SMALL GROUP LESSONS

Grade 4, Mission 1

Add, Subtract, and Round

Lessons

YOUR NOTES

Topic A: Place Value of Multi-Digit Whole Numbers

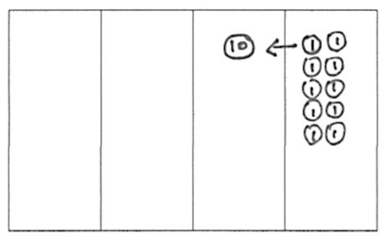
The place value chart is fundamental to Topic A. Building upon their previous knowledge of bundling, students learn that 10 hundreds can be composed into 1 thousand, and therefore, 30 hundreds can be composed into 3 thousands because a digit's value is 10 times what it would be one place to its right.

Lesson 1

Interpret a multiplication equation as a comparison.

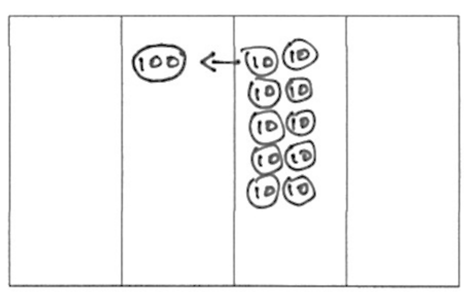
Materials: (T) Place value disks: ones, tens, hundreds, and thousands; unlabeled thousands place value chart (Template) (S) Personal white board, unlabeled thousands place value chart (Template)

Problem 1: 1 ten is 10 times as much as 1 one.



T: (Have a place value chart ready. Draw or place 1 unit into the ones place.)
T: How many units do I have?
S: 1.
T: What is the name of this unit?
S: A one.
T: Count the ones with me. (Draw ones as they do so.)
S: 1 one, 2 ones, 3 ones, 4 ones, 5 ones…,10 ones.
T: 10 ones. What larger unit can I make?
S: 1 ten.
T: I change 10 ones for 1 ten. We say, "1 ten is 10 times as much as 1 one." Tell your partner what we say and what that means. Use the model to help you.
S: 10 ones make 1 ten. → 10 times 1 one is 1 ten or 10 ones. → We say 1 ten is 10 times as many as 1 one.

Problem 2: One hundred is 10 times as much as 1 ten.

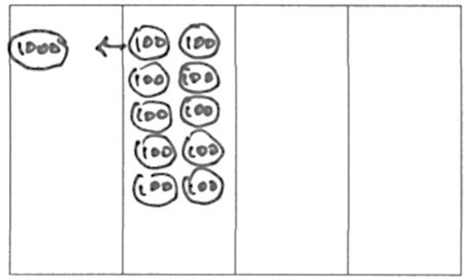


Quickly repeat the process from Problem 1 with 10 copies of 1 ten.

YOUR NOTES

Problem 3: One thousand is 10 times as much as 1 hundred.

Quickly repeat the process from Problem 1 with 10 copies of 1 hundred.



T: Discuss the patterns you have noticed with your partner.

S: 10 ones make 1 ten. 10 tens make 1 hundred. 10 hundreds make 1 thousand. → Every time we get 10, we bundle and make a bigger unit. → We copy a unit 10 times to make the next larger unit. → If we take any of the place value units, the next unit on the left is ten times as many.

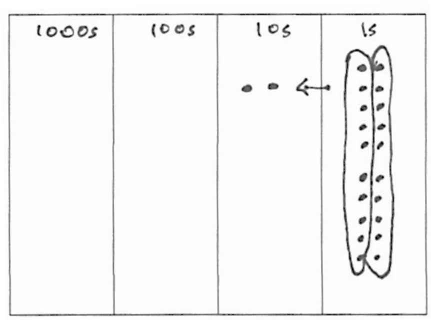
T: Let's review, in words, the multiplication pattern that matches our models and 10 times as many.

Display the following information for student reference:

- 1 ten = 10 × 1 one (Say, "1 ten is 10 times as much as 1 one.")
- 1 hundred = 10 × 1 ten (Say, "1 hundred is 10 times as much as 1 ten.")
- 1 thousand = 10 × 1 hundred (Say, "1 thousand is 10 times as much as 1 hundred.")

Problem 4: Model *10 times as much* as on the place value chart with an accompanying equation.

Note: Place value disks are used as models throughout the curriculum and can be represented in two different ways. A disk with a value labeled inside of it, such as in Problem 1, should be drawn or placed on a place value chart with no headings. The value of the disk in its appropriate column indicates the column heading. A place value disk drawn as a dot should be used on place value charts with headings, as in Problem 4. This type of representation is called the *chip model*. The chip model is a faster way to represent place value disks and is used as students move away from a concrete stage of learning.



(Model 2 tens is 10 times as much as 2 ones on the place value chart and as an equation.)

T: Draw place value disks as dots. Because you are using dots, label your columns with the unit value.

YOUR NOTES

T: Represent 2 ones. Solve to find 10 times as many as 2 ones. Work together.

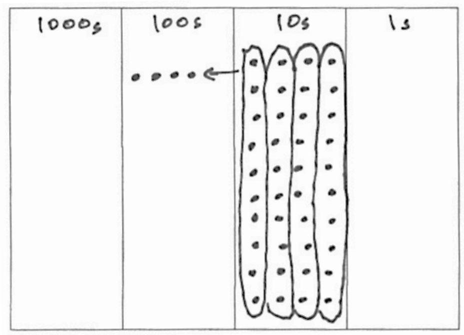
S: (Work together.)

T: 10 times as many as 2 ones is...?

S: 20 ones. → 2 tens.

T: Explain this equation to your partner using your model.

S: 10 × 2 ones = 20 ones = 2 tens.



Repeat the process with 10 times as many as 4 tens is 40 tens is 4 hundreds and 10 times as many as 7 hundreds is 70 hundreds is 7 thousands.

- 10 × 4 tens = 40 tens = 4 hundreds
- 10 × 7 hundreds = 70 hundreds = 7 thousands

Problem 5: Model as an equation 10 times as much as 9 hundreds is 9 thousands.

T: Write an equation to find the value of 10 times as many as 9 hundreds. (Circulate and assist students as necessary.)

T: Show me your board. Read your equation.

S: 10 × 9 hundreds = 90 hundreds = 9 thousands.

T: Yes. Discuss whether this is true with your partner. (Write 10 × 9 hundreds = 9 thousands.)

S: Yes, it is true because 90 hundreds equals 9 thousands, so this equation just eliminates that extra step. → Yes. We know 10 of a smaller unit equals 1 of the next larger unit, so we just avoided writing that step.

NOTES

Debrief Questions

- What are some ways you could model 10 times as many? What are the benefits and drawbacks of each way of modeling? (Money, base ten materials, disks, labeled drawings of disks, dots on a labeled place value chart, tape diagram.)
- Take two minutes to explain to your partner what we learned about the value of each unit as it moves from right to left on the place value chart.
- Write and complete the following statements:
  - _____ ten is _____ times as many as _____ one.
  - _____ hundred is _____ times as many as _____ ten.
  - _____ thousand is _____ times as many as _____ hundred.

Lesson 2

Recognize a digit represents 10 times the value of what it represents in the place to its right.

Materials: (S) Personal white board, unlabeled millions place value chart (Template)

YOUR NOTES

Problem 1: Multiply single units by 10 to build the place value chart to 1 million. Divide to reverse the process.

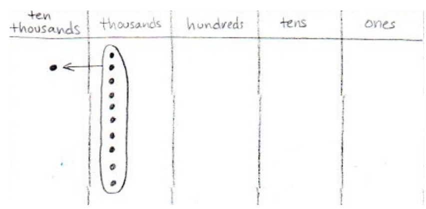
T: Label ones, tens, hundreds, and thousands on your place value chart.

T: On your personal white board, write the multiplication sentence that shows the relationship between 1 hundred and 1 thousand.

S: (Write 10 × 1 hundred = 10 hundreds = 1 thousand.)

T: Draw place value disks on your place value chart to find the value of 10 times 1 thousand.

T: (Circulate.) I saw that Tessa drew 10 disks in the thousands column. What does that represent?



S: 10 times 1 thousand equals 10 thousands. (10 × 1 thousand = 10 thousands.)

T: How else can 10 thousands be represented?

S: 10 thousands can be bundled because, when you have 10 of one unit, you can bundle them and move the bundle to the next column.

T: (Point to the place value chart.) Can anyone think of what the name of our next column after the thousands might be? (Students share. Label the **ten thousands** column.)

T: Now, write a complete multiplication sentence to show 10 times the value of 1 thousand. Show how you regroup.

S: (Write 10 × 1 thousand = 10 thousands = 1 ten thousand.)

T: On your place value chart, show what 10 times the value of 1 ten thousand equals. (Circulate and assist students as necessary.)

T: What is 10 times 1 ten thousand?

S: 10 ten thousands. → 1 **hundred thousand.**

T: That is our next larger unit. (Write 10 × 1 ten thousand = 10 ten thousands = 1 hundred thousand.)

T: To move another column to the left, what would be my next 10 times statement?

S: 10 times 1 hundred thousand.

T: Solve to find 10 times 1 hundred thousand. (Circulate and assist students as necessary.)

T: 10 hundred thousands can be bundled and represented as **1 million**. Title your column, and write the multiplication sentence.

S: (Write 10 × 1 hundred thousand = 10 hundred thousands = 1 million.)

YOUR NOTES

After having built the place value chart by multiplying by ten, quickly review the process simply moving from right to left on the place value chart and then reversing and moving left to right (e.g., 2 tens times 10 equals 2 hundreds; 2 hundreds times 10 equals 2 thousands; 2 thousands divided by 10 equals 2 hundreds; 2 hundreds divided by 10 equals 2 tens).

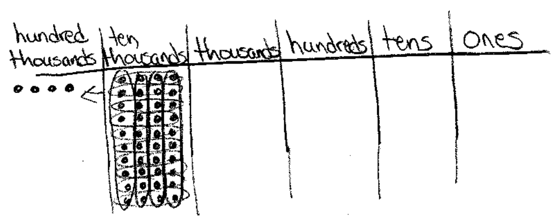
Problem 2: Multiply multiple copies of one unit by 10.

T: Draw place value disks, and write a multiplication sentence to show the value of 10 times 4 ten thousands.

T: 10 times 4 ten thousands is...?

S: 40 ten thousands. → 4 hundred thousands.

T: (Write 10 × 4 ten thousands = 40 ten thousands = 4 hundred thousands.) Explain to your partner how you know this equation is true.



Repeat with 10 × 3 hundred thousands.

Problem 3: Divide multiple copies of one unit by 10.

T: (Write 2 thousands ÷ 10.) What is the process for solving this division expression?

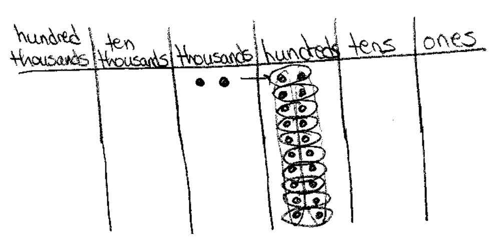
S: Use a place value chart. → Represent 2 thousands on a place value chart. Then, change them for smaller units so we can divide.

T: What would our place value chart look like if we changed each thousand for 10 smaller units?

S: 20 hundreds. → 2 thousands can be changed to be 20 hundreds because 2 thousands and 20 hundreds are equal.

T: Solve for the answer.

S: 2 hundreds. → 2 thousands ÷ 10 is 2 hundreds because 2 thousands unbundled becomes 20 hundreds. → 20 hundreds divided by 10 is 2 hundreds. → 2 thousands ÷ 10 = 20 hundreds ÷ 10 = 2 hundreds.



Repeat with 3 hundred thousands ÷ 10.

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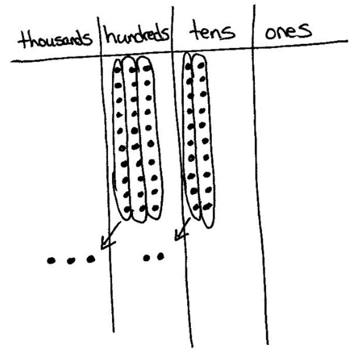
Problem 4: Multiply and divide multiple copies of two different units by 10.

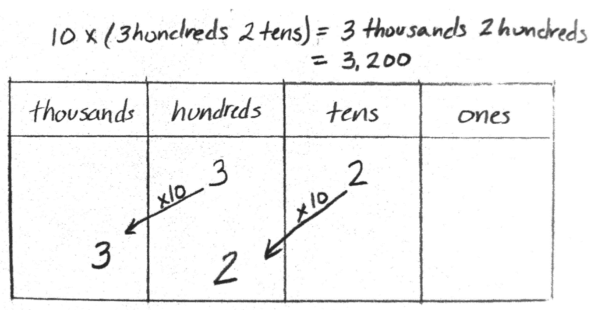


T: Draw place value disks to show 3 hundreds and 2 tens.

T: (Write 10 × (3 hundreds 2 tens).) Work in pairs to solve this expression. I wrote 3 hundreds 2 tens in parentheses to show it is one number. (Circulate as students work. Clarify that both hundreds and tens must be multiplied by 10.)

T: What is your product?

S: 3 thousands 2 hundreds.

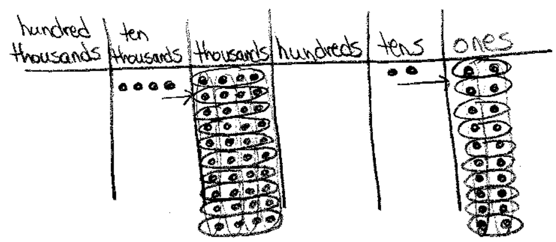


T: (Write 10 × (3 hundreds 2 tens) = 3 thousands 2 hundreds.) How do we write this in standard form?

