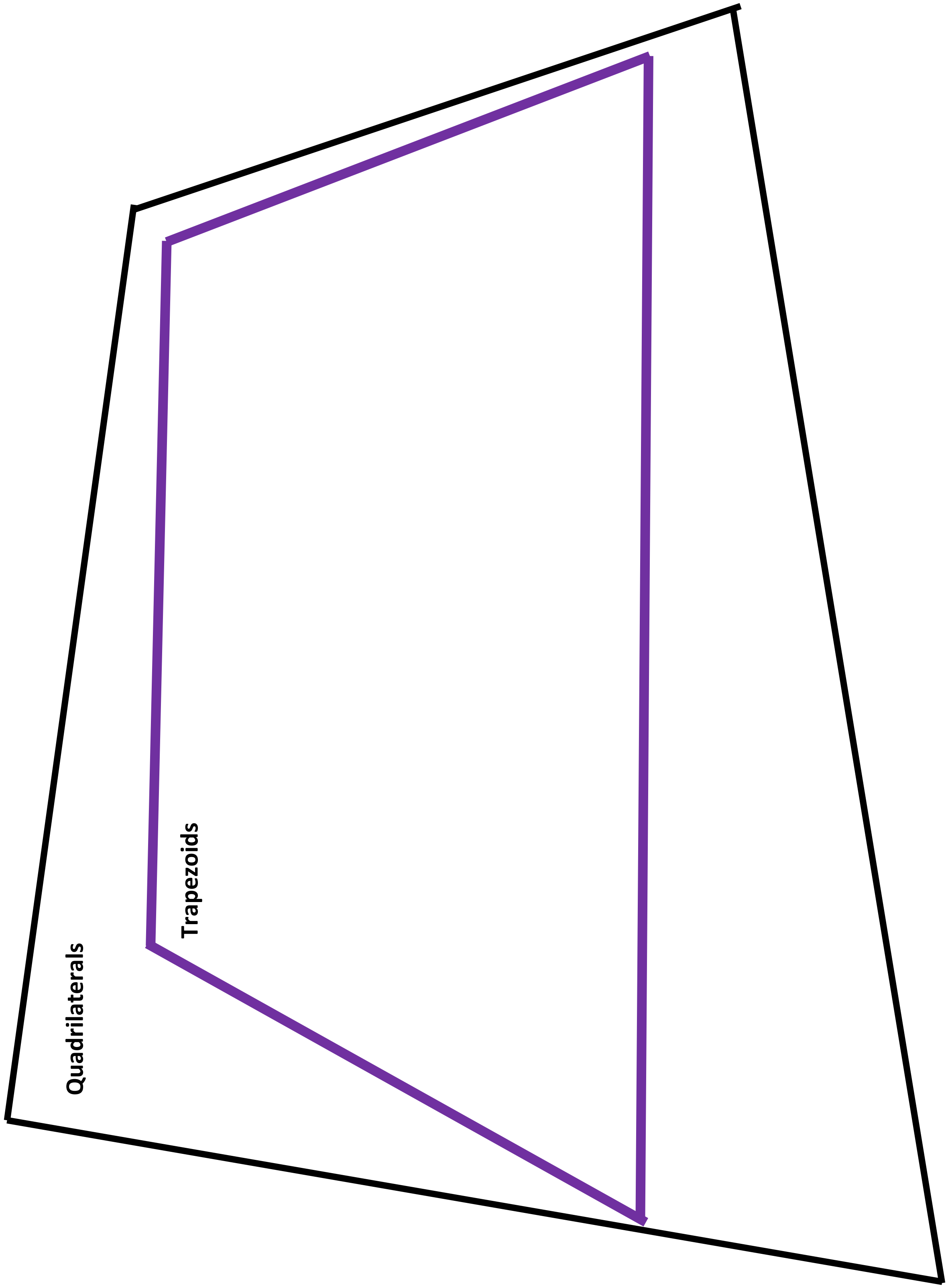
(See next page)


A
B
C
D
E
F
G
H
I
J
K
L
M
N
O

Quadrilateral Hierarchy (Template 2)

