ZEARN Math

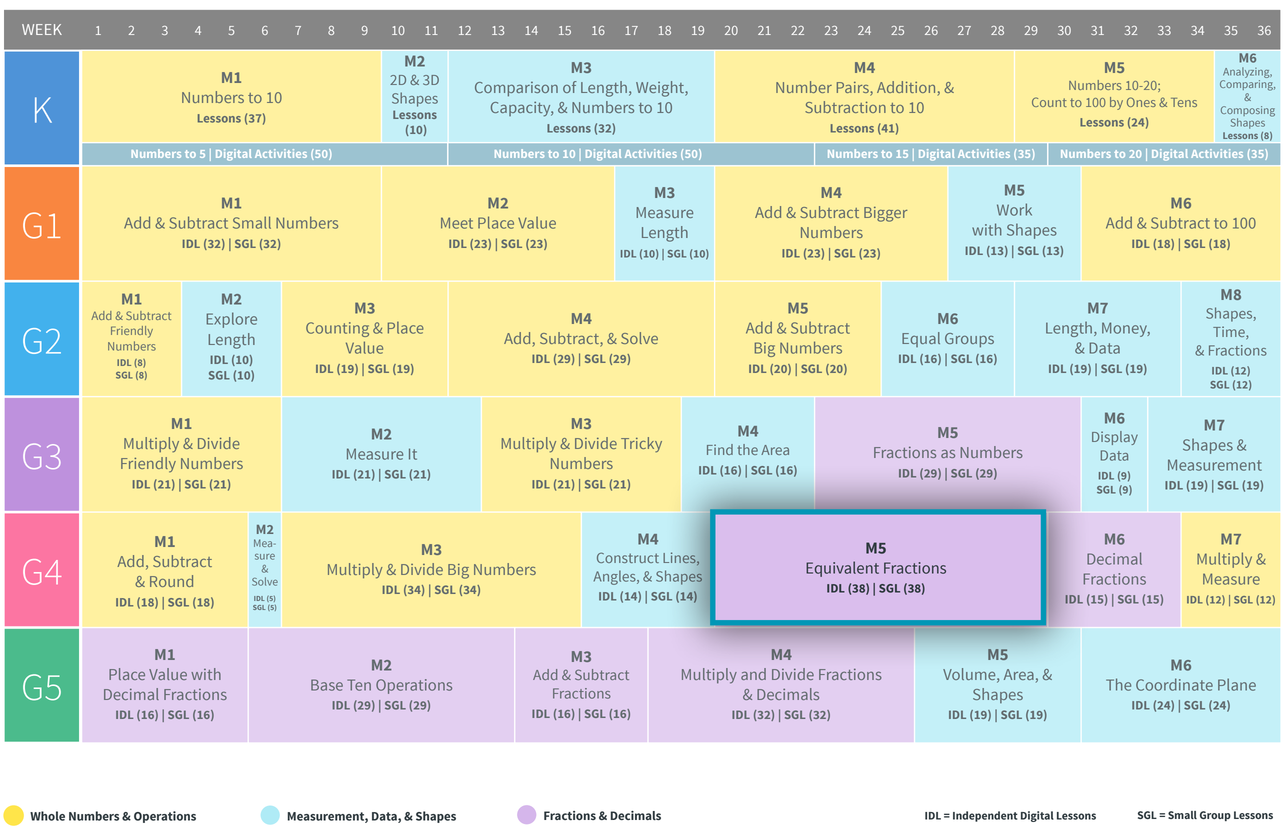
MISSION OVERVIEW

Grade 4, Mission 5

Equivalent Fractions

This Mission teaches students how to manipulate fractions. Students compare fractions, evaluate equivalence, and learn that the same methods they used for whole number operations can be used to add, subtract, and multiply fractions.

CURRICULUM MAP



Overview of Topics and Lesson Objectives

Each mission is broken down into topics. A topic is a group of lessons that teach the same concept. For each topic, Zearn offers Whole Group Fluencies, Whole Group Word Problems, Small Group Lessons, and Independent Digital Lessons. There is a balance of Independent Digital Lessons and Small Group Lessons in each topic of a mission to ensure every student learns with a mix of modalities, feedback, and support while engaging in grade-level content. Throughout each mission, students work on grade-level content with embedded remediation to fill gaps in prior knowledge.

	Objective	INDEPENDENT DIGITAL LESSON	SMALL GROUP LESSON
Topic A	**Decomposition and Fraction Equivalence** **4.NF.3b, 4.NF.4a,** 4.NF.3a		
Lesson 1	Decompose fractions as a sum of unit fractions using tape diagrams.	✓	✓
Lesson 2	Decompose fractions as a sum of unit fractions using tape diagrams.	✓	✓
Lesson 3	Decompose non-unit fractions and represent them as a whole number times a unit fraction using tape diagrams.	✓	✓
Lesson 4	Decompose fractions into sums of smaller unit fractions using tape diagrams.	✓	✓
Lesson 5	Decompose unit fractions using area models to show equivalence.	✓	✓
Lesson 6	Decompose fractions using area models to show equivalence.	✓	✓
Topic B	**Fraction Equivalence Using Multiplication and Division** **4.NF.1,** 4.NF.3b		
Lesson 7	Use the area model and multiplication to show the equivalence of two fractions.	✓	✓
Lesson 8	Use the area model and multiplication to show the equivalence of two fractions.	✓	✓
Lesson 9	Use the area model and division to show the equivalence of two fractions.	✓	✓
Lesson 10	Use the area model and division to show the equivalence of two fractions.	✓	✓

	Objective	INDEPENDENT DIGITAL LESSON	SMALL GROUP LESSON
Lesson 11	Explain fraction equivalence using a tape diagram and the number line, and relate that to the use of multiplication and division.	✓	✓
Topic C	**Fraction Comparison** **4.NF.2**		
Lesson 12	Reason using benchmarks to compare two fractions on the number line.	✓	✓
Lesson 13	Reason using benchmarks to compare two fractions on the number line.	✓	✓
Lesson 14	Find common units or number of units to compare two fractions.	✓	✓
Lesson 15	Find common units or number of units to compare two fractions.	✓	✓
Topic D	**Fraction Addition and Subtraction** **4.NF.3ad,** 4.NF.1, 4.MD.2		
Lesson 16	Use visual models to add and subtract two fractions with the same units.	✓	✓
Lesson 17	Use visual models to add and subtract two fractions with the same units, including subtracting from one whole.	✓	✓
Lesson 18	Add and subtract more than two fractions.	✓	✓
Lesson 19	Solve word problems involving addition and subtraction of fractions.	✓	✓
Lesson 20	Use visual models to add two fractions with related units using the denominators 2, 3, 4, 5, 6, 8, 10, and 12.	✓	✓
Lesson 21	Use visual models to add two fractions with related units using the denominators 2, 3, 4, 5, 6, 8, 10, and 12.	✓	✓
	Mid-Mission Assessment: Topics A-D		
Topic E	**Extending Fraction Equivalence to Fractions Greater Than 1** **4.NF.2, 4.NF.3, 4.MD.4,** 4.NBT.6, 4.NF.1, 4.NF.4a		
Lesson 22	Add a fraction less than 1 to, or subtract a fraction less than 1 from, a whole number using decomposition and visual models.	✓	✓
Lesson 23	Add and multiply unit fractions to build fractions greater than 1 using visual models.	✓	✓