S: 3,200.

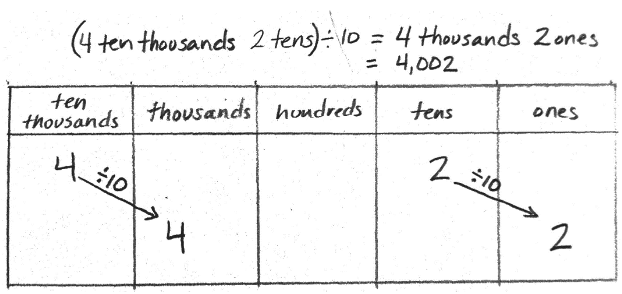
T: (Write 10 × (3 hundreds 2 tens) = 3 thousands 2 hundreds = 3,200.)

T: (Write (4 ten thousands 2 tens) ÷ 10.) In this expression, we have two units. Explain how you will find your answer.



S: We can use the place value chart again and represent the unbundled units and then divide. (Represent in the place value chart, and record the number sentence (4 ten thousands 2 tens) ÷ 10 = 4 thousands 2 ones = 4,002.)

T: Watch as I represent numbers in the place value chart to multiply or divide by ten instead of drawing disks.



Repeat with 10 × (4 thousands 5 hundreds) and (7 hundreds 9 tens) ÷ 10.

NOTES

Debrief Questions

- How did we use patterns to predict the increasing units on the place value chart up to **1 million**? Can you predict the unit that is 10 times 1 million? 100 times 1 million?
- What happens when you multiply a number by 10? 1 **ten thousand** is what times 10? 1 **hundred thousand** is what times 10?
- Gail said she noticed that when you multiply a number by 10, you shift the digits one place to the left and put a zero in the ones place. Is she correct?
- How can you use multiplication and division to describe the relationship between units on the place value chart?

Multiple Means of Representation

Scaffold student understanding of the place value pattern by recording the following sentence frames:

- 10 × 1 one is 1 ten
- 10 × 1 ten is 1 hundred
- 10 × 1 hundred is 1 thousand
- 10 × 1 thousand is 1 ten thousand
- 10 × 1 ten thousand is 1 hundred thousand

Students may benefit from speaking this pattern chorally. Deepen understanding with prepared visuals (perhaps using an interactive whiteboard).

YOUR NOTES

Lesson 3

Name numbers within 1 million by building understanding of the place value chart and placement of commas for naming base thousand units.

Materials: (S) Personal white board, unlabeled millions place value chart (Lesson 2 Template)

Note: Students will go beyond 1 million to establish a pattern within the base ten units.

Introduction: Patterns of the base ten system.

T: In the last lesson, we extended the place value chart to 1 million. Take a minute to label the place value headings on your place value chart. (Circulate and check all headings.)

T: Excellent. Now, talk with your partner about similarities and differences you see in those heading names.

S: I notice some words repeat, like *ten*, *hundred*, and *thousand*, but ones appears once. → I notice the thousand unit repeats 3 times—thousands, ten thousands, hundred thousands.

T: That's right! Beginning with thousands, we start naming new place value units by how many one thousands, ten thousands, and hundred thousands we have. What do you think the next unit might be called after 1 million?

S: **Ten millions.**

T: (Extend chart to the ten millions.) And the next?

S: **Hundred millions.**

T: (Extend chart again.) That's right! Just like with thousands, we name new units here in terms of how many one millions, ten millions, and hundred millions we have. 10 hundred millions gets renamed as 1 billion. Talk with your partner about what the next two place value units should be.

S: Ten billions and hundred billions. → It works just like it does for thousands and millions.

OPTIONAL FOR FLEX DAY: PROBLEM 1

Problem 1: Placing commas in and naming numbers.

T: You've noticed a pattern: ones, tens, and hundreds; one thousands, ten thousands, and hundred thousands; one millions, ten millions, and hundred millions; and so on. We use commas to indicate this grouping of units, taken 3 at a time. For example, ten billion would be written: 10,000,000,000.

T: (Write 608430325.) Record this number, and place the commas to show our groupings

YOUR NOTES

of units.

S: (Record the number and place the commas.)

T: (Show 430,325 on a place value chart.) How many thousands are in this number?

S: 430.

T: 430 what?

S: 430 thousands.

T: Correct. We read this number as "four hundred thirty thousand, three hundred twenty-five."

T: (Extend chart, and show 608,430,325.) How many millions are there in this number?

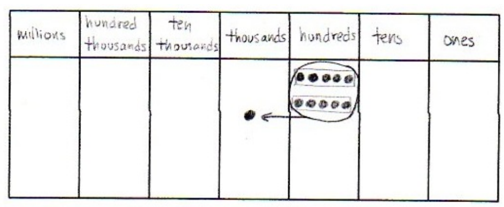
S: 608 millions.

T: Using what you know about our pattern in naming units, talk with your partner about how to name this number.

S: Six hundred eight million, four hundred thirty thousand, three hundred twenty-five.

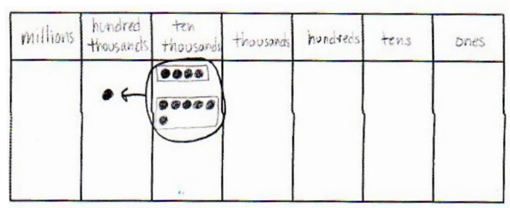
Problem 2: Add to make 10 of a unit and bundling up to 1 million.

T: What would happen if we combined 2 groups of 5 hundreds? With your partner, draw place value disks to solve. Use the largest unit possible to express your answer.



S: 2 groups of 5 hundreds equals 10 hundreds. → It would make 10 hundreds, which can be bundled to make 1 thousand.

T: Now, solve for 5 thousands plus 5 thousands. Bundle in order to express your answer using the largest unit possible.



S: 5 thousands plus 5 thousands equals 10 thousands. We can bundle 10 thousands to make 1 ten thousand.

T: Solve for 4 ten thousands plus 6 ten thousands. Express your answer using the largest unit possible.

S: 4 ten thousands plus 6 ten thousands equals 10 ten thousands. We can bundle 10 ten thousands to make 1 hundred thousand.

Continue renaming problems, showing regrouping as necessary.

- 3 hundred thousands + 7 hundred thousands
- 23 thousands + 4 ten thousands
- 43 ten thousands + 11 thousands

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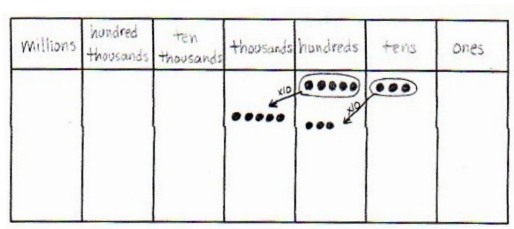
Problem 3: 10 times as many with multiple units.



T: On your place value chart, model 5 hundreds and 3 tens with place value disks. What is 10 times 5 hundreds 3 tens?

S: (Show charts.) 5 thousands 3 hundreds.

T: Model 10 times 5 hundreds 3 tens with digits on the place value chart. Record your answer in standard form.

S: (Show 10 times 5 hundreds is 5 thousands and 10 times 3 tens is 3 hundreds as digits.) 5,300.

T: Check your partner's work, and remind him of the comma's role in this number.

T: (Write 10 × 1 ten thousand 5 thousands 3 hundreds 9 ones = ______.) With your partner, solve this problem, and write your answer in standard form.

S: 10 × 15,309 = 153,090.

NOTES

Debrief Questions

- How does place value understanding and the role of commas help you to read the value in the millions period that is represented by the number of millions, **ten millions**, and **hundred millions**?
- When might it be useful to omit commas? (Please refer to the notes on commas to guide your discussion.)

Note

In this lesson, students extend past 1 million to establish a pattern of ones, tens, and hundreds within each base ten unit (thousands, millions, billions, trillions). Calculations in following lessons are limited to less than or equal to 1 million. If students are not ready for this step, omit establishing the pattern and internalize the units of the thousands period.

Commas

Commas are optional for 4-digit numbers, as omitting them supports visualization of the total amount of each unit. For example, in the number 3247, 32 hundreds or 324 tens is easier to visualize when 3247 is written without a comma. In Grade 3, students understand 324 as 324 ones, 32 tens 4 ones, or 3 hundreds 2 tens 4 ones. This flexible thinking allows for seeing simplifying strategies (e.g., to solve 3247 – 623, rather than decompose 3 thousands, students might subtract 6 hundreds from 32 hundreds: 32 hundreds – 6 hundreds + 47 ones – 23 ones is 26 hundreds and 24 ones or 2624).

Multiple Means of Action and Expression

Scaffold reading numbers into the hundred thousands with questioning such as:

T: What's the value of the 3?
S: 30 thousand.
T: How many thousands altogether?
S: 36 thousands.
T: What's the value of the 8?
S: 80.
T: Add the remaining ones.
S: 89.
T: Read the whole number.
S: Thirty-six thousand, eighty-nine.

Continue with similar numbers until students reach fluency. Alternate the student recording numbers, modeling, and reading.

Multiple Means of Action and Expression

Scaffold partner talk with sentence frames such as:

- "I notice _____."
- "The place value headings are alike because _____."
- "The place value headings are not alike because _____."
- "The pattern I notice is _____."
- "I notice the units _______."

YOUR NOTES

Lesson 4

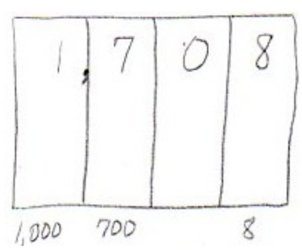
Read and write multi-digit numbers using base ten numerals, number names, and expanded form.

Materials: (S) Personal white board, unlabeled millions place value chart (Lesson 2 Template)

OPTIONAL FOR FLEX DAY: PROBLEM 1

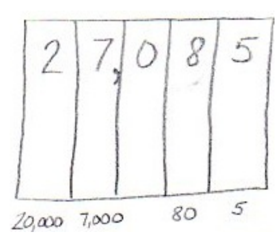
Problem 1: Write a four-digit number in expanded form.

T: On your place value chart, write 1,708.
T: What is the value of the 1?
S: 1 thousand.
T: (Record 1,000 under the thousands column.) What is the value of the 7?
S: 7 hundred.
T: (Record 700 under the hundreds column.) What value does the zero have?
S: Zero. → Zero tens.
T: What is the value of the 8?
S: 8 ones.
T: (Record 8 under the ones column.) What is the value of 1,000 and 700 and 8?
S: 1,708.
T: So, 1,708 is the same as 1,000 plus 700 plus 8.
T: Record that as a number sentence.
S: (Write 1,000 + 700 + 8 = 1,708.)



Problem 2: Write a five-digit number in word form and expanded form.

T: Now, erase your values, and write this number: 27,085.
T: Show the value of each digit at the bottom of your place value chart.
S: (Write 20,000, 7,000, 80, and 5.)
T: Why is there no term representing the hundreds?
S: Zero stands for nothing. → Zero added to a number doesn't change the value.
T: With your partner, write an addition sentence to represent 27,085.



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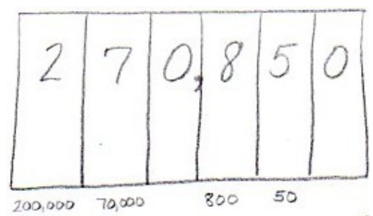
S: 20,000 + 7,000 + 80 + 5 = 27,085.
T: Now, read the number sentence with me.
S: Twenty thousand plus seven thousand plus eighty plus five equals twenty-seven thousand, eighty-five.
T: (Write the number as you speak.) You said "twenty-seven thousand, eighty-five."
T: What do you notice about where I placed a comma in both the standard form and word form?
S: It is placed after 27 to separate the thousands in both the standard form and word form.

Problem 3: Transcribe a number in word form to standard and expanded form.

Display two hundred seventy thousand, eight hundred fifty.

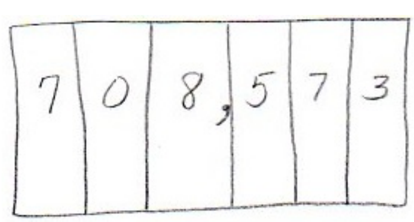


T: Read this number. (Students read.) Tell your partner how you can match the word form to the standard form.
S: Everything you say, you should write in words. → The comma helps to separate the numbers in the thousands from the numbers in the hundreds, tens, and ones.
T: Write this number in your place value chart. Now, write this number in expanded form. Tell your partner the number sentence.
S: 200,000 plus 70,000 plus 800 plus 50 equals 270,850.

Repeat with sixty-four thousand, three.

Problem 4: Convert a number in expanded form to word and standard form.

Display 700,000 + 8,000 + 500 + 70 + 3.



T: Read this expression. (Students read.) Use digits to write this number in your place value chart.
T: My sum is 78,573. Compare your sum with mine.
S: Your 7 is in the wrong place. → The value of the 7 is 700,000. Your 7 has a value of 70,000.
T: Read this number in standard form with me.
S: Seven hundred eight thousand, five hundred seventy-three.
T: Write this number in words. Remember to check for correct use of commas and hyphens.

Repeat with 500,000 + 30,000 + 10 + 3.