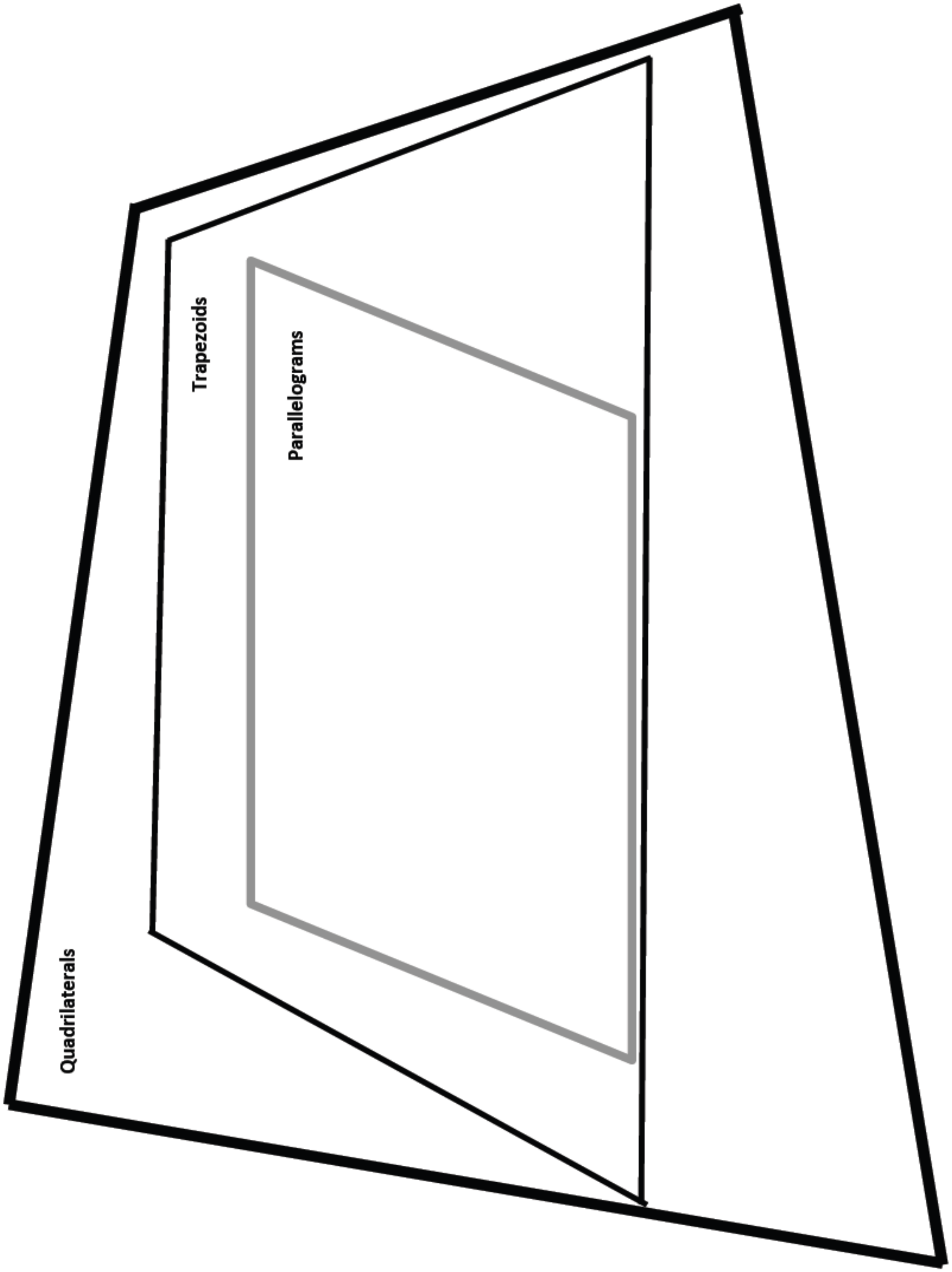


Quadrilateral Hierarchy: Color (Template 3)