	Objective	INDEPENDENT DIGITAL LESSON	SMALL GROUP LESSON
Lesson 24	Decompose and compose fractions greater than 1 to express them in various forms.	✓	✓
Lesson 25	Decompose and compose fractions greater than 1 to express them in various forms.	✓	✓
Lesson 26	Compare fractions greater than 1 by reasoning using benchmark fractions.	✓	✓
Lesson 27	Compare fractions greater than 1 by creating common numerators or denominators.	✓	✓
Lesson 28	Solve word problems with line plots.	✓	✓
Topic F	**Addition and Subtraction of Fractions by Decomposition** **4.NF.3c,** 4.MD.2		
Lesson 29	Estimate sums and differences using benchmark numbers.	✓	✓
Lesson 30	Add a mixed number and a fraction.	✓	OPTIONAL
Lesson 31	Add mixed numbers.	✓	✓
Lesson 32	Subtract a fraction from a mixed number.	✓	✓
Lesson 33	Subtract a mixed number from a mixed number.	✓	✓
Lesson 34	Subtract mixed numbers.	X	✓
Topic G	**Repeated Addition of Fractions as Multiplication** **4.NF.4,** 4.OA.2, 4.MD.2, 4.MD.4		
Lesson 35	Represent the multiplication of n times a/b as $(n \times a)/b$ using the associative property and visual models.	✓	✓
Lesson 36	Represent the multiplication of n times a/b as $(n \times a)/b$ using the associative property and visual models.	✓	OPTIONAL
Lesson 37	Find the product of a whole number and a mixed number using the distributive property.	✓	✓
Lesson 38	Find the product of a whole number and a mixed number using the distributive property.	X	✓
Lesson 39	Solve multiplicative comparison word problems involving fractions.	✓	OPTIONAL

	Objective	INDEPENDENT DIGITAL LESSON	SMALL GROUP LESSON
Lesson 40	Solve word problems involving the multiplication of a whole number and a fraction including those involving line plots.	✓	✓
Topic H	**Exploring a Fraction Pattern** **4.OA.5**		
Lesson 41	Find and use a pattern to calculate the sum of all fractional parts between 0 and 1. Share and critique peer strategies.	X	✓
	End-of-Mission Assessment: Topics E-H		

Foundational Missions

For each mission, Zearn Math highlights the foundational missions, the earlier content where concepts are introduced and developed. Teachers can access foundational missions directly from the mission page of their Teacher Account to address any gaps in prior knowledge. Zearn recommends that teachers assign foundational missions during Flex Day or during additional non-core instruction time. It is important to use a foundational mission to support a struggling student, rather than an unaligned mission, because the content students learn in each foundational mission supports their Core Day learning.

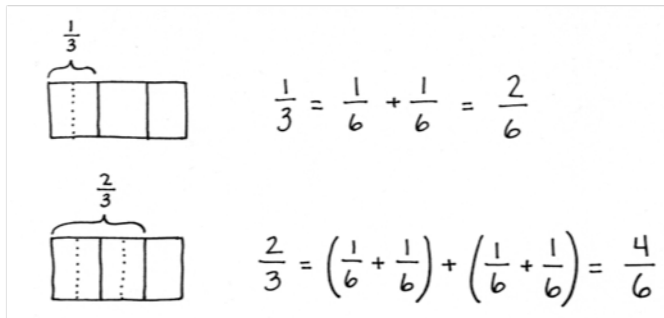
Foundational Missions for G4M5: G2M8 Shapes, Time, and Fractions, G3M5 Fractions as Numbers

Mission Overview

In this mission, students build on their Grade 3 work with unit fractions as they explore fraction equivalence and extend this understanding to mixed numbers. This leads to the comparison of fractions and mixed numbers and the representation of both in a variety of models. Benchmark fractions play an important part in students' ability to generalize and reason about relative fraction and mixed number sizes. Students then have the opportunity to apply what they know to be true for whole number operations to the new concepts of fraction and mixed number operations.

Students begin **Topic A** by decomposing fractions and creating tape diagrams to represent them as sums of fractions with the same denominator in different ways (e.g., $\frac{3}{5} = \frac{1}{5} + \frac{1}{5} + \frac{1}{5} = \frac{2}{5} + \frac{1}{5}$) (**4.NF.3b**). They proceed to see that representing a fraction as the repeated addition of a unit fraction is the same as multiplying that unit fraction by a whole number. This is already a familiar fact in other contexts.

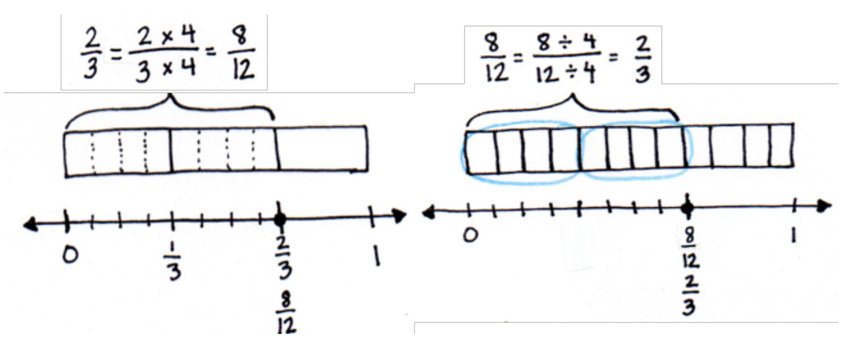
For example, just as 3 twos = 2 + 2 + 2 = 3 × 2, so does 3 fourths $\frac{1}{4} + \frac{1}{4} + \frac{1}{4} = 3 \times \frac{1}{4}$.



The introduction of multiplication as a record of the decomposition of a fraction (**4.NF.4a**) early in the mission allows students to become familiar with the notation before they work with more complex problems. As students continue working with decomposition, they represent familiar unit fractions as the sum of smaller unit fractions. A folded paper activity allows them to see that, when the number of fractional parts in a whole increases, the size of the parts decreases. They proceed to investigate this concept with the use of tape diagrams and area models. Reasoning enables them to explain why two different fractions can represent the same portion of a whole (**4.NF.1**).

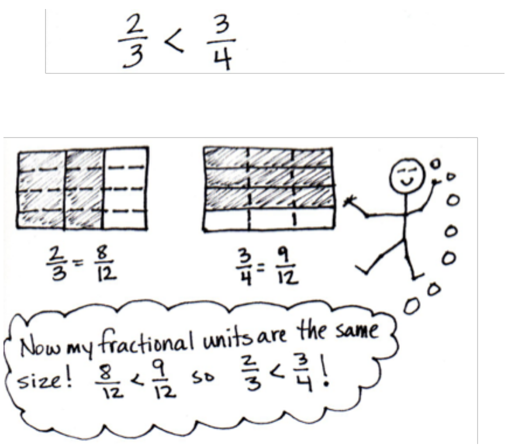
In **Topic B**, students use tape diagrams and area models to analyze their work from earlier in the mission and begin using multiplication to create an equivalent fraction that comprises smaller units, e.g., $\frac{2}{3} = \frac{(2 \times 4)}{(3 \times 4)} = \frac{8}{12}$ (**4.NF.1**). Based on the use of multiplication, they reason that division can be used to create a fraction that comprises larger units (or a single unit) equivalent to a given fraction (e.g., $\frac{8}{12} = \frac{(8 \div 4)}{(12 \div 4)} = \frac{2}{3}$). Their work is justified using area models and tape diagrams and, conversely, multiplication is used to test for and/or verify equivalence. Students use the tape diagram to transition to modeling equivalence on the number line.

They see that, by multiplying, any unit fraction length can be partitioned into *n* equal lengths and that doing so multiplies both the total number of fractional units (the denominator) and number of selected units (the numerator) by *n*. They also see that there are times when fractional units can be grouped together, or divided, into larger fractional units. When that occurs, both the total number of fractional units and number of selected units are divided by the same number.