NOTES

Debrief Questions

- Two students discussed the importance of zero. Nate said that zero is not important while Jill said that zero is extremely important. Who is right? Why do you think so?
- What role can zero play in a number?
- How is the expanded form related to the standard form of a number?
- When might you use expanded form to solve a calculation?

Multiple Means of Action and Expression

Scaffold student composition of number words with the following options:

- Provide individual cards with number words that can be easily copied.
- Allow students to abbreviate number words.
- Set individual goals for writing number words.
- Allow English language learners their language of choice for expressing number words.

YOUR NOTES

Topic B: Comparing Multi-Digit Whole Numbers

In Topic B, students use place value as a basis for comparing whole numbers. Although this is not a new concept, it becomes more complex as the numbers become larger.

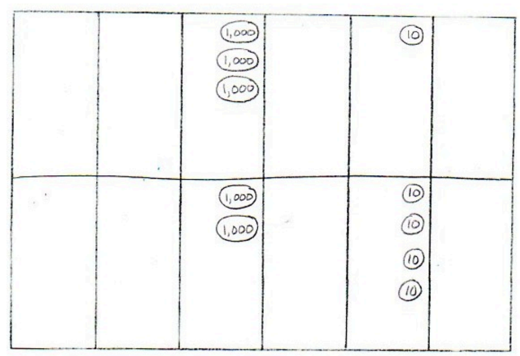
Lesson 5

Compare numbers based on meanings of the digits using >, <, or = to record the comparison.

Materials: (S) Personal white board, unlabeled hundred thousands place value chart (Template)

Problem 1: Comparing two numbers with the same largest unit.

Display: 3,010 ⬤ 2,040.



T: Let's compare two numbers. Say the standard form to your partner, and model each number on your place value chart.

S: Three thousand, ten. Two thousand, forty.

T: What is the name of the unit with the greatest value?

S: Thousands.

T: Compare the value of the thousands.

S: 3 thousands is greater than 2 thousands. → 2 thousands is less than 3 thousands.

T: Tell your partner what would happen if we only compared tens rather than the unit with the greatest value.

S: We would say that 2,040 is greater than 3,010, but that isn't right. → The number with more of the largest unit being compared is greater. → We don't need to compare the tens because the thousands are different.

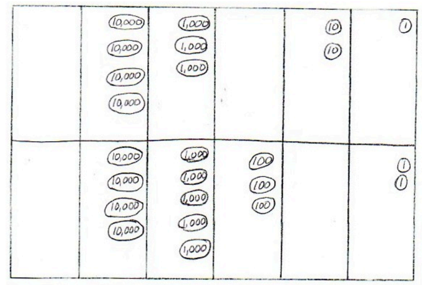
T: Thousands is our largest unit. 3 thousands is greater than 2 thousands, so 3,010 is greater than 2,040. (Write the comparison symbol > in the circle.) Write this comparison statement on your board, and say it to your partner in two different ways.

S: (Write 3,010 > 2,040.) 3,010 is greater than 2,040. 2,040 is less than 3,010.

Problem 2: Comparing two numbers with an equal amount of the largest units.

Display: 43,021 ⬤ 45,302.

YOUR NOTES



T: Model and read each number. How is this comparison different from our first comparison?

S: Before, our largest unit was thousands. Now, our largest unit is ten thousands. → In this comparison, both numbers have the same number of ten thousands.

T: If the digits of the largest unit are equal, how do we compare?

S: We compare the thousands. → We compare the next largest unit. → We compare the digit one place to the right.

T: Write your comparison statement on your board. Say the comparison statement in two ways.

S: (Write 43,021 < 45,302 and 45,302 > 43,021.) 43,021 is less than 45,302. 45,302 is greater than 43,021.

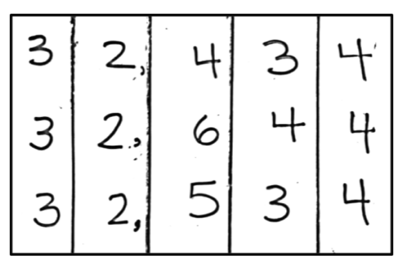
Repeat the comparison process using 2,305 and 2,530 and then 970,461 and 907,641.

T: Write your own comparison problem for your partner to solve. Create a two-number comparison problem in which the largest unit in both numbers is the same.

OPTIONAL FOR FLEX DAY: PROBLEM 3

Problem 3: Comparing values of multiple numbers using a place value chart.

Display: 32,434, 32,644, and 32,534.



T: Write these numbers in your place value chart. Whisper the value of each digit as you do so.

T: When you compare the value of these three numbers, what do you notice?

S: All three numbers have 3 ten thousands. → All three numbers have 2 thousands. → We can compare the hundreds because they are different.

T: Which number has the greatest value?

S: 32,644.

T: Tell your partner which number has the least value and how you know.

S: 32,434 is the smallest of the three numbers because it has the least number of hundreds.

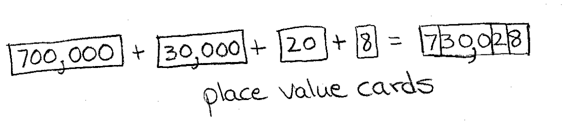
T: Write the numbers from greatest to least. Use comparison symbols to express the relationships of the numbers.

S: (Write 32,644 > 32,534 > 32,434.)

YOUR NOTES

Problem 4: Comparing numbers in different number forms.

Display: Compare 700,000 + 30,000 + 20 + 8 and 735,008.



T: Discuss with your partner how to solve and write your comparison.

S: I will write the numerals in my place value chart to compare. → Draw disks for each number. → I'll write the first number in standard form and then compare.

S: (Write 730,028 < 735,008.)

T: Tell your partner which units you compared and why.

S: I compared thousands because the larger units were the same. 5 thousands are greater than 0 thousands, so 735,008 is greater than 730,028.

Repeat with 4 hundred thousands 8 thousands 9 tens and 40,000 + 8,000 + 90.

NOTES

Debrief Questions

- When comparing numbers, which is more helpful to you: lining up digits or lining up place value disks in a place value chart? Explain.
- How does your understanding of place value help to compare and order numbers?
- How can ordering numbers apply to real life?
- What challenges arise in comparing numbers when the numbers are written in different forms?

Multiple Means of Representation

Provide sentence frames for students to refer to when using comparative statements.

Multiple Means of Action and Expression

For students who have difficulty converting numbers from expanded form into standard form, demonstrate using a place value chart to show how each number can be represented and then how the numbers can be added together. Alternatively, use place value cards (known as Hide Zero cards in the primary grades) to allow students to see the value of each digit that composes a number. The cards help students manipulate and visually display both the expanded form and the standard form of any number.

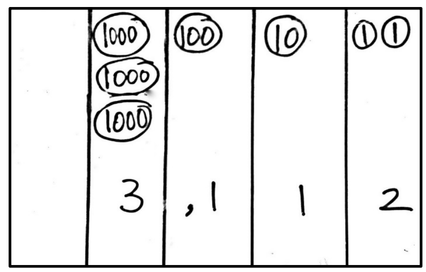
YOUR NOTES

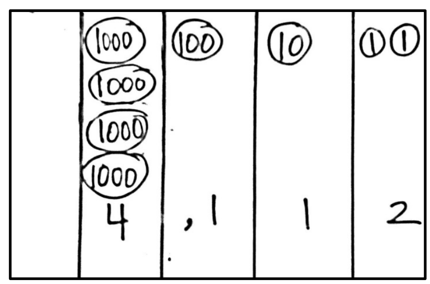
Lesson 6

Find 1, 10, and 100 thousand more and less than a given number.

Materials: (T) Unlabeled hundred thousands place value chart (Lesson 5 Template)
(S) Personal white board, unlabeled hundred thousands place value chart (Lesson 5 Template)

Problem 1: Find 1 thousand more and 1 thousand less.



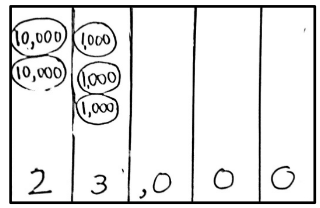


T: (Draw 2 thousands disks in the place value chart.) How many thousands do you count?
S: Two thousands.
T: What number is one thousand more? (Draw 1 more thousand.)
S: Three thousands.
T: (Write 3 thousands 112 ones.) Model this number with disks, and write its expanded and standard form.
S: (Write 3,000 + 100 + 10 + 2. 3,112.)
T: Draw 1 more unit of one thousand. What number is 1 thousand more than 3,112?
S: 4,112 is 1 thousand more than 3,112.
T: 1 thousand less than 3,112?
S: 2,112.
T: Draw 1 ten thousands disk. What number do you have now?
S: 14,112.
T: Show 1 less unit of 1 thousand. What number is 1 thousand less than 14,112?
S: 13,112.
T: 1 thousand more than 14,112?
S: 15,112.
T: Did the largest unit change? Discuss with your partner.
S: (Discuss.)
T: Show 19,112. (Pause as students draw.) What is 1 thousand less? 1 thousand more than 19,112?
S: 18,112. 20,112.
T: Did the largest unit change? Discuss with your partner.
S: (Discuss.)
T: Show 199,465. (Pause as they do so.) What is 1 thousand less? 1 thousand more than 199,465?

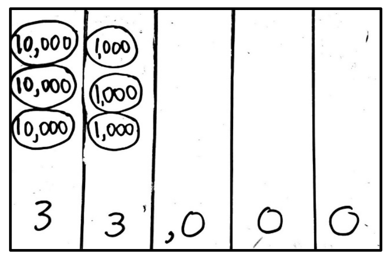
YOUR NOTES

S: 198,465. 200,465.
T: Did the largest unit change? Discuss with your partner.
S: (Discuss.)

Problem 2: Find 10 thousand more and 10 thousand less.



T: Use numbers and disks to model 2 ten thousands 3 thousands. Read and write the expanded form.
S: (Model, read, and write 20,000 + 3,000 = 23,000.)
T: What number is 10 thousand more than 2 ten thousands 3 thousands? Draw, read, and write the expanded form.
S: (Model, read, and write 20,000 + 10,000 + 3,000 = 33,000.)



T: (Display 100,000 + 30,000 + 4,000.) Use disks and numbers to model the sum. What number is 10 thousand more than 134,000? Say your answer as an addition sentence.
S: 10,000 plus 134,000 is 144,000.
T: (Display 25,130 – 10,000.) What number is 10 thousand less than 25,130? Work with your partner to use numbers and disks to model the difference. Write and whisper to your partner an equation in unit form to verify your answer.
S: (Model, read, and write 2 ten thousands 5 thousands 1 hundred 3 tens minus 1 ten thousand is 1 ten thousand 5 thousands 1 hundred 3 tens.)

Problem 3: Find 100 thousand more and 100 thousand less.

T: (Display 200,352.) Work with your partner to find the number that is 100 thousand more than 200,352. Write an equation to verify your answer.
S: (Write 200,352 + 100,000 = 300,352.)
T: (Display 545,000 and 445,000 and 345,000.) Read these three numbers to your partner. Predict the next number in my pattern, and explain your reasoning.
S: I predict the next number will be 245,000. I notice the numbers decrease by 100,000. 345,000 minus 100,000 is 245,000. → I notice the hundred thousand units decreasing: 5 hundred thousands, 4 hundred thousands, 3 hundred thousands. I predict the next number will have 2 hundred thousands. I notice the other units do not change, so the next number will be 2 hundred thousands 4 ten thousands 5 thousands.

NOTES

Debrief Questions

- How does your understanding of place value help you add or subtract 1,000, 10,000, and 100,000?
- What place value patterns have we discovered?

Multiple Means of Engagement

After students predict the next number in the pattern, ask students to create their own pattern using the strategy of one thousand more or less, ten thousand more or less, or one hundred thousand more or less. Then, ask students to challenge their classmates to predict the next number in the pattern.

YOUR NOTES

Topic C: Rounding Multi-Digit Whole Numbers

Grade 4 students moving into Topic C learn to round to any place value, initially using the vertical number line though ultimately moving away from the visual model altogether. Topic C also includes word problems where students apply rounding to real life situations.

Lesson 7

Round multi-digit numbers to the thousands place using the vertical number line.

Materials: (S) Personal white board

Problem 1: Use a vertical number line to round four-digit numbers to the nearest thousand.

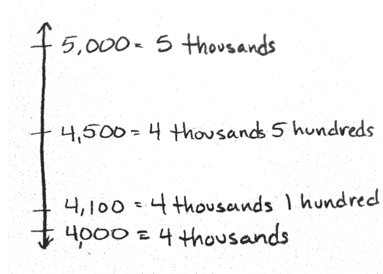
T: (Draw a vertical number line with 2 endpoints.) We are going to round 4,100 to the nearest thousand. How many thousands are in 4,100?

S: 4 thousands.

T: (Mark the lower endpoint with 4 thousands.) And 1 more thousand would be?

S: 5 thousands.

T: (Mark the upper endpoint with 5 thousands.) What's halfway between 4 thousands and 5 thousands?



S: 4,500.

T: (Label 4,500 on the number line.) Where should I label 4,100? Tell me where to stop. (Move your marker up the line.)

S: Stop!

T: (Label 4,100 on the number line.) Is 4,100 nearer to 4 thousands or 5 thousands?

S: 4,100 is nearer to 4 thousands.

T: True. We say 4,100 rounded to the nearest thousand is 4,000.

T: (Label 4,700 on the number line.) What about 4,700?

S: 4,700 is nearer to 5 thousands.

T: Therefore, we say 4,700 rounded to the nearest thousand is 5,000.

Problem 2: Use a vertical number line to round five- and six-digit numbers to the nearest thousand.