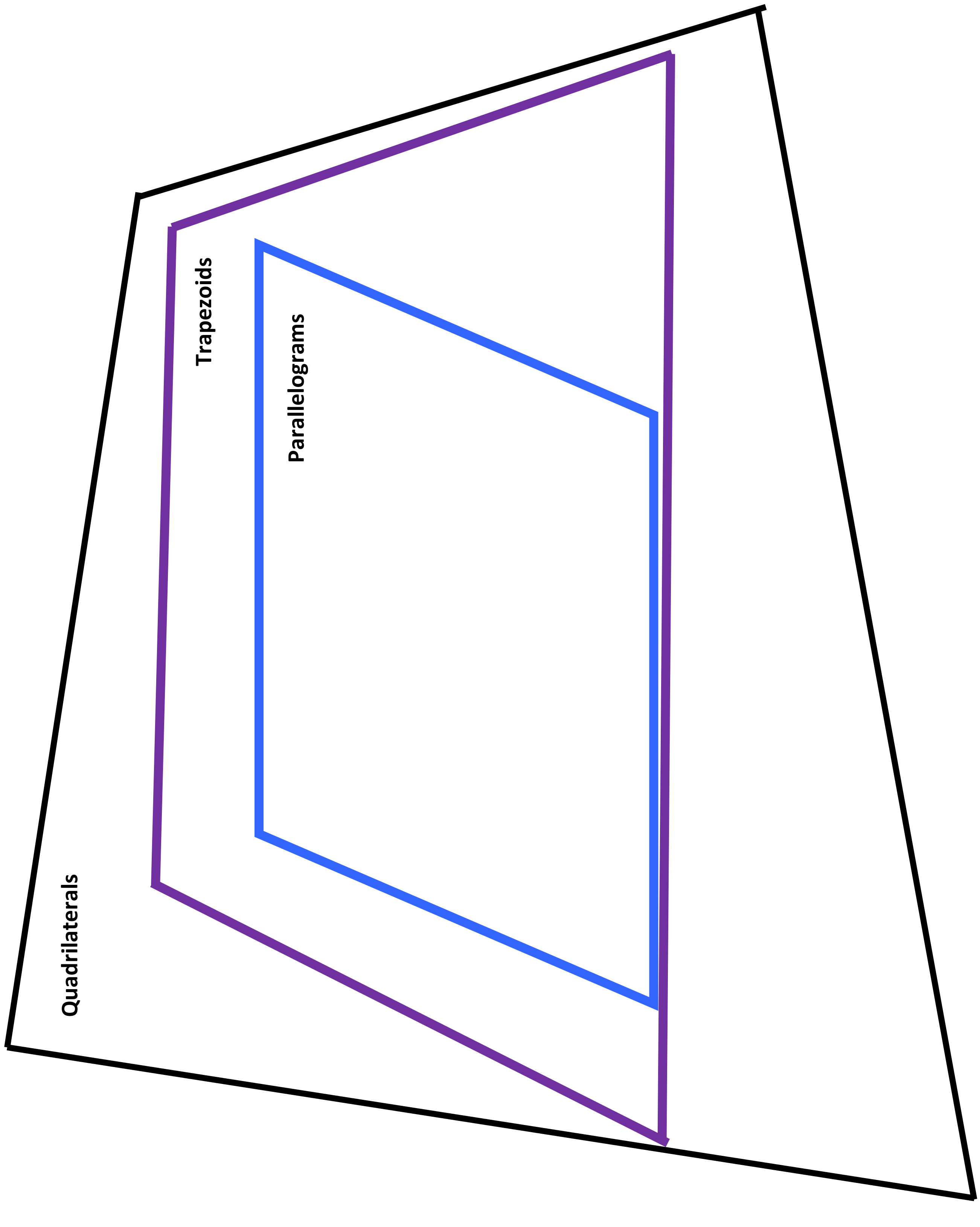


Lesson 17

Quadrilateral Hierarchy with Parallelogram (Template 1)