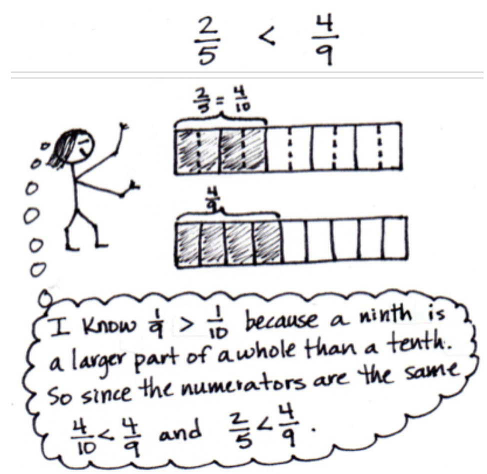


In Grade 3, students compared fractions using fraction strips and number lines with the same denominators. In **Topic C**, they expand on comparing fractions by reasoning about fractions with unlike denominators. Students use the relationship between the numerator and denominator of a fraction to compare to a known benchmark (e.g., 0, ½, or 1) on the number line. Alternatively, students compare using the same numerators. They find that the fraction with the greater denominator is the lesser fraction since the size of the fractional unit is smaller as the whole is decomposed into more equal parts (e.g., ⅕ > ⅒; therefore ⅗ > 3⁄10). Throughout the process, their reasoning is supported using tape diagrams and number lines in cases where one numerator or denominator is a factor of the other, such as ⅕ and ⅒ or ⅔ and ⅚. When the units are unrelated, students use area models and multiplication, the general method pictured below to the left, whereby two fractions are expressed in terms of the same denominators. Students also reason that comparing fractions can only be done when referring to the same whole, and they record their comparisons using the comparison symbols <, >, and = (**4.NF.2**).

Comparison Using Like Denominators

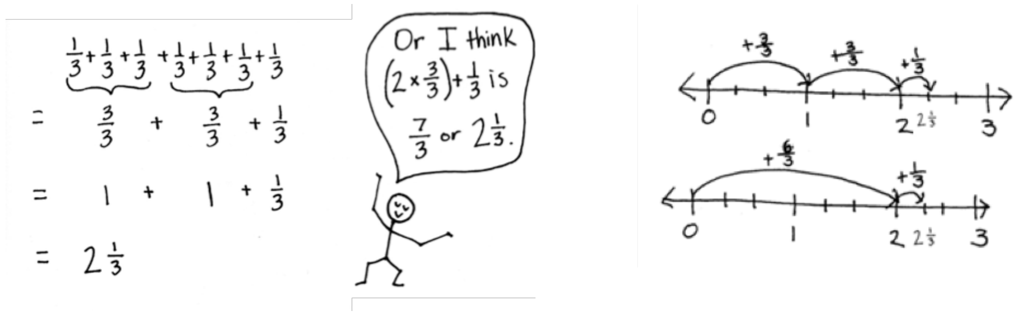


Comparison Using Like Numerators



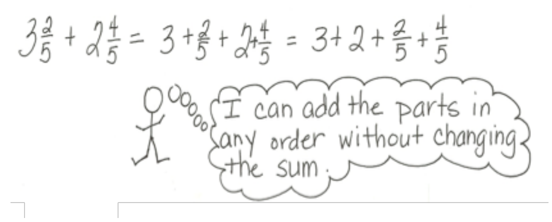
In **Topic D**, students apply their understanding of whole number addition (the combining of like units) and subtraction (finding an unknown part) to work with fractions (**4.NF.3a**). They see through visual models that, if the units are the same, computation can be performed immediately, e.g., 2 bananas + 3 bananas = 5 bananas and 2 eighths + 3 eighths = 5 eighths. They see that, when subtracting fractions from one whole, the whole is decomposed into the same units as the part being subtracted, e.g., 1 – ⅗ = 5⁄5 – ⅗ = ⅖. Students practice adding more than two fractions and model fractions in word problems using tape diagrams (**4.NF.3d**). As an extension of the Grade 4 standards, students apply their knowledge of decomposition from earlier topics to add fractions with related units using tape diagrams and area models to support their numerical work. To find the sum of ½ and ¼, for example, one simply decomposes 1 half into 2 smaller equal units, fourths, just as in Topics A and B. Now the addition can be completed: 2⁄4 + ¼ = ¾. Though not assessed, this work is warranted because, in Mission 6, students are asked to add tenths and hundredths when working with decimal fractions and decimal notation.

At the beginning of **Topic E**, students use decomposition and visual models to add and subtract fractions less than 1 to or from whole numbers, e.g., 4 + ¾ = 4 ¾ and 4 – ¾ = (3 + 1) – ¾. They use addition and multiplication to build fractions greater than 1 and represent them on the number line.

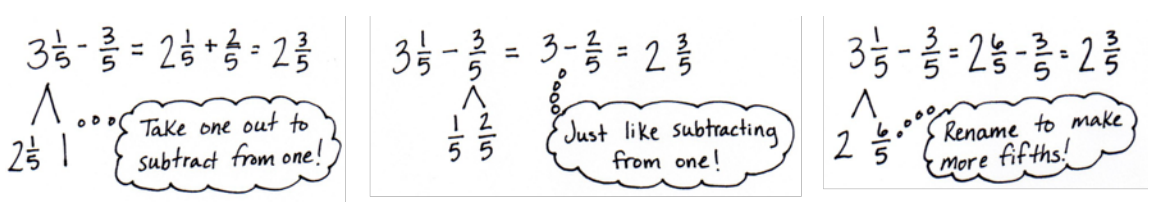


Students then use these visual models and decompositions to reason about the various forms in which a fraction greater than or equal to 1 may be presented, both as fractions and mixed numbers. They practice converting between these forms and begin understanding the usefulness of each form in different situations. Through this understanding, the common misconception that every improper fraction must be converted to a mixed number is avoided. Next, students compare fractions greater than 1, building on their rounding skills and using understanding of benchmarks to reason about which of two fractions is greater (**4.NF.2**). This activity continues to build understanding of the relationship between the numerator and denominator of a fraction. Students progress to finding and using like denominators or numerators to compare and order mixed numbers. They apply their skills of comparing numbers greater than 1 by solving word problems (**4.NF.3d**) requiring the interpretation of data presented in line plots (**4.MD.4**). Students use addition and subtraction strategies to solve the problems, as well as decomposition and modeling to compare numbers in the data sets.

In **Topic F**, students estimate sums and differences of mixed numbers, rounding before performing the actual operation to determine what a reasonable outcome is. They proceed to use decomposition to add and subtract mixed numbers (**4.NF.3c**). This work builds on their understanding of a mixed number being the sum of a whole number and fraction.



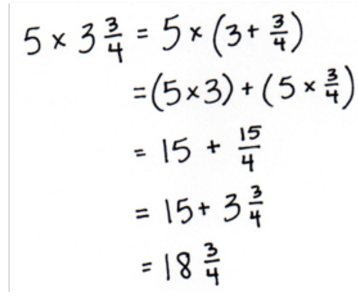
Using unit form, students add and subtract like units first (e.g., ones and ones, fourths and fourths). Students use decomposition, shown with number bonds, in mixed number addition to make one from fractional units before finding the sum. When subtracting, students learn to decompose the minuend or subtrahend when there are not enough fractional units from which to subtract. Alternatively, students can rename the subtrahend, giving more units to the fractional units, which connects to whole number subtraction when renaming 9 tens 2 ones as 8 tens 12 ones.



In **Topic G**, students build on the concept of representing repeated addition as multiplication, applying this familiar concept to work with fractions (**4.NF.4a**, **4.NF.4b**). They use the associative property and their understanding of decomposition. Just as with whole numbers, the unit remains unchanged.

$$4 \times \frac{3}{5} = 4 \times \left(3 \times \frac{1}{5}\right) = (4 \times 3) \times \frac{1}{5} = \frac{4 \times 3}{5} = \frac{12}{5}$$

This understanding connects to students' work with place value and whole numbers. Students proceed to explore the use of the distributive property to multiply a whole number by a mixed number. They recognize that they are multiplying each part of a mixed number by the whole number and use efficient strategies to do so. The topic closes with solving multiplicative comparison word problems involving fractions (**4.NF.4c**) as well as problems involving the interpretation of data presented on a line plot.