T: Let's round 14,500 to the nearest thousand. How many thousands are there in 14,500?

YOUR NOTES

S: 14 thousands.

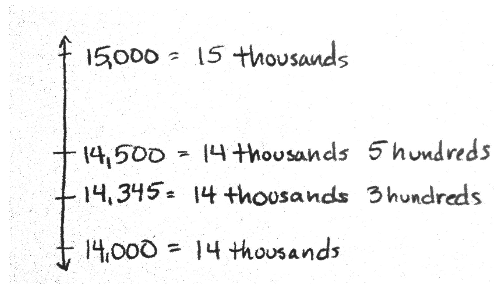
T: What's 1 more thousand?

S: 15 thousands.

T: Designate the endpoints on your number line. What is halfway between 14,000 and 15,000?

S: 14,500. Hey, that's the number that we are trying to round to the nearest thousand.

T: True. 14,500 is right in the middle. It is the halfway point. It is not closer to either number. The rule is that we round up. 14,500 rounded to the nearest thousand is 15,000.



T: With your partner, mark 14,990 on your number line, and round it to the nearest thousand.

S: 14,990 is nearer to 15 thousands or 15,000.

T: Mark 14,345 on your number line. Talk with your partner about how to round it to the nearest thousand.

S: 14,345 is nearer to 14 thousands. → 14,345 is nearer to 14,000. → 14,345 rounded to the nearest thousand is 14,000.

T: Is 14,345 greater than or less than the halfway point?

S: Less than.

T: We can look to see if 14,345 is closer to 14,000 or 15,000, and we can also look to see if it is greater than or less than the halfway point. If it is less than the halfway point, it is closer to 14,000.

Repeat using the numbers 215,711 and 214,569. Round to the nearest thousand, and name how many thousands are in each number.

NOTES

Debrief Questions

- What makes 5 special in rounding?
- How does the number line help you round numbers? Is there another way you prefer? Why?
- What is the purpose of rounding?
- When might we use rounding or estimation?

Multiple Means of Representation

For those students who have trouble conceptualizing halfway, demonstrate halfway using students as models.

Two students represent the thousands. A third student represents halfway. A fourth student represents the number being rounded.

Discuss: Where do they belong? To whom are they nearer? To which number would they round?

YOUR NOTES

Lesson 8

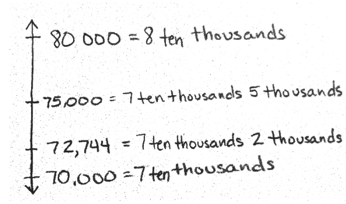
Round multi-digit numbers to any place using the vertical number line.

Materials: (S) Personal white board

OPTIONAL FOR FLEX DAY: PROBLEM 1

Problem 1: Use a vertical number line to round five- and six-digit numbers to the nearest ten thousand.

(Display a number line with endpoints 70,000 and 80,000.)



T: We are going to round 72,744 to the nearest ten thousand. How many ten thousands are in 72,744?
S: 7 ten thousands.
T: (Mark the lower endpoint with 7 ten thousands.) And 1 more ten thousand would be...?
S: 8 ten thousands.
T: (Mark the upper endpoint with 8 ten thousands.) What's halfway between 7 ten thousands and 8 ten thousands?
S: 7 ten thousands 5 thousands. → 75,000.
T: (Mark 75,000 on the number line.) Where should I label 72,744? Tell me where to stop. (Move your marker up the line.)
S: Stop.
T: (Mark 72,744 on the number line.)
T: Is 72,744 nearer to 70,000 or 80,000?
S: 72,744 is nearer to 70,000.
T: We say 72,744 rounded to the nearest ten thousand is 70,000.

Repeat with 337,601 rounded to the nearest ten thousand.

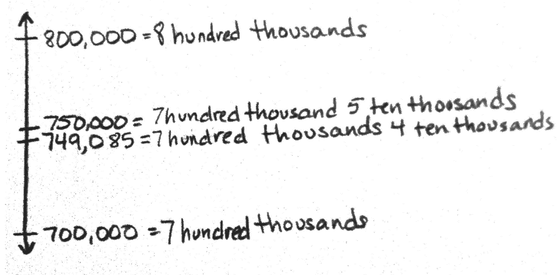
OPTIONAL FOR FLEX DAY: PROBLEM 2

Problem 2: Use a vertical number line to round six-digit numbers to the nearest hundred thousand.

T: (Draw a number line to round 749,085 to the nearest hundred thousand.) We are going to round 749,085 to the nearest hundred thousand. How many hundred thousands are

YOUR NOTES

in 749,085?



S: 7 hundred thousands.

T: What's 1 more hundred thousand?

S: 8 hundred thousands.

T: Label your endpoints on the number line. What is halfway between 7 hundred thousands and 8 hundred thousands?

S: 7 hundred thousands 5 ten thousands. → 750,000.

T: Designate the midpoint on the number line. With your partner, mark 749,085 on the number line, and round it to the nearest hundred thousand.

S: 749,085 is nearer to 7 hundred thousands. → 749,085 is nearest to 700,000. → 749,085 rounded to the nearest hundred thousand is 700,000.

Repeat with 908,899 rounded to the nearest hundred thousand.

Problem 3: Estimating with addition and subtraction.

T: (Write 505,341 + 193,841.) Without finding the exact answer, I can estimate the answer by first rounding each addend to the nearest hundred thousand and then adding the rounded numbers.

T: Use a number line to round both numbers to the nearest hundred thousand.

S: (Round 505,341 to 500,000. Round 193,841 to 200,000.)

T: Now add 500,000 + 200,000.

S: 700,000.

T: So, what's a good estimate for the sum of 505,341 and 193,841?

S: 700,000.

T: (Write 35,555 – 26,555.) How can we use rounding to estimate the answer?

S: Let's round each number before we subtract.

T: Good idea. Discuss with your partner how you will round to estimate the difference.

S: I can round each number to the nearest ten thousand. That way I'll have mostly zeros in my numbers. 40,000 minus 30,000 is 10,000. → 35,555 minus 26,555 is like 35 minus 26, which is 9. 35,000 minus 26,000 is 9,000. → It's more accurate to round up. 36,000 minus 27,000 is 9,000. Hey, it's the same answer!

T: What did you discover?

S: It's easier to find an estimate rounded to the largest unit. → We found the same estimate even though you rounded up and I rounded down. → We got two different estimates!

T: Which estimate do you suppose is closer to the actual difference?

S: I think 9,000 is closer because we changed fewer numbers when we rounded.

T: How might we find an estimate even closer to the actual difference?

S: We could round to the nearest hundred or ten..

NOTES

Debrief Questions

- Tell your partner your steps for rounding a number. Which step is most difficult for you? Why?
- Write and complete one of the following statements in your math journal:
  - The purpose of rounding addends is _____.
  - Rounding to the nearest _____ is best when _____.

Multiple Means of Representation

An effective scaffold when working in the thousands period is to first work with an analogous number in the ones period. For example:

T: Let's round 72 to the nearest ten.

T: How many tens are in 72?

S: 7 tens.

T: What is 1 more ten?

S: 8 tens.

T: 7 tens and 8 tens are the endpoints of my number line.

T: What is the value of the halfway point?

S: 7 tens 5 ones. → Seventy-five.

T: Tell me where to stop on my number line. (Start at 70 and move up.)

S: 7 tens.

S: Stop!

T: Is 72 less than halfway or more than halfway to 8 tens or 80?

S: Less than halfway.

T: We say 72 rounded to the nearest ten is 70.

T: We use the exact same process when rounding 72 thousand to the nearest ten thousand.

Multiple Means of Engagement

Make the lesson relevant to students' lives. Discuss everyday instances of estimation. Elicit examples of when a general idea about a sum or difference is necessary, rather than an exact answer. Ask, "When is it appropriate to estimate? When do we need an exact answer?"

YOUR NOTES

Lesson 9

Use place value understanding to round multi-digit numbers to any place value.

Materials: (S) Personal white board

OPTIONAL FOR FLEX DAY: PROBLEM 1

Problem 1: Rounding to the nearest thousand without using a number line.

T: (Write 4,333 ≈ ____.) Round to the nearest thousand. Between what two thousands is 4,333?
S: 4 thousands and 5 thousands.
T: What is halfway between 4,000 and 5,000?
S: 4,500.
T: Is 4,333 less than or more than halfway?
S: Less than.
T: So 4,333 is nearer to 4,000.
T: (Write 18,753 ≈____.) Round to the nearest thousand. Tell your partner between what two thousands 18,753 is located.
S: 18 thousands and 19 thousands.
T: What is halfway between 18 thousands and 19 thousands?
S: 18,500.
T: Round 18,753 to the nearest thousand. Tell your partner if 18,753 is more than or less than halfway.
S: 18,753 is more than halfway. 18,753 is nearer to 19,000. → 18,753 rounded to the nearest thousand is 19,000.

4,333 ≈ ?
___

Repeat with 346,560 rounded to the nearest thousand.

OPTIONAL FOR FLEX DAY: PROBLEM 2

Problem 2: Rounding to the nearest ten thousand or hundred thousand without using a vertical line.

T: (Write 65,600 ≈ ____.) Round to the nearest ten thousand. Between what two ten thousands is 65,600?
S: 6 ten thousands and 7 ten thousands.

YOUR NOTES

T: What is halfway between 60,000 and 70,000?
S: 65,000.
T: Is 65,600 less than or more than halfway?
S: 65,600 is more than halfway.
T: Tell your partner what 65,600 is when rounded to the nearest ten thousand.
S: 65,600 rounded to the nearest ten thousand is 70,000.

Repeat with 548,253 rounded to the nearest ten thousand.

T: (Write 676,000 ≈____.) Round 676,000 to the nearest hundred thousand. First tell your partner what your endpoints will be.
S: 600,000 and 700,000.
T: Determine the halfway point.
S: 650,000.
T: Is 676,000 greater than or less than the halfway point?
S: Greater than.
T: Tell your partner what 676,000 is when rounded to the nearest hundred thousand.
S: 676,000 rounded to the nearest hundred thousand is 700,000.
T: (Write 203,301 ≈____.) Work with your partner to round 203,301 to the nearest hundred thousand.
T: Explain to your partner how we use the midpoint to round without a number line.
S: We can't look at a number line, so we have to use mental math to find our endpoints and halfway point. → If we know the midpoint, we can see if the number is greater than or less than the midpoint. → When rounding, the midpoint helps determine which endpoint the rounded number is closer to.

Problem 3: Rounding to any value without using a number line.

147,591 ≈ 100,000
147,591 ≈ 150,000
147,591 ≈ 148,000
147,591 ≈ 147,600
147,591 ≈ 147,590

T: (Write 147,591 ≈____.) Whisper read this number to your partner in standard form. Now, round 147,591 to the nearest hundred thousand.
S: 100,000.
T: Excellent. (Write 147,591 ≈ 100,000. Point to 100,000.) 100,000 has zero ones in the ones place, zero tens in the tens place, zero hundreds in the hundreds place, zero thousands in the thousands place, and zero ten thousands in the ten thousands place. I could add, subtract, multiply, or divide with this rounded number much easier than with 147,591. True? But, what if I wanted a more accurate estimate? Give me a number closer to 147,591 that has (point) a zero in the ones, tens, hundreds, and thousands.
S: 150,000.
T: Why not 140,000?
S: 147,591 is closer to 150,000 because it is greater than the halfway point 145,000.

T: Great. 147,591 rounded to the nearest ten thousand is 150,000. Now let's round 147,591 to the nearest thousand.

S: 148,000.

T: Work with your partner to round 147,591 to the nearest hundred and then the nearest ten.

S: 147,591 rounded to the nearest hundred is 147,600. 147,591 rounded to the nearest ten is 147,590.

T: Compare estimates of 147,591 after rounding to different units. What do you notice? When might it be better to round to a larger unit? When might it be better to round to a smaller unit?

S: (Discuss.)

YOUR NOTES

NOTES

Debrief Questions

- How is rounding without a number line easier? How is it more challenging?
- How does knowing how to round mentally assist you in everyday life?
- What strategy do you use when observing a number to be rounded?

Multiple Means of Representation

Students who have difficulty visualizing 4,333 as having 4 thousands 3 hundreds could benefit from writing the number on their place value chart. In doing so, they will be able to see that 4,333 has 43 hundreds. This same strategy could also be used for other numbers.

Multiple Means of Engagement

Challenge students who are above grade level to look at the many ways that they rounded the number 147,591 to different place values. Have them discuss with a partner what they notice about the rounded numbers. Students should notice that when rounding to the hundred thousands, the answer is 100,000, but when rounding to all of the other places, the answers are closer to 150,000. Have them discuss what this can teach them about rounding.

YOUR NOTES

Lesson 10

Use place value understanding to round multi-digit numbers to any place value using real world applications.

Materials: (S) Personal white board

Problem 1: Round one number to multiple units.

T: Write 935,292 ≈ ____. Read it to your partner, and round to the nearest hundred thousand.

S: 900,000.

T: It is 900,000 when we round to the largest unit. What's the next largest unit we might round to?

S: Ten thousands.

T: Round to the ten thousands. Then, round to the thousands.

S: 940,000. 935,000.

935,292 ≈ 900,000
935,292 ≈ 940,000
935,292 ≈ 935,000

T: What changes about the numbers when rounding to smaller and smaller units? Discuss with your partner.

S: When you round to the largest unit, every other place will have a zero. → Rounding to the largest unit gives you the easiest number to add, subtract, multiply, or divide. → As you round to smaller units, there are not as many zeros in the number. → Rounding to smaller units gives an estimate that is closer to the actual value of the number.