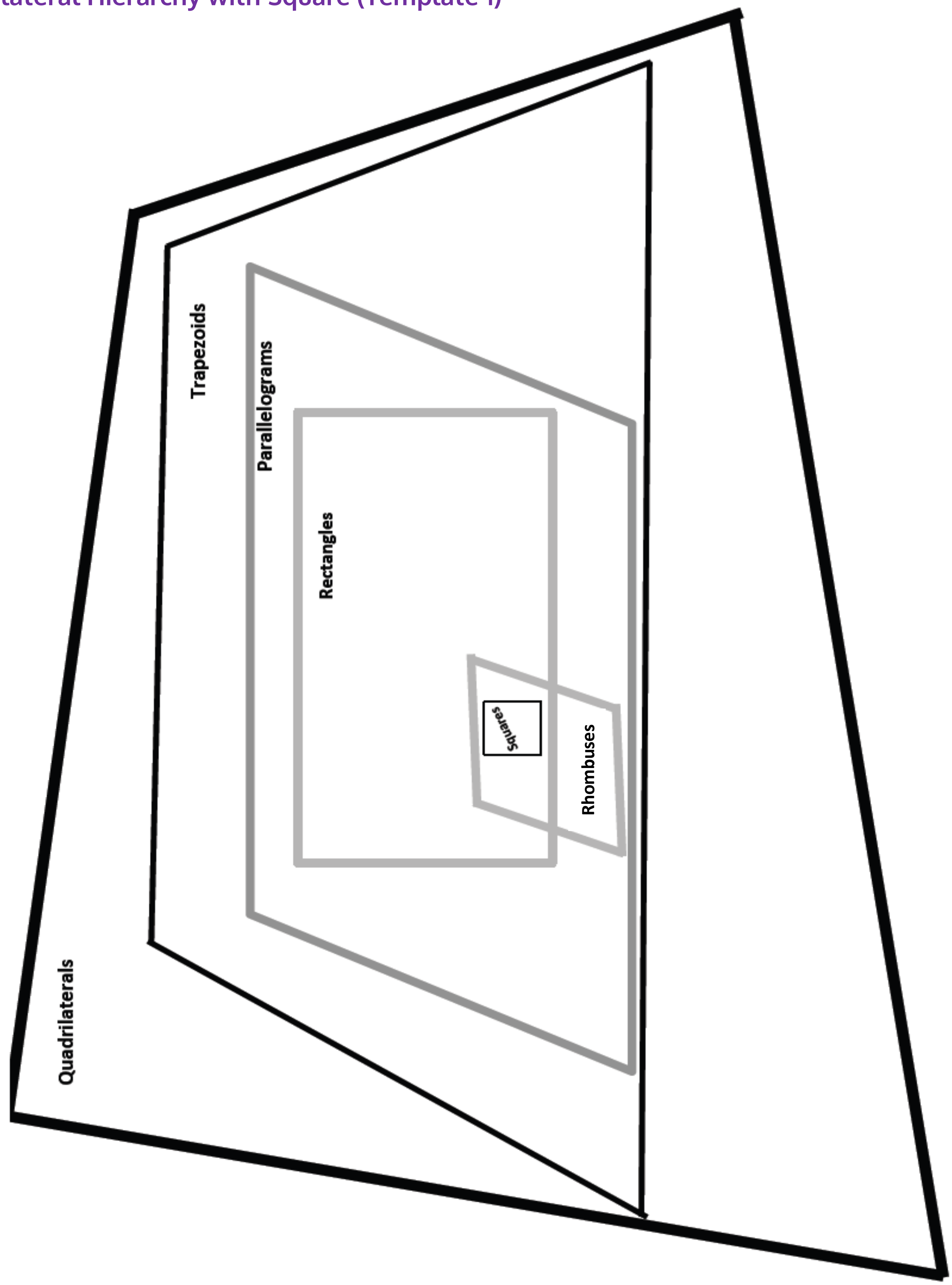


Quadrilateral Hierarchy with Parallelogram: Color (Template 2)