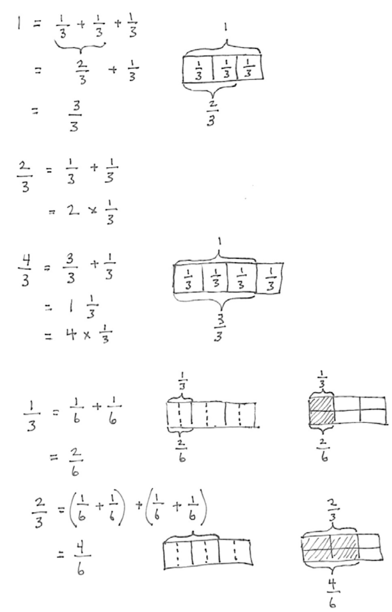


Topic H comprises an exploration lesson where students find the sum of all like denominators from $\frac{0}{n}$ to $\frac{n}{n}$. Students first work in teams with fourths, sixths, eighths, and tenths. For example, they might find the sum of all sixths from $\frac{0}{6}$ to $\frac{6}{6}$. Students discover that they can make pairs with a sum of 1 to add more efficiently, e.g., $\frac{0}{6} + \frac{6}{6}$, $\frac{1}{6} + \frac{5}{6}$, $\frac{2}{6} + \frac{4}{6}$, and there is one fraction, $\frac{3}{6}$, without a pair. They then extend this to similarly find sums of thirds, fifths, sevenths, and ninths, observing patterns when finding the sum of odd and even denominators (**4.OA.5**).

The Mid-Mission Assessment follows Topic D, and the End-of-Mission Assessment follows Topic H.

Topic A: Decomposition and Fraction Equivalence

LESSONS 1-6



Topic A builds on Grade 3 work with unit fractions. Students explore fraction equivalence through the decomposition of non-unit fractions into unit fractions, as well as the decomposition of unit fractions into smaller unit fractions. They represent these decompositions, and prove equivalence, using visual models.

In Lesson 1, students use paper strips to represent the decomposition of a whole into parts. In Lessons 1 and 2, students decompose fractions as unit fractions, drawing tape diagrams to represent them as sums of fractions with the same denominator in different ways, e.g., $\frac{3}{5} = \frac{1}{5} + \frac{1}{5} + \frac{1}{5} = \frac{1}{5} + \frac{2}{5}$.

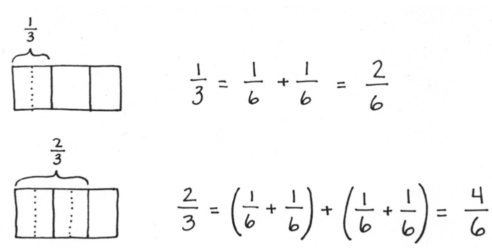
In Lesson 3, students see that representing a fraction as the repeated addition of a unit fraction is the same as multiplying that unit fraction by a whole number. This is already a familiar fact in other contexts. An example is as follows:

3 bananas = 1 banana + 1 banana + 1 banana = 3 × 1 banana

3 twos = 2 + 2 + 2 = 3 × 2

3 fourths = 1 fourth + 1 fourth + 1 fourth = 3 × 1 fourth

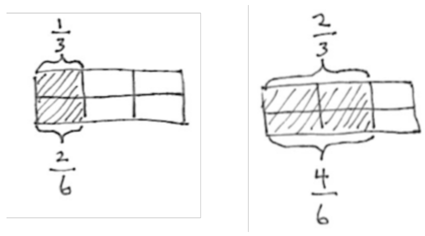
$\frac{3}{4} = \frac{1}{4} + \frac{1}{4} + \frac{1}{4} = 3 \times \frac{1}{4}$



By introducing multiplication as a record of the decomposition of a fraction early in the mission, students are accustomed to the notation by the time they work with more complex problems in Topic G.

Students continue with decomposition in Lesson 4, where they use tape diagrams to represent fractions, e.g., ½, ⅓, and ⅔, as the sum of smaller unit fractions. Students record the results as a number sentence, e.g., ½ = ¼ + ¼ = (⅛ + ⅛) + (⅛ + ⅛) = ⁴⁄₈.

In Lesson 5, this idea is further investigated as students represent the decomposition of unit fractions in area models. In Lesson 6, students use the area model for a second day, this time to represent fractions with different numerators. They explain why two different fractions represent the same portion of a whole.

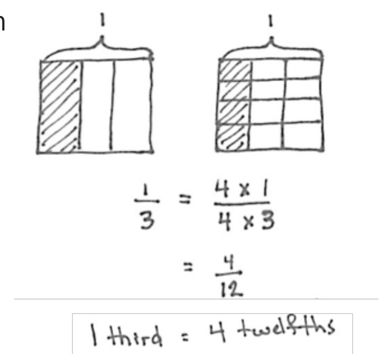


Topic B: Fraction Equivalence Using Multiplication and Division

LESSONS 7-11

In Topic B, students begin generalizing their work with fraction equivalence. In Lessons 7 and 8, students analyze their earlier work with tape diagrams and the area model in Lessons 3 through 5 to begin using multiplication to create an equivalent fraction that comprises smaller units, e.g., $\frac{2}{3} = \frac{(2 \times 4)}{(3 \times 4)} = \frac{8}{12}$. Conversely, students reason, in Lessons 9 and 10, that division can be used to create a fraction that comprises larger units (or a single unit) equivalent to a given fraction, e.g., $\frac{8}{12} = \frac{(8 \div 4)}{(12 \div 4)} = \frac{2}{3}$. The numerical work of Lessons 7 through 10 is introduced and supported using area models and tape diagrams.

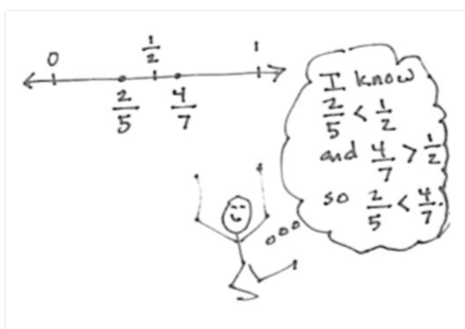
In Lesson 11, students use tape diagrams to transition their knowledge of fraction equivalence to the number line. They see that any unit fraction length can be partitioned into *n* equal lengths. For example, each third in the interval from 0 to 1 may be partitioned into 4 equal parts. Doing so multiplies both the total number of fractional units (the denominator) and the number of selected units (the numerator) by 4. Conversely, students see that, in some cases, fractional units may be grouped together to form some number of larger fractional units. For example, when the interval from 0 to 1 is partitioned into twelfths, one may group 4 twelfths at a time to make thirds. By doing so, both the total number of fractional units and number of selected units are divided by 4.



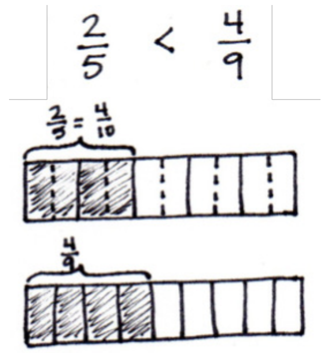
Topic C: Fraction Comparison

LESSONS 12-15

In Topic C, students use benchmarks and common units to compare fractions with different numerators and different denominators. The use of benchmarks is the focus of Lessons 12 and 13 and is modeled using a number line. Students use the relationship between the numerator and denominator of a fraction to compare to a known benchmark (e.g., 0, ½, or 1) and then use that information to compare the given fractions. For example, when comparing ⁴⁄₇ and ⅖, students reason that 4 sevenths is more than 1 half, while 2 fifths is less than 1 half. They then conclude that 4 sevenths is greater than 2 fifths.