Problem 2: Determine the best estimate to solve a word problem.

Display: In the year 2012, there were 935,292 visitors to the White House.

T: Let's read together. Assume that each visitor is given a White House map. Now, use this information to predict the number of White House maps needed for visitors in 2013. Discuss with your partner how you made your estimate.

S: I predict 940,000 maps are needed. I rounded to the nearest ten thousands place in order to make sure every visitor has a map. It is better to have more maps than not enough maps. → I predict more people may visit the White House in 2013, so I rounded to the nearest ten thousand—940,000—the only estimate that is greater than the actual number.

YOUR NOTES

Display: In the year 2011, there were 998,250 visitors to the White House.

T: Discuss with your partner how these data may change your estimate.
S: These data show the number of visitors decreased from 2011 to 2012. It may be wiser to predict 935,000 or 900,000 maps needed for 2013.
T: How can you determine the best estimate in a situation?
S: I can notice patterns or find data that might inform my estimate.

Problem 3: Choose a unit of rounding to solve a word problem.

Display: 2,837 students attend Lincoln Elementary school.

T: Discuss with your partner how you would estimate the number of chairs needed in the school.
S: I would round to the nearest thousand for an estimate of 3,000 chairs needed. If I rounded to the nearest hundred—2,800—some students may not have a seat. → I disagree. 3,000 is almost 200 too many. I would round to the nearest hundred because some students might be absent.
T: Compare the effect of rounding to the largest unit in this problem and Problem 2.
S: In Problem 2, rounding to the largest unit resulted in a number less than the actual number. By contrast, when we rounded to the largest unit in Problem 3, our estimate was greater.

$2{,}837 \approx 3{,}000$

$2{,}837 \approx 2{,}800$

$2{,}837 \approx 2{,}840$

T: What can you conclude?
S: Rounding to the largest unit may not always be a reliable estimate. → I will choose the unit based on the situation and what is most reasonable.

NOTES

Debrief Questions

- How do you choose a best estimate? What is the advantage of rounding to smaller and larger units?
- Why might you round up, even though the numbers tell you to round down?

Multiple Means of Representation

For English language learners, define unfamiliar words and experiences, such as the White House. Give an alternative example using a more familiar tourist attraction, perhaps from their cultural experience.

Multiple Means of Engagement

Challenge students working above grade level to think of at least two situations similar to that of Problem 3, where choosing the unit to which to round is important to the outcome of the problem. Have them share and discuss.

YOUR NOTES

Topic D: Multi-Digit Whole Number Addition

In Topics D and E, students focus on single like-unit calculations (ones with ones, thousands with thousands, etc.), at times requiring the composition of greater units when adding (10 hundreds are composed into 1 thousand) and decomposition into smaller units when subtracting (1 thousand is decomposed into 10 hundreds).

Lesson 11

Use place value understanding to fluently add multi-digit whole numbers using the standard addition algorithm, and apply the algorithm to solve word problems using tape diagrams.

Materials: (T) Millions place value chart (Template) (S) Personal white board, millions place value chart (Template)

Note: Using the template provided within this lesson in upcoming lessons provides students with space to draw a tape diagram and record an addition or a subtraction problem below the place value chart. Alternatively, the unlabeled millions place value chart template from Lesson 2 could be used along with paper and pencil.

Problem 1: Add, renaming once, using place value disks in a place value chart.

T: (Project vertically: 3,134 + 2,493.) Say this problem with me.

S: Three thousand, one hundred thirty-four plus two thousand, four hundred ninety-three.

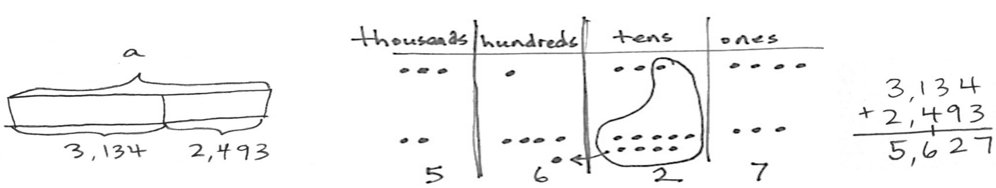
T: Draw a tape diagram to represent this problem. What are the two parts that make up the whole?

S: 3,134 and 2,493.

T: Record that in the tape diagram.

T: What is the unknown?

S: In this case, the unknown is the whole.



YOUR NOTES

T: Show the whole above the tape diagram using a bracket and label the unknown quantity with an *a*. When a letter represents an unknown number, we call that letter a **variable**.

T: (Draw place value disks on the place value chart to represent the first part, 3,134.) Now, it is your turn. When you are done, add 2,493 by drawing more disks on your place value chart.

T: (Point to the problem.) 4 ones plus 3 ones equals?

S: 7 ones. (Count 7 ones in the chart, and record 7 ones in the problem.)

T: (Point to the problem.) 3 tens plus 9 tens equals?

S: 12 tens. (Count 12 tens in the chart.)

T: We can bundle 10 tens as 1 hundred. (Circle 10 tens disks, draw an arrow to the hundreds place, and draw the 1 hundred disk to show the regrouping.)

T: We can represent this in writing. (Write 12 tens as 1 hundred, crossing the line, and 2 tens in the tens column so that you are writing 12 and not 2 and 1 as separate numbers. Refer to the visual above.)

T: (Point to the problem.) 1 hundred plus 4 hundreds plus 1 hundred equals?

S: 6 hundreds. (Count 6 hundreds in the chart, and record 6 hundreds in the problem.)

T: (Point to the problem.) 3 thousands plus 2 thousands equals?

S: 5 thousands. (Count 5 thousands in the chart, and record 5 thousands in the problem.)

T: Say the equation with me: 3,134 plus 2,493 equals 5,627. Label the whole in the tape diagram, above the bracket, with *a* = 5,627.

Problem 2: Add, renaming in multiple units, using the standard algorithm and the place value chart.

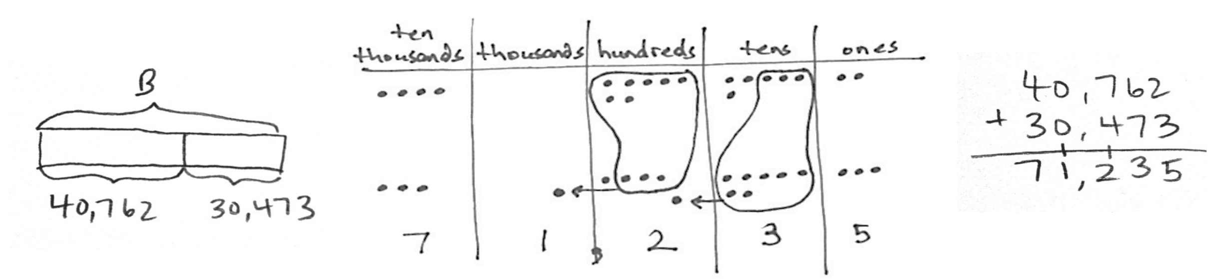
T: (Project vertically: 40,762 + 30,473.) With your partner, draw a tape diagram to model this problem, labeling the two known parts and the unknown whole, using the variable *B* to represent the whole. (Circulate and assist students.)

T: With your partner, write the problem, and draw disks for the first addend in your chart. Then, draw disks for the second addend.

T: (Point to the problem.) 2 ones plus 3 ones equals?

S: 5 ones. (Count the disks to confirm 5 ones, and write 5 in the ones column.)

T: 6 tens plus 7 tens equals?



YOUR NOTES

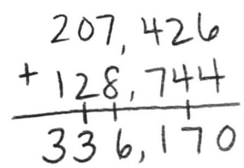
S: 13 tens. → We can group 10 tens to make 1 hundred. → We do not write two digits in one column. We can change 10 tens for 1 hundred leaving us with 3 tens.

T: (Regroup the disks.) Watch me as I record the larger unit using the addition problem. (First, record the 1 on the line in the hundreds place, and then record the 3 in the tens so that you are writing 13, not 3 then 1.)

T: 7 hundreds plus 4 hundreds plus 1 hundred equals 12 hundreds. Discuss with your partner how to record this. (Continue adding, regrouping, and recording across other units.)

T: Say the equation with me. 40,762 plus 30,473 equals 71,235. Label the whole in the tape diagram with 71,235, and write *B* = 71,235.

Problem 3: Add, renaming multiple units using the standard algorithm.



T: (Project: 207,426 + 128,744.) Draw a tape diagram to model this problem. Record the numbers on your personal white board.

T: With your partner, add units right to left, regrouping when necessary using the standard algorithm.

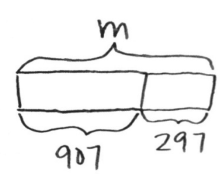
S: 207,426 + 128,744 = 336,170.

Problem 4: Solve a one-step word problem using the standard algorithm modeled with a tape diagram.

The Lane family took a road trip. During the first week, they drove 907 miles. The second week they drove the same amount as the first week plus an additional 297 miles. How many miles did they drive during the second week?

T: What information do we know?

S: We know they drove 907 miles the first week. We also know they drove 297 miles more during the second week than the first week.



T: What is the unknown information?

S: We do not know the total miles they drove in the second week.

T: Draw a tape diagram to represent the amount of miles in the first week, 907 miles. Since the Lane family drove an additional 297 miles in the second week, extend the bar for 297 more miles. What does the tape diagram represent?

S: The number of miles they drove in the second week.

T: Use a bracket and label the unknown with the variable *m* for miles.

T: How do we solve for *m*?

S: 907 + 297 = *m*.

T: (Check student work to see they are recording the regrouping of 10 of a smaller unit for 1 larger unit.)

T: Solve. What is *m*?

S: $m = 1{,}204$. (Write $m = 1{,}204$.)

T: Write a statement that tells your answer.

S: (Write: The Lane family drove 1,204 miles during the second week.)

YOUR NOTES

NOTES

Debrief Questions

- When we are writing a sentence to express our answer, what part of the original problem helps us to tell our answer using the correct words and context?
- What purpose does a tape diagram have? How does it support your work?
- What does a **variable** help us do when drawing a tape diagram?
- If you have 2 addends, can you ever have enough ones to make 2 tens or enough tens to make 2 hundreds or enough hundreds to make 2 thousands? Try it out with your partner. What if you have 3 addends?
- How is recording the regrouped number in the next column when using the standard algorithm related to bundling disks?

Multiple Means of Action and Expression

English language learners benefit from further explanation of the Problem 4 word problem. Have a conversation around the following: "What do we do if we do not understand a word in the problem? What thinking can we use to figure out the answer anyway?" In this case, students do not need to know what a road trip is in order to solve. Discuss, "How is the tape diagram helpful to us?" It may be helpful to use the RDW approach: Read important information. Draw a picture. Write an equation to solve. Write the answer as a statement.

YOUR NOTES

Lesson 12

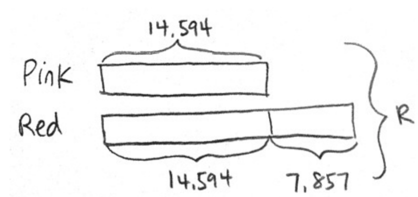
Solve multi-step word problems using the standard addition algorithm modeled with tape diagrams, and assess the reasonableness of answers using rounding.

Materials: (S) Personal white board

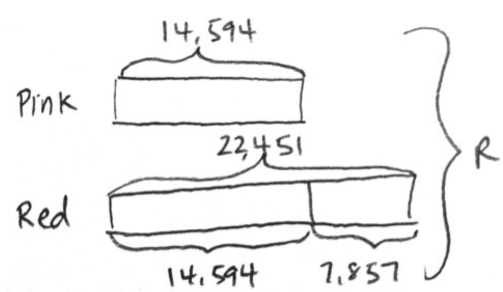
Problem 1: Solve a multi-step word problem using a tape diagram.

The city flower shop sold 14,594 pink roses on Valentine's Day. They sold 7,857 more red roses than pink roses. How many pink and red roses did the city flower shop sell altogether on Valentine's Day? Use a tape diagram to show the work.

T: Read the problem with me. What information do we know?
S: We know they sold 14,594 pink roses.
T: To model this, let's draw one tape to represent the pink roses. Do we know how many red roses were sold?
S: No, but we know that there were 7,857 more red roses sold than pink roses.
T: A second tape can represent the number of red roses sold. (Model on the tape diagram.)



T: What is the problem asking us to find?
S: The total number of roses.
T: We can draw a bracket to the side of both tapes. Let's label it *R* for pink and red roses.
T: First, solve to find how many red roses were sold.
S: (Solve 14,594 + 7,857 = 22,451.)
T: What does the bottom tape represent?
S: The bottom tape represents the number of red roses, 22,451.
T: (Bracket and label 22,451 to show the total number of red roses.) Now, we need to find the total number of roses sold. How do we solve for *R*?
S: Add the totals for both tapes together. 14,594 + 22,451 = R.

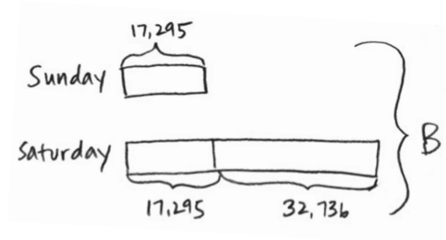


T: Solve with me. What does *R* equal?
S: *R* equals 37,045.
T: (Write *R* = 37,045.) Let's write a statement of the answer.
S: (Write: The city flower shop sold 37,045 pink and red roses on Valentine's Day.)