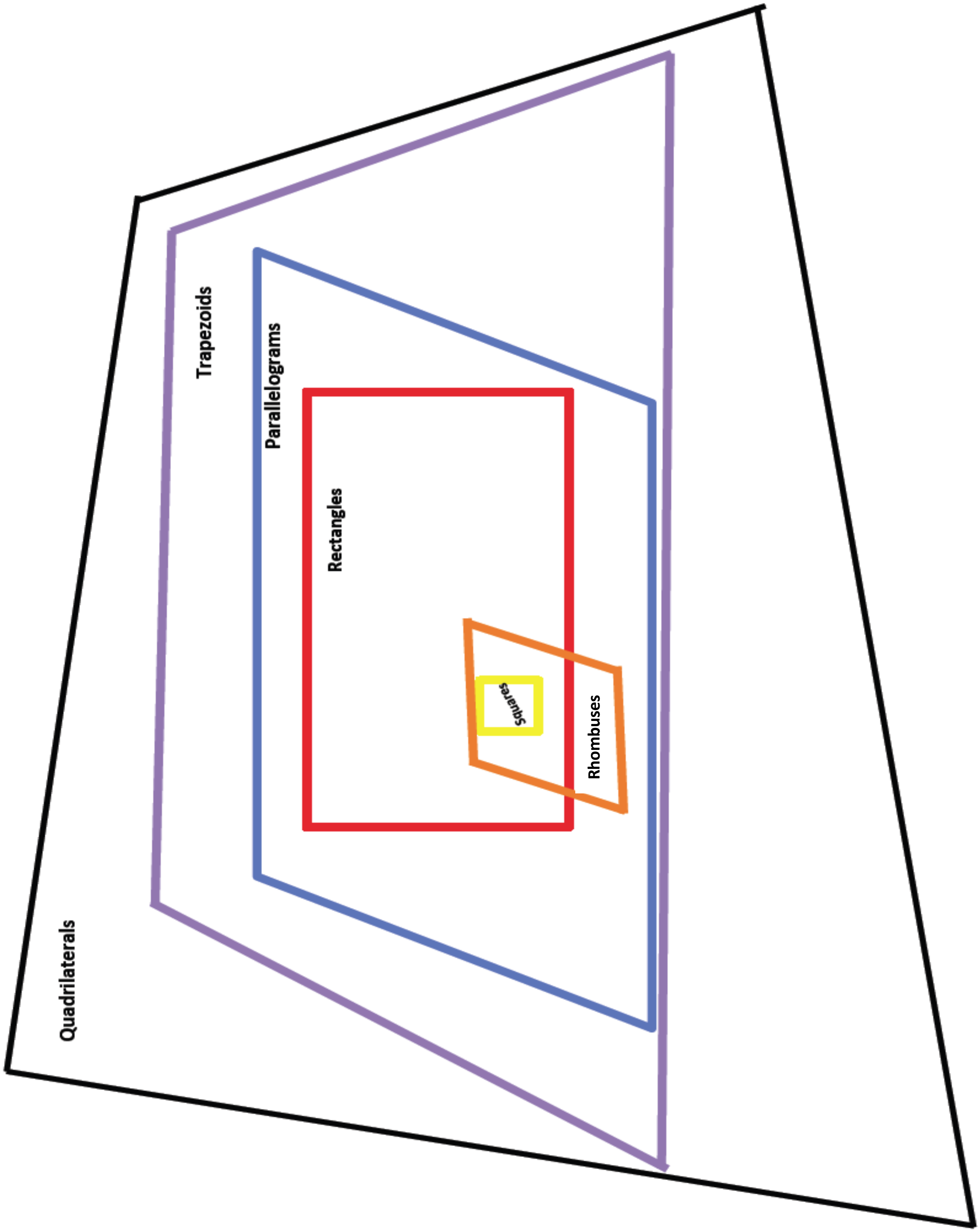


Lesson 18

Quadrilateral Hierarchy with Square (Template 1)