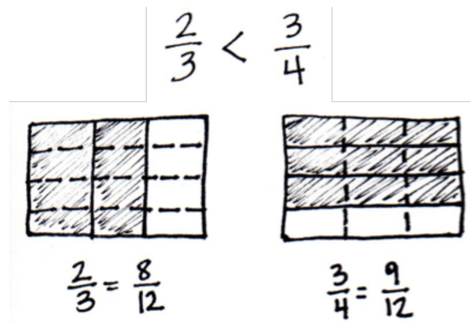


In Lesson 14, students reason that they can also use like numerators based on what they know about the size of the fractional units. They begin at a simple level by reasoning, for example, that 3 fifths is less than 3 fourths because fifths are smaller than fourths. They then see that it is easy to make like numerators at times to compare, e.g., $\frac{2}{5} < \frac{4}{9}$ because $\frac{2}{5} = \frac{4}{10}$, and $\frac{4}{10} < \frac{4}{9}$ because $\frac{1}{10} < \frac{1}{9}$. Using their experience with fractions in Grade 3, they know the larger the denominator of a unit fraction, the smaller the size of the fractional unit.



Like numerators are modeled using tape diagrams directly above each other, where one fractional unit is partitioned into smaller unit fractions. The lesson then moves to comparing fractions with related denominators, such as $\frac{2}{3}$ and $\frac{5}{6}$, wherein one denominator is a factor of the other, using both tape diagrams and the number line. In Lesson 15, students compare fractions by using an area model to express two fractions, wherein one denominator is not a factor of the other, in terms of the same unit using multiplication, e.g., $\frac{2}{3} < \frac{3}{4}$ because $\frac{2}{3} = \frac{(2 \times 4)}{(3 \times 4)} = \frac{8}{12}$ and $\frac{3}{4} = \frac{(3 \times 3)}{(4 \times 3)} = \frac{9}{12}$ and $\frac{8}{12} < \frac{9}{12}$. The area for $\frac{2}{3}$ is partitioned vertically, and the area for $\frac{3}{4}$ is partitioned horizontally.

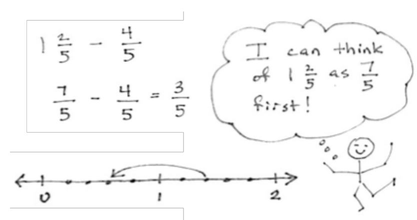
To find the equivalent fraction and create the same size units, the areas are decomposed horizontally and vertically, respectively. Now the unit fractions are the same in each model or equation, and students can easily compare. The topic culminates with students comparing pairs of fractions and, by doing so, deciding which strategy is either necessary or efficient: reasoning using benchmarks and what they know about units, drawing a model (such as a number line, a tape diagram, or an area model), or the general method of finding like denominators through multiplication.



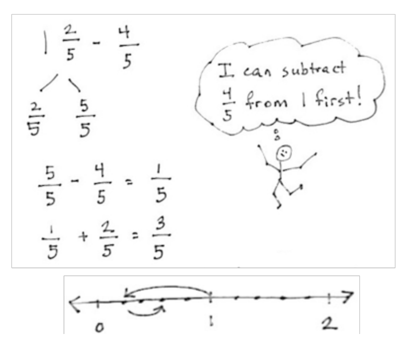
Topic D: Fraction Addition and Subtraction

LESSONS 16-21

Topic D bridges students' understanding of whole number addition and subtraction to fractions. Everything that they know to be true of addition and subtraction with whole numbers now applies to fractions. Addition is finding a total by combining like units. Subtraction is finding an unknown part. Implicit in the equations $3 + 2 = 5$ and $2 = 5 - 3$ is the assumption that the numbers are referring to the same units.

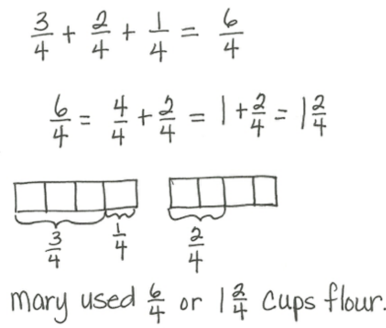


In Lessons 16 and 17, students generalize familiar facts about whole number addition and subtraction to work with fractions. Just as 3 apples – 2 apples = 1 apple, students note that 3 fourths – 2 fourths = 1 fourth. Just as 6 days + 3 days = 9 days = 1 week 2 days, students note that $\frac{6}{7} + \frac{3}{7} = \frac{9}{7} = \frac{7}{7} + \frac{2}{7} = 1\frac{2}{7}$. In Lesson 17, students decompose a whole into a fraction having the same denominator as the subtrahend. For example, 1 – 4 fifths becomes 5 fifths – 4 fifths = 1 fifth, connecting with Topic B skills. They then see that, when solving $1\frac{2}{5} - \frac{4}{5}$, they have a choice of subtracting $\frac{4}{5}$ from $\frac{7}{5}$ or from 1 (as pictured to the right). Students model with tape diagrams and number lines to understand and then verify their numerical work.

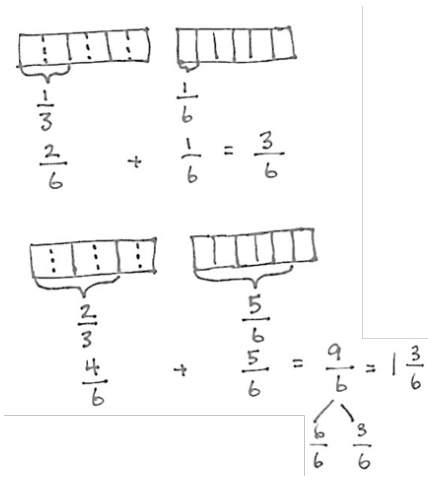


In Lesson 18, students add more than two fractions and see sums of more than one whole, such as $\frac{2}{8} + \frac{5}{8} + \frac{7}{8} = 1\frac{6}{8}$. As students move into problem solving in Lesson 19, they create tape diagrams or number lines to represent and solve fraction addition and subtraction word problems (see the example below). These problems bridge students into work with mixed numbers, which follows the Mid-Mission Assessment.

Mary mixed ¾ cup of wheat flour, ²⁄₄ cup of rice flour, and ¼ cup of oat flour for her bread dough. How many cups of flour did she put in her bread in all?



In Lessons 20 and 21, students add fractions with related units, where one denominator is a multiple (or factor) of the other. To add such fractions, a decomposition is necessary. Decomposing one unit into another is familiar territory: Students have had ample practice composing and decomposing in Topics A and B when working with place value units, converting units of measurement, and using the distributive property. For example, they have converted between equivalent measurement units (e.g., 100 cm = 1 m), and they have used such conversions to do arithmetic (e.g., 1 meter – 54 centimeters). With fractions, the concept is the same. To find the sum of ½ and ¼, one simply renames (converts, decomposes) ½ as ²⁄₄ and adds: ²⁄₄ + ¼ = ¾. All numerical work is accompanied by visual models that allow students to use and apply their known skills and understandings. The addition of fractions with related units is also foundational to decimal work when adding tenths and hundredths in Mission 6. Please note that addition of fractions with related denominators is not assessed.