YOUR NOTES

Problem 2: Solve a two-step word problem using a tape diagram, and assess the reasonableness of the answer.

On Saturday, 32,736 more bus tickets were sold than on Sunday. On Sunday, only 17,295 tickets were sold. How many people bought bus tickets over the weekend? Use a tape diagram to show the work.



T: Tell your partner what information we know.

S: We know how many people bought bus tickets on Sunday, 17,295. We also know how many more people bought tickets on Saturday. But we do not know the total number of people that bought tickets on Saturday.

T: Let's draw a tape for Sunday's ticket sales and label it. How can we represent Saturday's ticket sales?

S: Draw a tape the same length as Sunday's, and extend it further for 32,736 more tickets.

T: What does the problem ask us to solve for?

S: The number of people that bought tickets over the weekend.

T: With your partner, finish drawing a tape diagram to model this problem. Use B to represent the total number of tickets bought over the weekend.

T: Before we solve, estimate to get a general sense of what our answer will be. Round each number to the nearest ten thousand.

S: (Write 20,000 + 20,000 + 30,000 = 70,000.) About 70,000 tickets were sold over the weekend.

T: Now, solve with your partner to find the actual number of tickets sold over the weekend.

S: (Solve.)

S: B equals 67,326.

T: (Write B = 67,326.)

T: Now, let's look back at the estimate we got earlier and compare with our actual answer. Is 67,326 close to 70,000?

S: Yes, 67,326 rounded to the nearest ten thousand is 70,000.

T: Our answer is reasonable.

T: Write a statement of the answer.

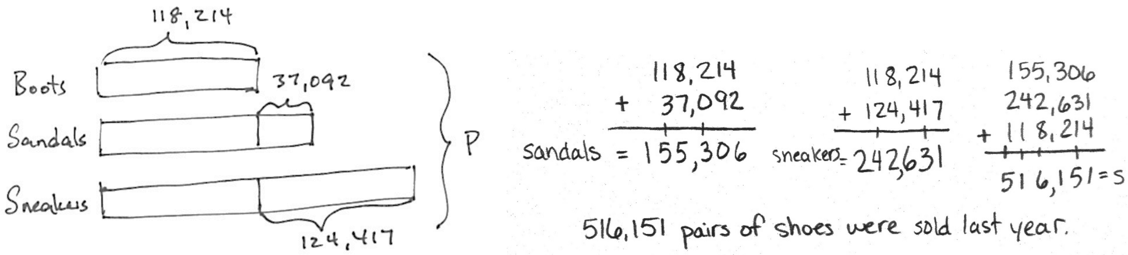
S: (Write: There were 67,326 people who bought bus tickets over the weekend.)

Problem 3: Solve a multi-step word problem using a tape diagram, and assess reasonableness.

Last year, Big Bill's Department Store sold many pairs of footwear. 118,214 pairs of boots were sold, 37,092 more pairs of sandals than pairs of boots were sold, and 124,417 more pairs of sneakers than pairs of boots were sold. How many pairs of footwear were sold last year?

YOUR NOTES

T: Discuss with your partner the information we have and the unknown information we want to find.



S: (Discuss.)

T: With your partner, draw a tape diagram to model this problem. How do you solve for *P*?

S: The tape shows me I could add the number of pairs of boots 3 times, and then add 37,092 and 124,417. → You could find the number of pairs of sandals, find the number of pairs of sneakers, and then add those totals to the number of pairs of boots.

Have students then round each addend to get an estimated answer, calculate precisely, and compare to see if their answers are reasonable.

NOTES

Debrief Questions

- Explain why we should test to see if our answers are reasonable. (Show an example of one of the above lesson problems solved incorrectly to show how checking the reasonableness of an answer is important.)
- When might you need to use an estimate in real life?
- What are the next steps if your estimate is not near the actual answer? Consider the example we discussed earlier where the problem was solved incorrectly. Because we had estimated an answer, we knew that our solution was not reasonable.

Multiple Means of Representation

Students working below grade level may have difficulty conceptualizing word problems. Use smaller numbers or familiar contexts for problems. Have students make sense of the problem, and direct them through the process of creating a tape diagram.

> "The pizza shop sold five pepperoni pizzas on Friday. They sold ten more sausage pizzas than pepperoni pizzas. How many pizzas did they sell?"

Have a discussion about the two unknowns in the problem and about which unknown needs to be solved first. Students may draw a picture to help them solve. Then, relate the problem to that with bigger numbers and numbers that involve regrouping. Relay the message that it is the same process. The difference is that the numbers are bigger.

Multiple Means of Representation

English language learners may need direction in creating their answer in the form of a sentence. Direct them to look back at the question and then to verbally answer the question using some of the words in the question. Direct them to be sure to provide a label for their numerical answer.

YOUR NOTES

Topic E: Multi-Digit Whole Number Subtraction

In Topics D and E, students focus on single like-unit calculations (ones with ones, thousands with thousands, etc.), at times requiring the composition of greater units when adding (10 hundreds are composed into 1 thousand) and decomposition into smaller units when subtracting (1 thousand is decomposed into 10 hundreds).

Lesson 13

Use place value understanding to decompose to smaller units once using the standard subtraction algorithm, and apply the algorithm to solve word problems using tape diagrams.

Materials: (T) Millions place value chart (Lesson 11 Template) (S) Personal white board, millions place value chart (Lesson 11 Template)

Problem 1: Use a place value chart and place value disks to model subtracting alongside the algorithm, regrouping 1 hundred into 10 tens.

Display 4,259 – 2,171 vertically on the board.

T: Say this problem with me. (Read problem together.)

T: Watch as I draw a tape diagram to represent this problem. What is the whole?

S: 4,259.

T: We record that above the tape as the whole and record the known part of 2,171 under the tape. It is your turn to draw a tape diagram. Mark the unknown part of the diagram with the variable *A*.

T: Model the whole, 4,259, using place value disks on your place value chart.

T: Do we model the part we are subtracting?

S: No, just the whole.

T: First, let's determine if we are ready to subtract. We look across the top number, from right to left, to see if there are enough units in each column. Let's look at the ones column. Are there enough ones in the top number to subtract the ones in the bottom number? (Point to the 9 and the 1 in the problem.)

S: Yes, 9 is greater than 1.

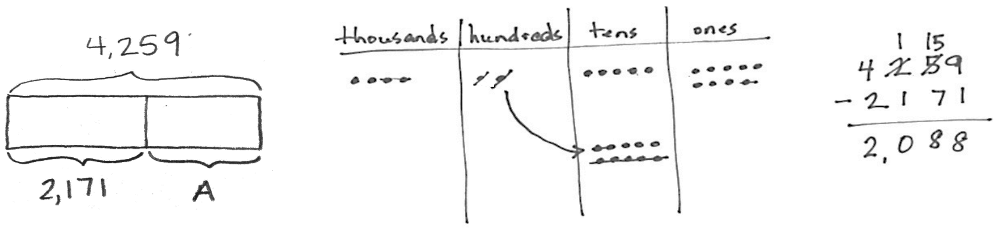
T: That means we are ready to subtract in the ones column. Let's look at the tens column. Are there enough tens in the top number to subtract the tens in the bottom number?

S: No, 5 is less than 7.

YOUR NOTES

T: (Show regrouping on the place value chart.) We ungroup or unbundle 1 unit from the hundreds to make 10 tens. I now have 1 hundred and 15 tens. Let's rename and represent the change in writing using the algorithm. (Cross out the hundreds and tens to rename them in the problem.)

T: Show the change with your disks. (Cross off 1 hundred, and change it for 10 tens as shown below.)



T: Are there enough hundreds in the top number to subtract the hundreds in the bottom number?

S: Yes, 1 is equal to 1.

T: Are there enough thousands in the top number to subtract the thousands in the bottom number?

S: Yes, 4 is greater than 2.

T: Are we ready to subtract?

S: Yes, we are ready to subtract.

T: (Point to the problem.) 9 ones minus 1 one?

S: 8 ones.

T: (Cross off 1 disk; write an 8 in the problem.)

T: 15 tens minus 7 tens?

S: 8 tens.

T: (Cross off 7 disks; write an 8 in the problem.)

Continue subtracting through the hundreds and thousands.

T: Say the number sentence.

S: 4,259 – 2,171 = 2,088.

T: The value of the *A* in our tape diagram is 2,088. We write *A* = 2,088 below the tape diagram. What can be added to 2,171 to result in the sum of 4,259?

S: 2,088.

OPTIONAL FOR FLEX DAY: REPEAT THE PROCESS

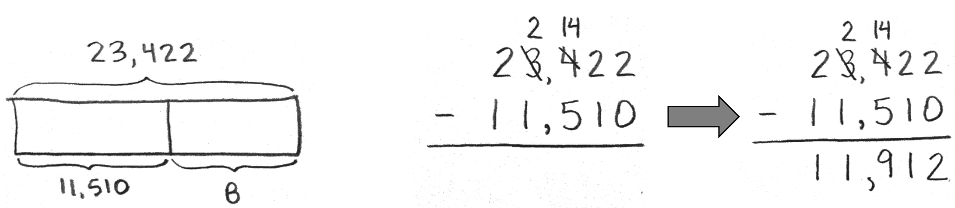
Repeat the process for 6,314 – 3,133.

YOUR NOTES

Problem 2: Regroup 1 thousand into 10 hundreds using the subtraction algorithm.

Display 23,422 – 11,510 vertically on the board.

T: With your partner, read this problem and draw a tape diagram. Label the whole, the known part, and use the variable *B* for the unknown part.
T: Record the problem on your personal white board.
T: Look across the digits. Are we ready to subtract?
S: No.
T: Are there enough ones in the top number to subtract the ones in the bottom number? (Point to the 2 and the 0.)
S: Yes, 2 is greater than 0.
T: Are there enough tens in the top number to subtract the tens in the bottom number?
S: Yes, 2 is greater than 1.
T: Are there enough hundreds in the top number to subtract the hundreds in the bottom number?
S: No, 4 is less than 5.
T: Tell your partner how to make enough hundreds to subtract.
S: I unbundle 1 thousand to make 10 hundreds. I now have 2 thousands and 14 hundreds. → I change 1 thousand for 10 hundreds. → I rename 34 hundreds as 20 hundreds and 14 hundreds.
T: Watch as I record that. Now it is your turn.



Repeat questioning for the thousands and ten thousands columns.

T: Are we ready to subtract?
S: Yes, we are ready to subtract.
T: 2 ones minus 0 ones?
S: 2 ones. (Record 2 in the ones column.)

Continue subtracting across the number from right to left, always naming the units.

T: Tell your partner what must be added to 11,510 to result in the sum of 23,422.
T: How do we check a subtraction problem?
S: We can add the difference to the part we knew at first to see if the sum we get equals the whole.

YOUR NOTES

T: Please add 11,912 and 11,510. What sum do you get?

S: 23,422, so our answer to the subtraction problem is correct.

T: Label your tape diagram as B = 11,912.

OPTIONAL FOR FLEX DAY: REPEAT THE PROCESS

Repeat for 29,014 – 7,503.

Problem 3: Solve a subtraction word problem, regrouping 1 ten thousand into 10 thousands.

The paper mill produced 73,658 boxes of paper. 8,052 boxes have been sold. How many boxes remain?

T: Draw a tape diagram to represent the boxes of paper produced and sold. I will use the letter P to represent the boxes of paper remaining. Record the subtraction problem. Check to see that you lined up all units.

T: Am I ready to subtract?

S: No.

T: Work with your partner, asking if there are enough units in each column to subtract. Regroup when needed. Then ask, "Am I ready to subtract?" before you begin subtracting. Use the standard algorithm. (Students work.)

S: 73,658 – 8,052 = 65,606.

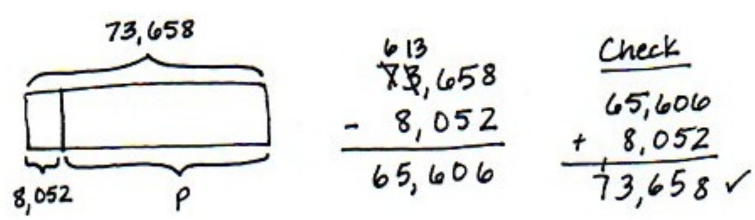
T: The value of P is 65,606. In a statement, tell your partner how many boxes remain.

S: 65,606 boxes remain.

T: To check and see if your answer is correct, add the two values of the tape, 8,052 and your answer of 65,606, to see if the sum is the value of the tape, 73,658.

S: (Add to find that the sum matches the value of the tape.)

Repeat with the following: The library has 50,819 books. 4,506 are checked out. How many books remain in the library?



NOTES

Debrief Questions

- Why do we ask, “Are we ready to subtract?”
- After we get our top number ready to subtract, do we have to subtract in order from right to left?
- When do we need to unbundle to subtract?
- What are the benefits to modeling subtraction using place value disks?
- Why must the units line up when subtracting? How might our answer change if the digits were not aligned?
- What happens when there is a zero in the top number of a subtraction problem?
- What happens when there is a zero in the bottom number of a subtraction problem?
- When you are completing word problems, how can you tell that you need to subtract?

Multiple Means of Engagement

Ask students to look at the numbers in the subtraction problem and to think about how the numbers are related. Ask them how they might use their discovery to check to see if their answer is correct. Use the tape diagram to show if 8,052 was subtracted from 73,658 to find the unknown part of the tape diagram, the value of the unknown, 65,606, can be added to the known part of the tape diagram, 8,052. If the sum is the value of the whole tape diagram, the answer is correct.