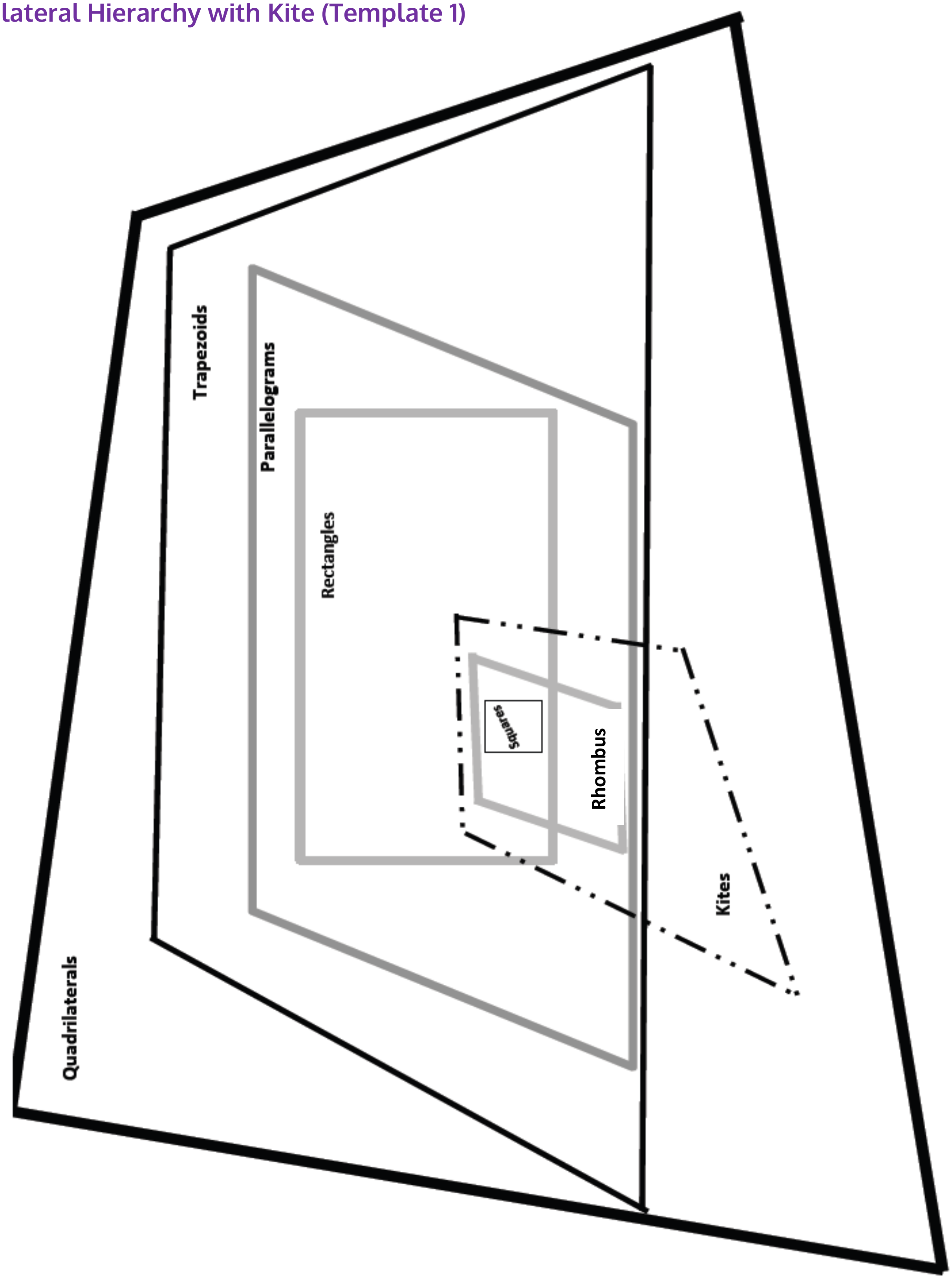


Quadrilateral Hierarchy with Square: Color (Template 2)