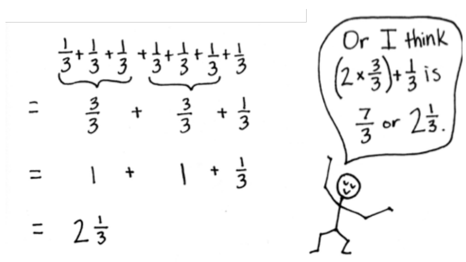


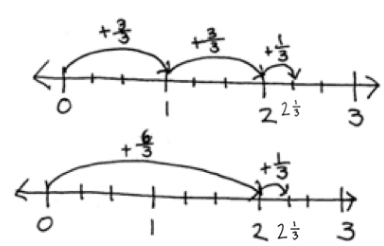
Topic E: Extending Fraction Equivalence to Fractions Greater Than 1

LESSONS 22-28

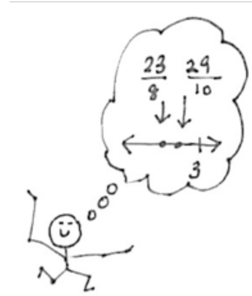
In Topic E, students study equivalence involving both ones and fractional units. In Lesson 22, they use decomposition and visual models to add and subtract fractions less than 1 to and from whole numbers, e.g., 4 + ¾ = 4 ¾ and 4 – ¾ = (3 + 1) – ¾, subtracting the fraction from 1 using a number bond and a number line.

Lesson 23 has students use addition and multiplication to build fractions greater than 1 and then represent them on the number line. Fractions can be expressed both in mixed units of a whole number and a fraction or simply as a fraction, as pictured below, e.g., 7 × ⅓ = ³⁄₃ + ³⁄₃ + ⅓ = 2 × ³⁄₃ + ⅓ = ⁷⁄₃ = 2 ⅓.





In Lessons 24 and 25, students use decompositions to reason about the various equivalent forms in which a fraction greater than or equal to 1 may be presented, both as fractions and as mixed numbers. In Lesson 24, they decompose, for example, 11 fourths into 8 fourths and 3 fourths, $\frac{11}{4} = \frac{8}{4} + \frac{3}{4}$, or they can think of it as $\frac{11}{4} = \frac{4}{4} + \frac{4}{4} + \frac{3}{4} = 2 \times \frac{4}{4} + \frac{3}{4} = 2\frac{3}{4}$. In Lesson 25, students are then able to decompose the two wholes into 8 fourths so their original number can then be looked at as $\frac{8}{4} + \frac{3}{4}$ or $\frac{11}{4}$. In this way, they see that $2\frac{3}{4} = \frac{11}{4}$. This fact is further reinforced when they plot $\frac{11}{4}$ on the number line and see that it is at the same point as $2\frac{3}{4}$. Unfortunately, the term *improper fraction* carries some baggage. As many have observed, there is nothing *improper* about an improper fraction. Nevertheless, as a mathematical term, it is useful for describing a particular form in which a fraction may be presented (i.e., a fraction is improper if the numerator is greater than or equal to the denominator). Students do need practice in terms of converting between the various forms a fraction may take, but take care not to foster the misconception that every improper fraction *must* be converted to a mixed number.

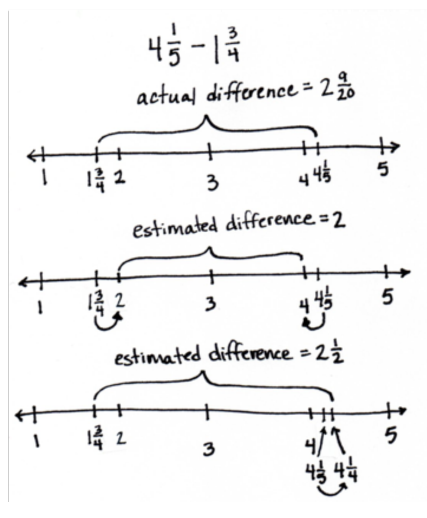


Students compare fractions greater than 1 in Lessons 26 and 27. They begin by using their understanding of benchmarks to reason about which of two fractions is greater. This activity builds on students' rounding skills, having them identify the whole numbers and the halfway points between them on the number line. The relationship between the numerator and denominator of a fraction is a key concept here as students consider relationships to whole numbers; e.g., a student might reason that $\frac{23}{8}$ is less than $\frac{29}{10}$ because $\frac{23}{8}$ is 1 eighth less than 3, but $\frac{29}{10}$ is 1 tenth less than 3. They know each fraction is 1 fractional unit away from 3, and since $\frac{1}{8} > \frac{1}{10}$, then $\frac{23}{8} < \frac{29}{10}$. Students progress to finding and using like denominators to compare and order mixed numbers. Once again, students must use reasoning skills as they determine that, when they have two fractions with the same numerator, the larger fraction has a larger unit (or smaller denominator). Conversely, when they have two fractions with the same denominator, the larger one has the larger number of units (or larger numerator).

Lesson 28 concludes the topic with word problems requiring the interpretation of data presented in line plots. Students create line plots to display a given data set that includes fraction and mixed number values. To do this, they apply their skill in comparing mixed numbers, both through reasoning and the use of common numerators or denominators. For example, a data set might contain both $1\frac{5}{8}$ and $\frac{14}{8}$ giving students the opportunity to determine that they must be plotted at the same point. They also use addition and subtraction to solve the problems.

Topic F: Addition and Subtraction of Fractions by Decomposition

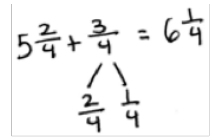
LESSONS 29-34

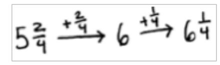


Topic F provides students with the opportunity to use their understandings of fraction addition and subtraction as they explore mixed number addition and subtraction by decomposition.

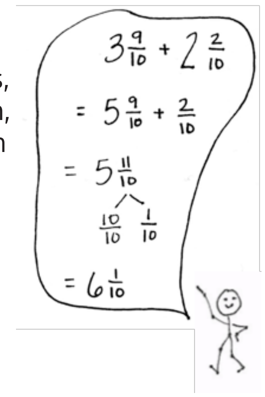
Lesson 29 focuses on the process of using benchmark numbers to estimate sums and differences of mixed numbers. Students once again call on their understanding of benchmark fractions as they determine, prior to performing the actual operation, what a reasonable outcome will be. One student might use benchmark whole numbers and reason, for example, that the difference between $4\frac{1}{5}$ and $1\frac{3}{4}$ is close to 2 because $4\frac{1}{5}$ is closer to 4 than 5, $1\frac{3}{4}$ is closer to 2 than 1, and the difference between 4 and 2 is 2. Another student might use familiar benchmark fractions and reason that the answer will be closer to $2\frac{1}{2}$ since $4\frac{1}{5}$ is about $\frac{1}{4}$ more than 4 and $1\frac{3}{4}$ is about $\frac{1}{4}$ less than 2, making the difference about a half more than 2, or 2 ½.

In Lesson 30, students begin adding a mixed number to a fraction using unit form. They add like units, applying their Grades 1 and 2 understanding of completing a unit to add when the sum of the fractional units exceeds 1. Students ask, “How many more do we need to make one?” rather than “How many more do we need to make ten?” as was the case in Grade 1. A number bond decomposes the fraction to make one and can be modeled on the number line or using the arrow way, as shown to the right. Alternatively, a number bond can be used after adding like units, when the sum results in a mixed number with a fraction greater than 1, to decompose the fraction greater than 1 into ones and fractional units.