YOUR NOTES

Lesson 14

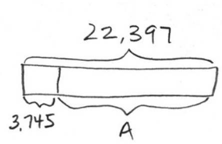
Use place value understanding to decompose to smaller units up to three times using the standard subtraction algorithm, and apply the algorithm to solve word problems using tape diagrams.

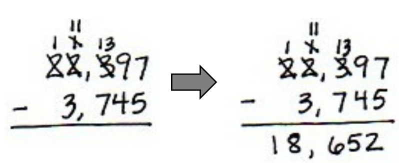
Materials: (S) Personal white board

Problem 1: Subtract, decomposing twice.

Write 22,397 – 3,745 vertically on the board.

T: Let's read this subtraction problem together. Watch as I draw a tape diagram labeling the whole, the known part, and the unknown part using a variable, *A*. Now, it is your turn.
T: Record the problem on your personal white board.
T: Look across the digits. Am I ready to subtract?
S: No.
T: We look across the top number to see if I have enough units in each column. Are there enough ones in the top number to subtract the ones in the bottom number?
S: Yes, 7 ones is greater than 5 ones.
T: Are there enough tens in the top number to subtract the tens in the bottom number?
S: Yes, 9 tens is greater than 4 tens.
T: Are there enough hundreds in the top number to subtract the hundreds in the bottom number?
S: No, 3 hundreds is less than 7 hundreds. We can unbundle 1 thousand as 10 hundreds to make 1 thousand and 13 hundreds. I can subtract the hundreds column now.
T: Watch as I record that. Now, it is your turn to record the change.
T: Are there enough thousands in the top number to subtract the thousands in the bottom number?
S: No, 1 thousand is less than 3 thousands. We can unbundle 1 ten thousand to 10 thousands to make 1 ten thousand and 11 thousands. I can subtract in the thousands column now.
T: Watch as I record. Now, it is your turn to record the change.
T: Are there enough ten thousands in the top number to subtract the ten thousands in the bottom number?
S: Yes.
T: Are we ready to subtract?
S: Yes, we are ready to subtract.
T: 7 ones minus 5 ones?





YOUR NOTES

S: 2 ones. (Record 2 in the ones column.)

Continue subtracting across the problem, always naming the units.

T: Say the equation with me.
S: 22,397 minus 3,745 equals 18,652.
T: Check your answer using addition.
S: Our answer is correct because 18,652 plus 3,745 equals 22,397.
T: What is the value of *A* in the tape diagram?
S: *A* equals 18,652.

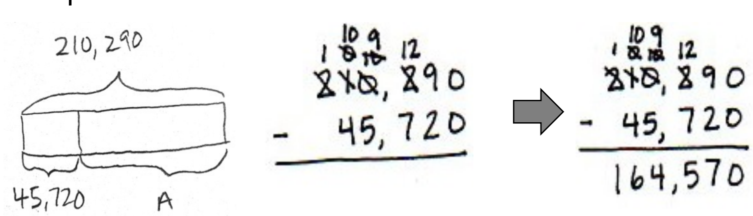
Problem 2: Subtract, decomposing three times.

Write 210,290 – 45,720 vertically on the board.

T: With your partner, draw a tape diagram to represent the whole, the known part, and the unknown part.
T: Record the subtraction problem on your board.
T: Look across the digits. Are we ready to subtract?
S: No.
T: Look across the top number's digits to see if we have enough units in each column. Are there enough ones in the top number to subtract the ones in the bottom number? (Point to the zeros in the ones column.)
S: Yes, 0 equals 0.
T: We are ready to subtract in the ones column. Are there enough tens in the top number to subtract the tens in the bottom number?
S: Yes, 9 is greater than 2.
T: We are ready to subtract in the tens column. Are there enough hundreds in the top number to subtract the hundreds in the bottom number?
S: No, 2 hundreds is less than 7 hundreds.
T: There are no thousands to unbundle, so we look to the ten thousands. We can unbundle 1 ten thousand to 10 thousands. Unbundle 10 thousands to make 9 thousands and 12 hundreds. Now we can subtract the hundreds column.

Repeat questioning for the thousands, ten thousands, and hundred thousands place, recording the renaming of units in the problem.

T: Are we ready to subtract?
S: Yes, we are ready to subtract.
T: 0 ones minus 0 ones?
S: 0 ones.



YOUR NOTES

T: 9 tens minus 2 tens?
S: 7 tens.

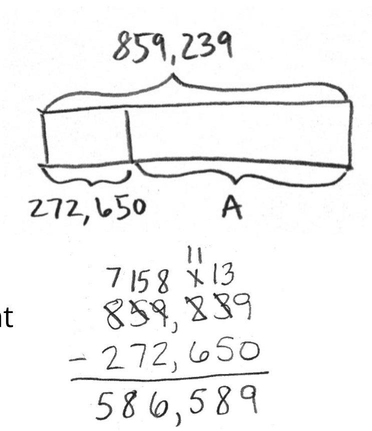
Have partners continue using the algorithm, reminding them to work right to left, always naming the units.

T: Read the equation to your partner and complete your tape diagram by labeling the variable.
S: 210,290 minus 45,720 is 164,570. (*A* = 164,570.)

OPTIONAL FOR FLEX DAY: PROBLEM 3

Problem 3: Use the subtraction algorithm to solve a word problem, modeled with a tape diagram, decomposing units 3 times.

Bryce needed to purchase a large order of computer supplies for his company. He was allowed to spend $859,239 on computers. However, he ended up only spending $272,650. How much money was left?



T: Read the problem with me. Tell your partner the information we know.
S: We know he can spend $859,239, and we know he only spent $272,650.
T: Draw a tape diagram to represent the information in the problem. Label the whole, the known part, and the unknown part using a variable.
T: Tell me the problem we must solve, and write it on your board.
S: $859,239 – $272,650.
T: Work with your partner to move across the digits. Are there enough in each column to subtract? Regroup when needed. Then ask, "Are we ready to subtract?" before you begin subtracting. Use the standard algorithm.
S: $859,239 – $272,650 = $586,589.
T: Say your answer as a statement.
S: $586,589 was left.

NOTES

Debrief Questions

- How is the complexity of this lesson different from the complexity of Small Group Lesson 13?
- In which column can you begin subtracting when you are ready to subtract? (Any column.)
- You are using a variable, or a letter, to represent the unknown in each tape diagram. Tell your partner how you determine what variable to use and how it helps you to solve the problem.
- How can you check a subtraction problem?

YOUR NOTES

Lesson 15

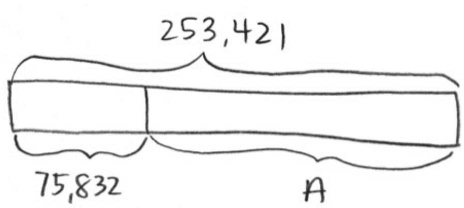
Use place value understanding to fluently decompose to smaller units multiple times in any place using the standard subtraction algorithm, and apply the algorithm to solve word problems using tape diagrams.

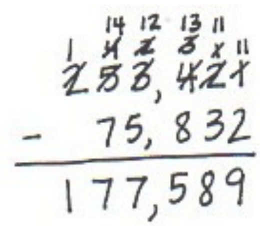
Materials: (T) Millions place value chart (Lesson 11) (S) Personal white board, millions place value chart (Lesson 11 Template)

Problem 1: Regroup units 5 times to subtract.

Write 253,421 – 75,832 vertically on the board.

T: Say this problem with me.
T: Work with your partner to draw a tape diagram representing this problem.
T: What is the whole amount?
S: 253,421.
T: What is the part?
S: 75,832.
T: Look across the top number, 253,421, to see if we have enough units in each column to subtract 75,832. Are we ready to subtract?





S: No.
T: Are there enough ones in the top number to subtract the ones in the bottom number? (Point to the 1 and 2 in the ones column.)
S: No, 1 one is less than 2 ones.
T: What should we do?
S: Change 1 ten for 10 ones. That means you have 1 ten and 11 ones.
T: Are there enough tens in the top number to subtract the tens in the bottom number? (Point to tens column.)
S: No, 1 ten is less than 3 tens.
T: What should we do?
S: Change 1 hundred for 10 tens. You have 3 hundreds and 11 tens.
T: The tens column is ready to subtract.

Have partners continue questioning if there are enough units to subtract in each column, regrouping where needed.

YOUR NOTES

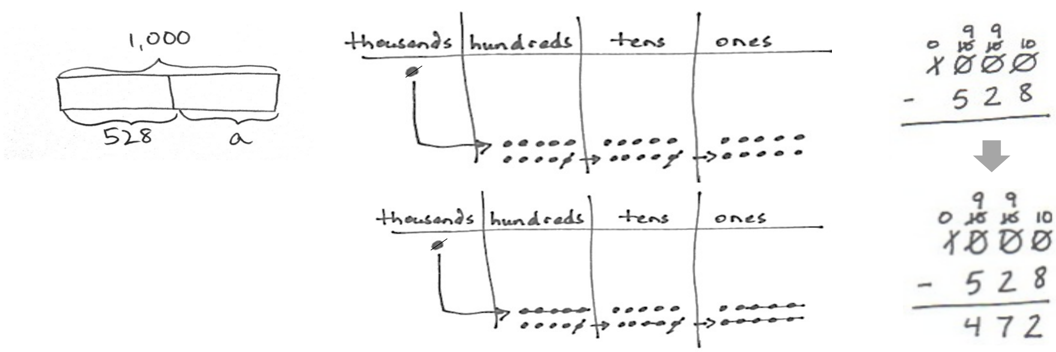
T: Are we ready to subtract?
S: Yes, we are ready to subtract.
T: Go ahead and subtract. State the difference to your partner. Label the unknown part in your tape diagram.
S: The difference between 253,421 and 75,832 is 177,589. (Label A = 177,589.)
T: Add the difference to the part you knew to see if your answer is correct.
S: It is. The sum of the parts is 253,421.

Problem 2: Decompose numbers from 1 thousand and 1 million into smaller units to subtract, modeled with place value disks.

Note: Be sure to discuss multiple ways of solving these problems. The standard algorithm may be the least efficient strategy in some problems. A more efficient strategy for some students may be using the arrow way or another mental math strategy.

Write 1,000 – 528 vertically on the board.

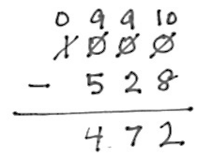
T: With your partner, read this problem, and draw a tape diagram. Label what you know and the unknown.



T: Record the problem on your personal white board.
T: Look across the units in the top number. Are we ready to subtract?
S: No.
T: Are there enough ones in the top number to subtract the ones in the bottom number? (Point to 0 and 8 in the ones column.)
S: No. 0 ones is less than 8 ones.
T: I need to ungroup 1 unit from the tens. What do you notice?
S: There are no tens to ungroup.
T: We can look to the hundreds. (There are no hundreds to ungroup either.)
T: In order to get 10 ones, we need to regroup 1 thousand. Watch as I represent the ungrouping in my subtraction problem. (Model using place value disks and, rename units in the problem simultaneously.) Now it is your turn.

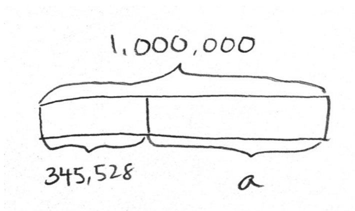
YOUR NOTES

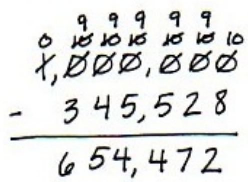
T: Are we ready to subtract?
S: Yes, we are ready to subtract.
T: Solve for 9 hundreds 9 tens 10 ones minus 5 hundreds 2 tens 8 ones.
S: 1,000 – 528 is 472.
T: Check our answer.
S: The sum of 472 and 528 is 1,000.



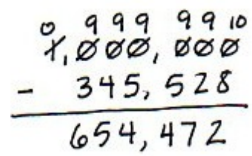
Write 1,000,000 – 345,528 vertically on the board.

T: Read this problem, and draw a tape diagram to represent the subtraction problem.
T: Record the subtraction problem on your board.
T: What do you notice when you look across the top number?





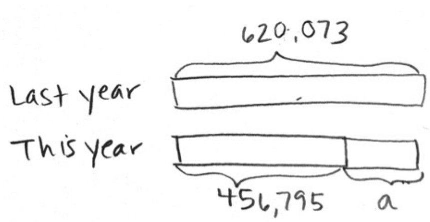
or

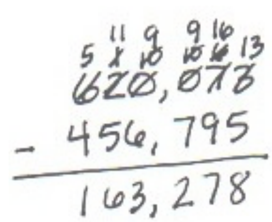


S: There are a lot more zeros. → We will have to regroup 6 times. → We are not ready to subtract. We will have to regroup 1 million to solve the problem.
T: Work with your partner to get 1,000,000 ready to subtract. Rename your units in the subtraction problem.
S: 9 hundred thousands 9 ten thousands 9 thousands 9 hundreds 9 tens 10 ones. We are ready to subtract.
S: 1,000,000 minus 345,528 equals 654,472.
T: To check your answer, add the parts to see if you get the correct whole amount.
S: We did! We got one million when we added the parts.

Problem 3: Solve a word problem, decomposing units multiple times.

Last year, there were 620,073 people in attendance at a local parade. This year, there were 456,795 people in attendance. How many more people were in attendance last year?





T: Read with me.
T: Represent this information in a tape diagram.
T: Work with your partner to write a subtraction problem using the information in the tape diagram.
T: Look across the units in the top number. Are you ready to subtract?

YOUR NOTES

S: No, I do not have enough ones in the top number. I need to unbundle 1 ten to make 10 ones. Then I have 6 tens and 13 ones.