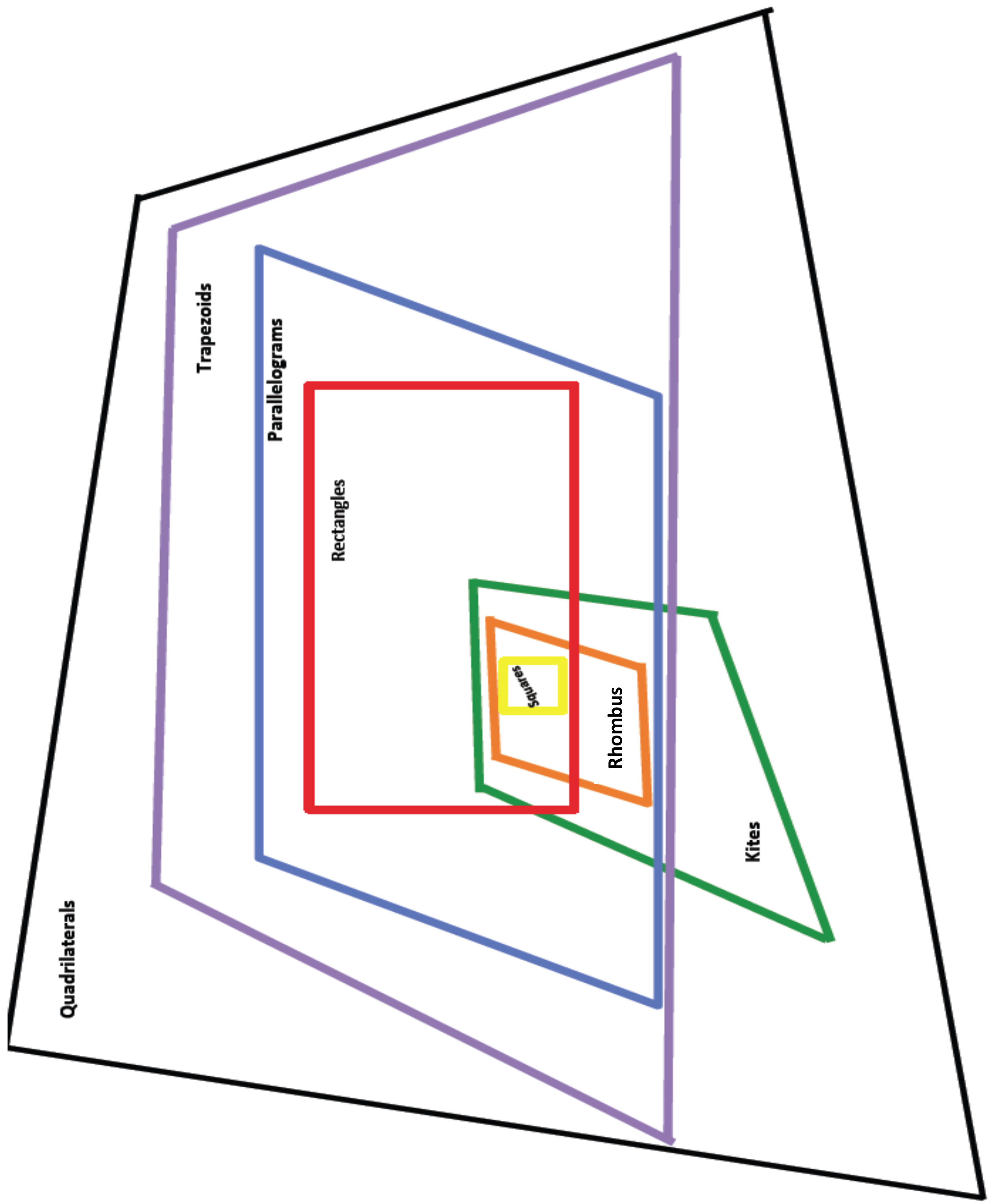


Lesson 19

Quadrilateral Hierarchy with Kite (Template 1)



Quadrilateral Hierarchy with Kite: Color (Template 2)