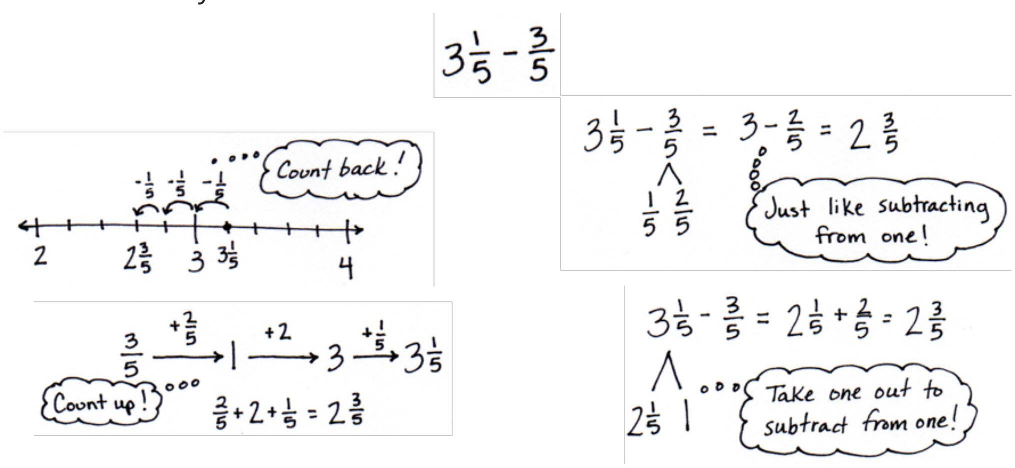




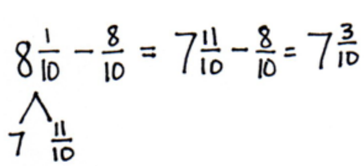
Directly applying what was learned in Lesson 30, Lesson 31 starts with adding like units, e.g., ones with ones and fourths with fourths, to add two mixed numbers. Students can, again, choose to make one before finding the sum or to decompose the sum to result in a proper mixed number.

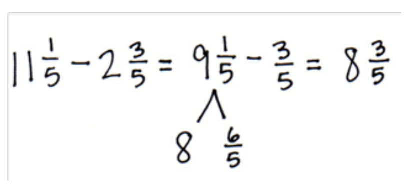


Lessons 32 and 33 follow the same sequence for subtraction. In Lesson 32, students simply subtract a fraction from a mixed number, using three main strategies both when there are and there are not enough fractional units. They count back or up, subtract from 1, or take one out to subtract from 1. In Lesson 33, students apply these strategies after subtracting the ones first. They model subtraction of mixed numbers using a number line or the arrow way.



In Lesson 34, students learn another strategy for subtraction by decomposing the total into a whole number and a fraction greater than one to either subtract a fraction or a mixed number.





Topic G: Repeated Addition of Fractions as Multiplication

LESSONS 35-40

Topic G extends the concept of representing repeated addition as multiplication, applying this familiar concept to work with fractions. Multiplying a whole number times a fraction was introduced in Topic A as students learned to decompose fractions, e.g., $\frac{3}{5} = \frac{1}{5} + \frac{1}{5} + \frac{1}{5} = 3 \times \frac{1}{5}$. In Lessons 35 and 36, students use the associative property, as exemplified below, to multiply a whole number times a mixed number.

3 bananas + 3 bananas + 3 bananas + 3 bananas

= 4 × 3 bananas

= 4 × (3 × 1 banana) = (4 × 3) × 1 banana = 12 bananas

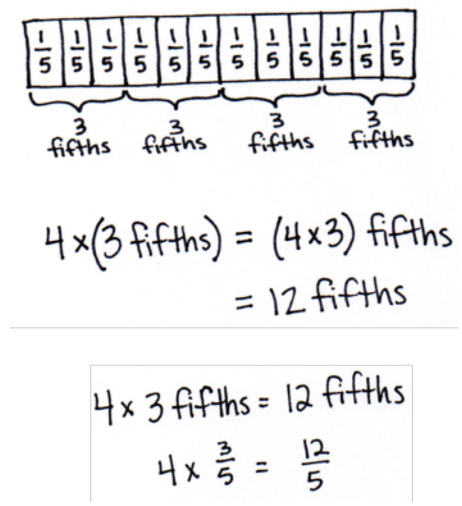
3 fifths + 3 fifths + 3 fifths + 3 fifths

= 4 × 3 fifths

= 4 × (3 fifths) = (4 × 3) fifths = 12 fifths

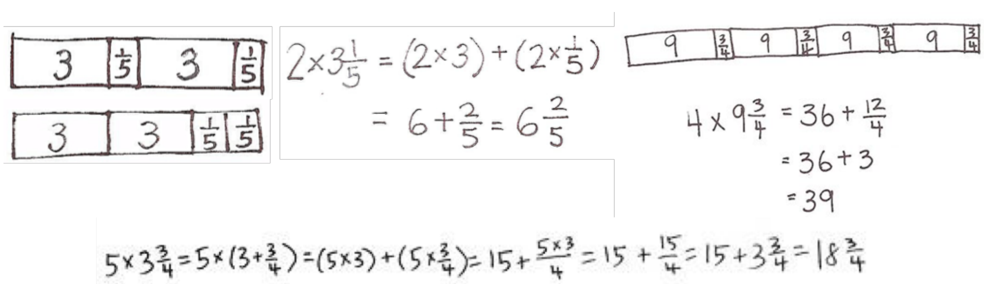
$4 \times \frac{3}{5}$

$4 \times (3 \times \frac{1}{5}) = (4 \times 3) \times \frac{1}{5} = \frac{(4\times3)}{5} = \frac{12}{5}$

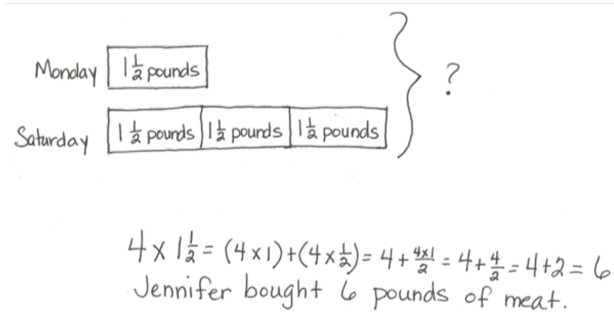


Students may have never before considered that 3 bananas = 3 × 1 banana, but it is an understanding that connects place value, whole number work, measurement conversions, and fractions, e.g., 3 hundreds = 3 × 1 hundred or 3 feet = 3 × (1 foot); 1 foot = 12 inches; therefore, 3 feet = 3 × (12 inches) = (3 × 12) inches = 36 inches.

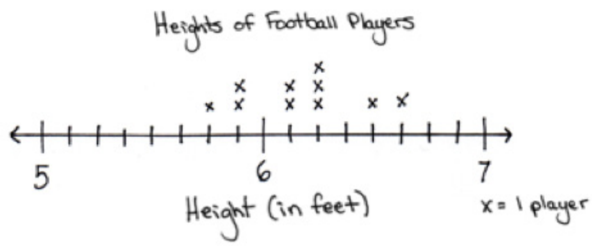
Students explore the use of the distributive property in Lessons 37 and 38 to multiply a whole number by a mixed number. They see the multiplication of each part of a mixed number by the whole number and use the appropriate strategies to do so. As students progress through each lesson, they are encouraged to record only as much as they need to keep track of the math. As shown below, there are multiple steps when using the distributive property, and students can become lost in those steps. Efficiency in solving is encouraged.



In Lesson 39, students build their problem-solving skills by solving multiplicative comparison word problems involving mixed numbers, e.g., "Jennifer bought 3 times as much meat on Saturday as she did on Monday. If she bought 1 ½ pounds on Monday, what is the total amount of meat bought for the two days?" They create and use tape diagrams to represent these problems before using various strategies to solve them numerically.



In Lesson 40, students solve word problems involving multiplication of a fraction by a whole number. Additionally, students work with data presented in line plots.



Topic H: Exploring a Fraction Pattern

LESSON 41

Topic H is an exploration lesson in which students find the sum of all like denominators from $\frac{0}{n}$ to $\frac{n}{n}$.