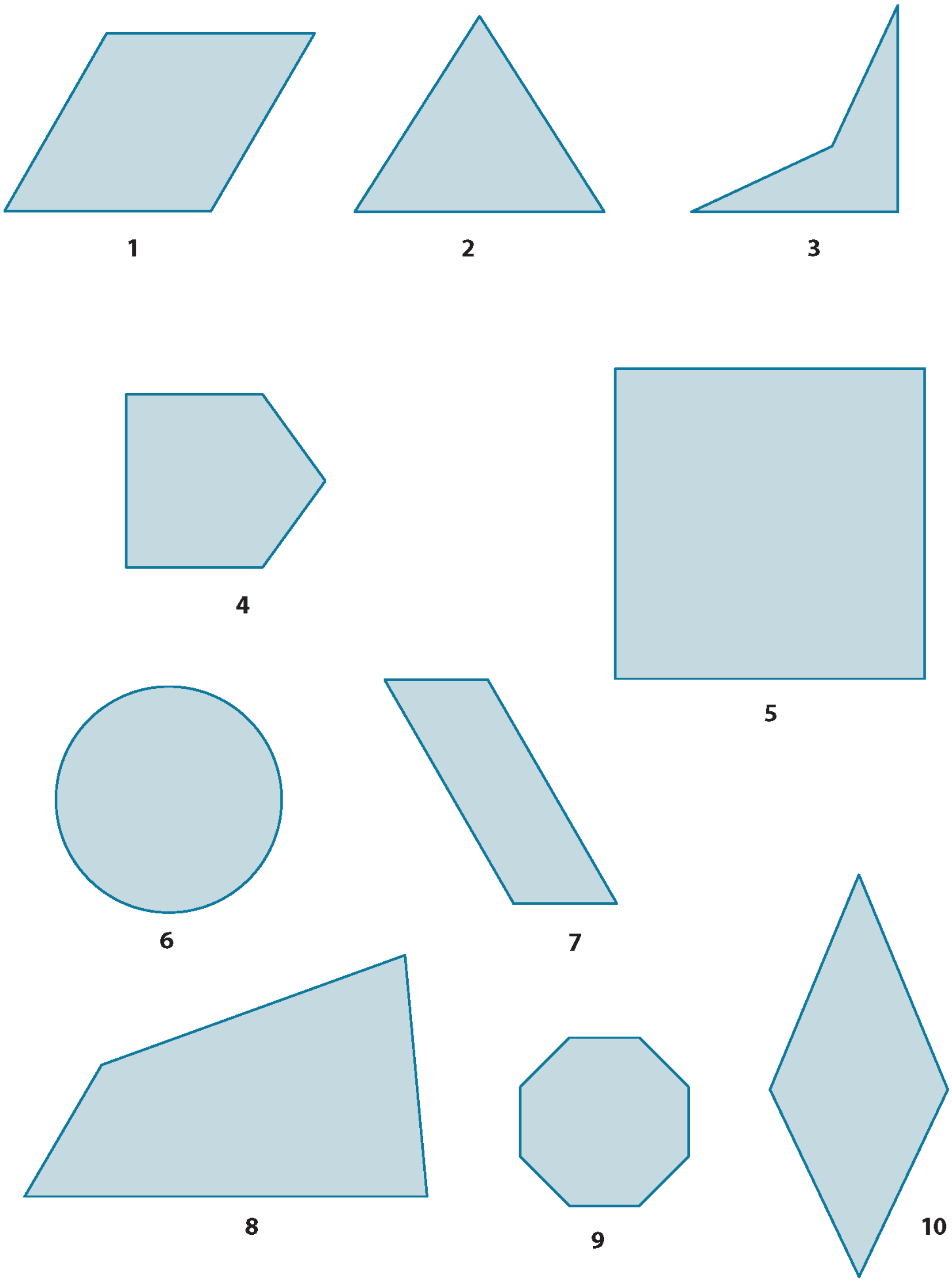


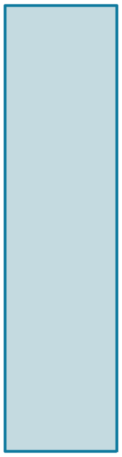
Lesson 20

Shape Name Cards (Template 1)

Quadrilaterals	Trapezoids
Parallelograms	Rectangles
Rhombuses	Kites
Squares	Polygons

Shapes for Sorting (Template 2)





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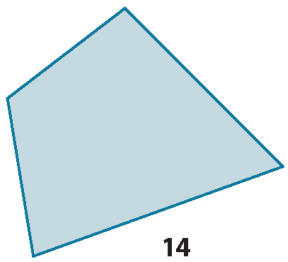
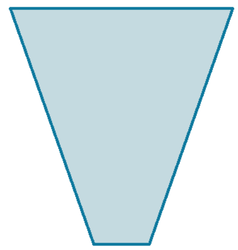
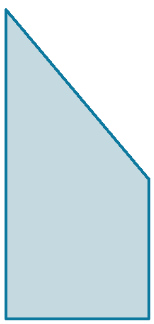


14



15



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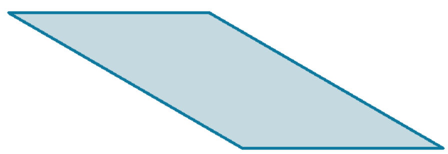
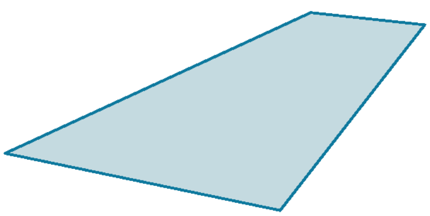


17



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Lesson 21

Problem Set

Name ______________________________ Date ______________

1. Write the number on your task card and a summary of the task in the blank. Then, draw the figure in the box. Label your figure with as many names as you can. Circle the most specific name.

Task #____: ______________________	Task #____: ______________________
Task #____: ______________________	Task #____: ______________________
Task #____: ______________________	Task #____: ______________________

2. John says that because rhombuses do not have perpendicular sides, they cannot be rectangles. Explain his error in thinking.

3. Jack says that because kites do not have parallel sides, a square is not a kite. Explain his error in thinking.

Task Cards 1-6 (Template 1)

Task 1: Draw a trapezoid with a right angle.	Task 2: Draw a rectangle with a length that is twice its width.	Task 3: Draw a quadrilateral with 2 pairs of equal sides and no parallel sides.
Task 4: Draw a rhombus with right angles.	Task 5: Draw a parallelogram with two pairs of perpendicular sides.	Task 6: Draw a rhombus with 4 equal angles.

Task Cards 7-12 (Template 2)

Task 7: Draw a quadrilateral with four equal sides.	Task 8: Draw a parallelogram with right angles.	Task 9: Draw a parallelogram with a side of 4 cm and a side of 6 cm.
Task 10: Draw an isosceles trapezoid.	Task 11: Draw a parallelogram with no right angles.	Task 12: Draw a rectangle that is also a rhombus.

Task Cards 13-18 (Template 3)

Task 13: Draw a quadrilateral that has at least one pair of equal opposite angles.	Task 14: Draw a quadrilateral that has only one pair of equal opposite angles.	Task 15: Draw a trapezoid with four right angles.
Task 16: Draw a kite that is also a parallelogram.	Task 17: Draw a parallelogram with a 60° angle.	Task 18: Draw a rectangle that is not a rhombus.

Task Cards 19-24 (Template 4)

Task 19: Draw a rhombus that is not a rectangle.	Task 22: Draw a quadrilateral whose diagonals bisect each other at a right angle.
Task 20: Draw a parallelogram that is not a rectangle.	Task 23: Draw a trapezoid that is not a parallelogram.
Task 21: Draw a kite that is not a parallelogram.	Task 24: Draw a quadrilateral whose diagonals do not bisect each other.