Students first work, in teams, with fourths, sixths, eighths, and tenths. For example, they might find the sum of all sixths from $\frac{0}{6}$ to $\frac{6}{6}$. Students discover that they can make pairs with a sum of 1 to add more efficiently, e.g., $\frac{0}{6} + \frac{6}{6}$, $\frac{1}{6} + \frac{5}{6}$, $\frac{2}{6} + \frac{4}{6}$, and there will be one fraction, $\frac{3}{6}$, without a pair. As students make this discovery, they share and compare their strategies within their teams. They then extend this to similarly find sums of thirds, fifths, sevenths, and ninths, observing patterns when finding the sum of odd and even denominators (**4.OA.5**). Through discussion of their strategies, students determine which are most efficient.

For enrichment, students can be challenged to find the sum of all hundredths from 0 hundredths to 100 hundredths.

Terminology

New or Recently Introduced Terms

- **Benchmark**
  Standard or reference point by which something is measured
- **Common denominator**
  When two or more fractions have the same denominator
- **Denominator**
  E.g., the 5 in $\frac{3}{5}$ names the fractional unit as fifths

- **Fraction greater than 1**
  A fraction with a numerator that is greater than the denominator

- **Line plot**
  Display of data on a number line, using an x or another mark to show frequency

- **Mixed number**
  Number made up of a whole number and a fraction

- **Numerator**
  E.g., the 3 in ⅗ indicates 3 fractional units are selected

Familiar Terms and Symbols[1]

- **=, <, >**
  Equal to, less than, greater than

- **Compose**
  Change a smaller unit for an equivalent of a larger unit, e.g., 2 fourths = 1 half, 10 ones = 1 ten; combining 2 or more numbers, e.g., 1 fourth + 1 fourth = 2 fourths, 2 + 2 + 1 = 5

- **Decompose**
  Change a larger unit for an equivalent of a smaller unit, e.g., 1 half = 2 fourths, 1 ten = 10 ones; partition a number into 2 or more parts, e.g., 2 fourths = 1 fourth + 1 fourth, 5 = 2 + 2 + 1

- **Equivalent fractions**
  Fractions that name the same size or amount

- **Fraction**
  E.g., ⅓, ⅔, 3⁄3, 4⁄3

- **Fractional unit**
  E.g., half, third, fourth

- **Multiple**
  Product of a given number and any other whole number

- **Non-unit fraction**
  Fractions with numerators other than 1

- **Unit fraction**
  Fractions with numerator 1

- **Unit interval**
  E.g., the interval from 0 to 1, measured by length

- **Whole**
  E.g., 2 halves, 3 thirds, 4 fourths

1 These are terms and symbols students have seen previously.

Suggested Tools and Representations

- **Area model**
- **Fraction strips**
  Made from paper, folded, and used to model equivalent fractions
- **Line plot**
- **Number line**
- **Rulers**
- **Tape diagram**

Focus Grade Level Standards

Generate and analyze patterns.

4.OA.5

Generate a number or shape pattern that follows a given rule. Identify apparent features of the pattern that were not explicit in the rule itself. *For example, given the rule "Add 3" and the starting number 1, generate terms in the resulting sequence and observe that the terms appear to alternate between odd and even numbers. Explain informally why the numbers will continue to alternate in this way.*

Extend understanding of fraction equivalence and ordering.

4.NF.1

Explain why a fraction a/b is equivalent to a fraction $(n \times a)/(n \times b)$ by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions.

4.NF.2

Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as ½. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.

Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.

4.NF.3

Understand a fraction a/b with $a > 1$ as a sum of fractions $1/b$.

a. Understand addition and subtraction of fractions as joining and separating parts referring to the same whole.
b. Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using a visual fraction model. *Examples:* $3/8 = 1/8 + 1/8 + 1/8$; $3/8 = 1/8 + 2/8$; $2\ 1/8 = 1 + 1 + 1/8 = 8/8 + 8/8 + 1/8$.
c. Add and subtract mixed numbers with like denominators, e.g., by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction.

d. Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators, e.g., by using visual fraction models and equations to represent the problem.

4.NF.4

Apply and extend previous understandings of multiplication to multiply a fraction by a whole number.

a. Understand a fraction a/b as a multiple of $1/b$. *For example, use a visual fraction model to represent 5/4 as the product 5 × (1/4), recording the conclusion by the equation 5/4 = 5 × (1/4).*
b. Understand a multiple of a/b as a multiple of $1/b$, and use this understanding to multiply a fraction by a whole number. *For example, use a visual fraction model to express 3 × (2/5) as 6 × (1/5), recognizing this product as 6/5. (In general,* $n \times (a/b) = (n \times a)/b$.*)*
c. Solve word problems involving multiplication of a fraction by a whole number, e.g., by using visual fraction models and equations to represent the problem. *For example, if each person at a party will eat 3/8 of a pound of roast beef, and there will be 5 people at the party, how many pounds of roast beef will be needed? Between what two whole numbers does your answer lie?*

Represent and interpret data.

4.MD.4

Make a line plot to display a data set of measurements in fractions of a unit (1/2, 1/4, 1/8). Solve problems involving addition and subtraction of fractions by using information presented in line plots. *For example, from a line plot find and interpret the difference in length between the longest and shortest specimens in an insect collection.*

Foundational Standards

3.NF.1

Understand a fraction $1/b$ as the quantity formed by 1 part when a whole is partitioned into *b* equal parts; understand a fraction a/b as the quantity formed by *a* parts of size $1/b$.

3.NF.2

Understand a fraction as a number on the number line; represent fractions on a number line diagram.

a. Represent a fraction $1/b$ on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into *b* equal parts. Recognize that each part has size $1/b$ and that the endpoint of the part based at 0 locates the number $1/b$ on the number line.
b. Represent a fraction a/b on a number line diagram by marking off *a* lengths $1/b$ from 0. Recognize that the resulting interval has size a/b and that its endpoint locates the number a/b on the number line.

3.NF.3

Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size.

a. Understand two fractions as equivalent (equal) if they are the same size, or the same point on a number line.
b. Recognize and generate simple equivalent fractions, e.g., 1/2 = 2/4, 4/6 = 2/3. Explain why the fractions are equivalent, e.g., by using a visual fraction model.
c. Express whole numbers as fractions, and recognize fractions that are equivalent to whole numbers. *Examples: Express 3 in the form 3 = 3/1; recognize that 6/1 = 6; locate 4/4 and 1 at the same point of a number line diagram.*
d. Compare two fractions with the same numerator or the same denominator by reasoning about their size. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.

3.MD.4

Generate measurement data by measuring lengths using rulers marked with halves and fourths of an inch. Show the data by making a line plot, where the horizontal scale is marked off in appropriate units—whole numbers, halves, or quarters.

3.G.2

Partition shapes into parts with equal areas. Express the area of each part as a unit fraction of the whole. *For example, partition a shape into 4 parts with equal area, and describe the area of each part as ¼ of the area of the shape.*

Focus Standards for Mathematical Practice

MP.2

Reason abstractly and quantitatively. Students reason both abstractly and quantitatively throughout this mission. They draw area models, number lines, and tape diagrams to represent fractional quantities, as well as word problems.

MP.3

Construct viable arguments and critique the reasoning of others. Much of the work in this mission is centered on multiple ways to solve fraction and mixed number problems. Students explore various strategies and participate in many turn and talk and explain to your partner activities. By doing so, they construct arguments to defend their choice of strategy, as well as think about and critique the reasoning of others.

MP.4

Model with mathematics. Throughout this mission, students represent fractions with various models. Area models are used to investigate and prove equivalence. The number line is used to compare and order fractions, as well as model addition and subtraction of fractions. Students also use models in problem solving as they create line plots to display given sets of fractional data and solve problems requiring the interpretation of data presented in line plots.

MP.7

Look for and make use of structure. As students progress through this fraction mission, they search for and use patterns and connections that help them build understanding of new concepts. They relate and apply what they know about operations with whole numbers to operations with fractions.