T: Continue to check if you are ready to subtract in each column. When you are ready to subtract, solve.

S: 620,073 minus 456,795 equals 163,278. There were 163,278 more people in attendance last year.

NOTES

Debrief Questions

- How do you know when you are ready to subtract across the problem?
- How can you check your answer when subtracting?
- Is there a number that you can subtract from 1,000,000 without decomposing across to the ones (other than 1,000,000)? 100,000? 10,000?
- How can decomposing multiple times be challenging?
- How does the tape diagram help you determine which operation to use to find the answer?

Multiple Means of Engagement

Students of all abilities will benefit from using addition to check subtraction. Students should see that if the sum does not match the whole, the subtraction (or calculation) is faulty. They must subtract again and then check with addition. Challenge students to think about how they use this check strategy in everyday life. We use it all of the time when we add up to another number.

Multiple Means of Action and Expression

Encourage students who notice a pattern of repeated nines when subtracting across multiple zeros to express this pattern in writing. Allow students to identify why this happens using manipulatives or in writing. Allow students to slowly transition into recording this particular unbundling across zeros as nines as they become fluent with using the algorithm.

YOUR NOTES

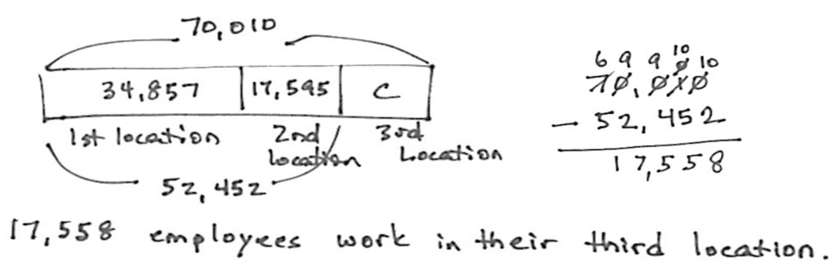
Lesson 16

Solve two-step word problems using the standard subtraction algorithm fluently modeled with tape diagrams, and assess the reasonableness of answers using rounding.

Materials: (S) Personal white board

Problem 1: Solve a two-step word problem, modeled with a tape diagram, assessing reasonableness of the answer using rounding.

A company has 3 locations with 70,010 employees altogether. The first location has 34,857 employees. The second location has 17,595 employees. How many employees work in the third location?



T: Read with me. Take 2 minutes to draw and label a tape diagram. (Circulate and encourage the students: "Can you draw something?" "What can you draw?")

T: (After 2 minutes.) Tell your partner what you understand and what you still do not understand.

S: We know the total number of employees and the employees at the first and second locations. We do not know how many employees are at the third location.

T: Use your tape diagram to estimate the number of employees at the third location. Explain your reasoning to your partner.

S: I rounded the number of employees. 30,000 + 20,000 = 50,000, and I know that the total number of employees is about 70,000. That means that there would be about 20,000 employees at the third location.

T: Now, find the precise answer. Work with your partner to do so. (Give students time to work.)

T: Label the unknown part on your diagram, and make a statement of the solution.

S: There are 17,558 employees at the third location.

T: Is your answer reasonable?

S: Yes, because 17,558 rounded to the nearest ten thousand is 20,000, and that was our estimate.

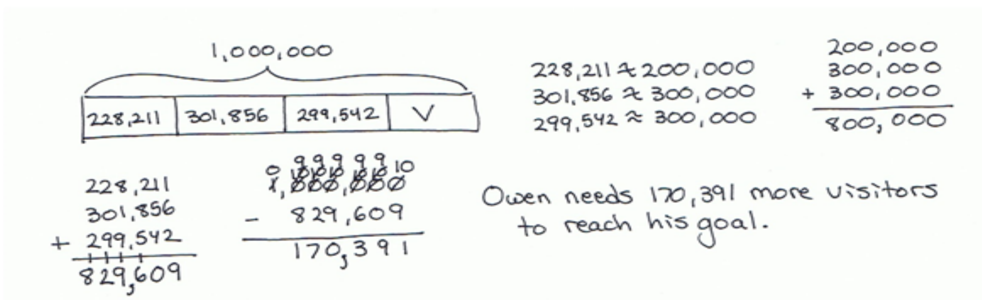
YOUR NOTES

OPTIONAL FOR FLEX DAY: PROBLEM 2

Problem 2: Solve two-step word problems, modeled with a tape diagram, assessing reasonableness of the answer using rounding.

Owen's goal is to have 1 million people visit his new website within the first four months of it being launched. Below is a chart showing the number of visitors each month. How many more visitors does he need in Month 4 to reach his goal?

Month	Month 1	Month 2	Month 3	Month 4
Visitors	228,211	301,856	299,542	



T: With your partner, draw a tape diagram. Tell your partner your strategy for solving this problem.

S: We can find the sum of the number of visitors during the first 3 months. Then, we subtract that from 1 million to find how many more visitors are needed to reach his goal.

T: Make an estimate for the number of visitors in Month 4. Explain your reasoning to your partner.

S: I can round to the nearest hundred thousand and estimate. Owen will need about 200,000 visitors to reach his goal. → I rounded to the nearest ten thousand to get a closer estimate of 170,000 visitors.

T: Find the total for the first 3 months. What is the precise sum?

S: 829,609.

T: Compare the actual and estimated solutions. s your answer reasonable?

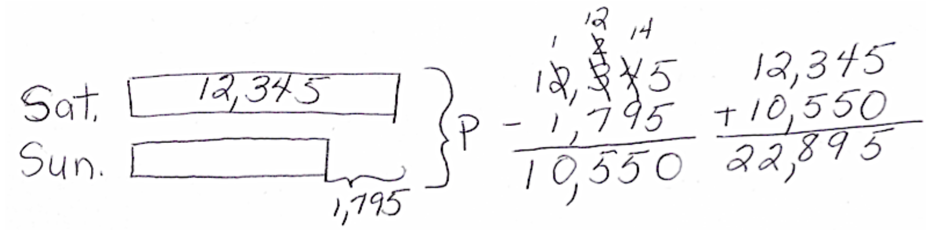
S: Yes, because our estimate of 200,000 is near 170,391. → Rounded to the nearest hundred thousand, 170,391 is 200,000. → 170,391 rounded to the nearest ten thousand is 170,000, which was also our estimate, so our solution is reasonable.

Problem 3: Solve a two-step, *compare with smaller unknown* word problem.

There were 12,345 people at a concert on Saturday night. On Sunday night, there were 1,795 fewer people at the concert than on Saturday night. How many people attended the concert on both nights?

YOUR NOTES

T: For 2 minutes, with your partner, draw a tape diagram. (Circulate and encourage students as they work. You might choose to call two pairs of students to draw on the board while others work at their seats. Have the pairs then present their diagrams to the class.)



T: Now how can you calculate to solve the problem?

S: We can find the number of people on Sunday night, and then add that number to the people on Saturday night.

T: Make an estimate of the solution. Explain your reasoning to your partner.

S: Rounding to the nearest thousand, the number of people on Saturday night was about 12,000. The number of people fewer on Sunday night can be rounded to 2,000, so the estimate for the number of people on Sunday is 10,000. 12,000 + 10,000 is 22,000.

T: Find the exact number of people who attended the concert on both nights. What is the exact sum?

S: 22,895.

T: Compare the actual and estimated solutions. Is your answer reasonable?

S: Yes, because 22,895 is near our estimate of 22,000.

T: Be sure to write a statement of your solution.

NOTES

Debrief Questions

- How do you determine what place value to round to when finding an estimate?
- What is the benefit of checking the reasonableness of your answer?
- Describe the difference between rounding and estimating.

Multiple Means of Action and Expression

Students working below grade level may not consider whether their answer makes sense. Guide students to choose the sensible operation and check their answers. Encourage students to reread the problem after solving and to ask themselves, “Does my answer make sense?” If not, ask, “What else can I try?”

Multiple Means of Engagement

Challenge students working above grade level to expand their thinking and to figure out another way to solve the two-step problem. Is there another strategy that would work?

YOUR NOTES

Topic F: Addition and Subtraction Word Problems

The mission culminates with multi-step word problems in Topic F. Tape diagrams are used throughout the topic to model *additive compare* problems. These diagrams facilitate deeper comprehension and serve as a way to support the reasonableness of an answer.

Lesson 17

Solve *additive compare* word problems modeled with tape diagrams.

Note: Today's lesson uses the Problem Set. Solutions for each problem are included below.

Materials: (S) Problem Set (see Appendix)

Suggested delivery of instruction for solving Topic F's word problems

Note: In Lessons 17-19, the Problem Set comprises word problems from the lesson and is, therefore, to be used during the lesson itself.

1. Model the problem.

   Have two pairs of students (choose as models those students who are likely to successfully solve the problem) work at the board while the others work independently or in pairs at their seats. Review the following questions before solving the first problem.

   - Can you draw something?
   - What can you draw?
   - What conclusions can you make from your drawing?

   As students work, circulate. Reiterate the questions above. After two minutes, have the two pairs of students share only their labeled diagrams. For about one minute, have the demonstrating students receive and respond to feedback and questions from their peers.

2. Calculate to solve and write a statement.

   Give everyone two minutes to finish work on the problem, sharing their work and thinking with a peer. All should then write their equations and statements for the answer.

YOUR NOTES

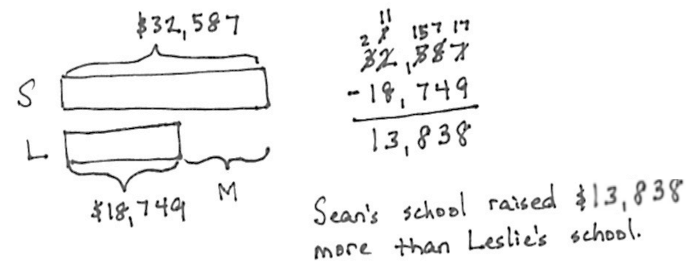
3. Assess the solution for reasonableness.

Give students one to two minutes to assess and explain the reasonableness of their solutions.

Problem 1: Solve a single-step word problem using *how much more.*

Sean's school raised $32,587. Leslie's school raised $18,749. How much more money did Sean's school raise?

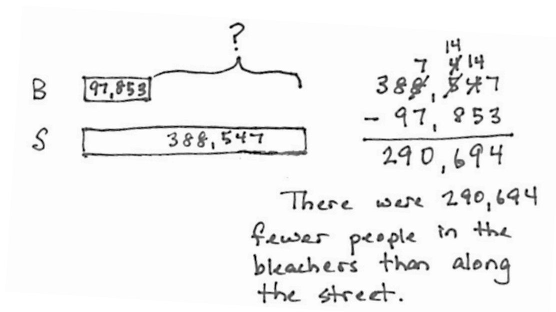
Support students in realizing that though the question is asking, "How much more?" the tape diagram shows that the unknown is a missing part, and therefore, subtraction is necessary to find the answer.



Problem 2: Solve a single-step word problem using *how many fewer.*

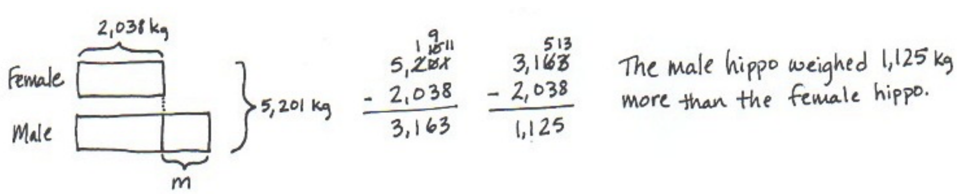
At a parade, 97,853 people sat in bleachers. 388,547 people stood along the street. How many fewer people were in the bleachers than standing along the street?

Circulate and support students to realize that the unknown number of how many fewer people is the difference between the two tape diagrams. Encourage them to write a statement using the word *fewer* when talking about separate things. For example, I have *fewer* apples than you do and *less* juice.



Problem 3: Solve a two-step problem using *how much more.*

A pair of hippos weighs 5,201 kilograms together. The female weighs 2,038 kilograms. How much more does the male weigh than the female?



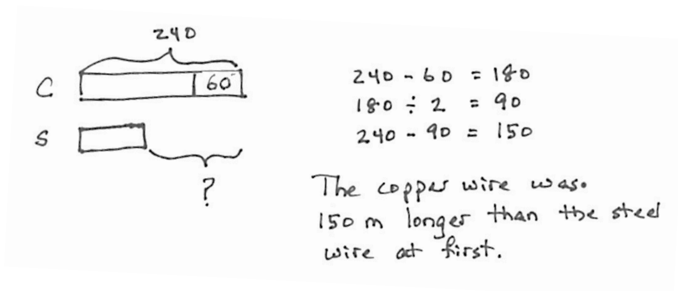
YOUR NOTES

Many students may want to draw this as a single tape showing the combined weight to start. That works. However, the second step most likely requires a new double tape to compare the weights of the male and female. If no one comes up with the model pictured, it can be shown quickly. Students generally do not choose to draw a bracket with the known total to the side until they are very familiar with two-step comparison models. However, be aware that students have modeled this problem type since Grade 2.

Problem 4: Solve a three-step problem using *how much longer.*

A copper wire was 240 meters long. After 60 meters was cut off, it was double the length of a steel wire. How much longer was the copper wire than the steel wire at first?

T: Read the problem, draw a model, write equations both to estimate and calculate precisely, and write a statement. I'll give you five minutes.



Circulate, using the bulleted questions to guide students. When students get stuck, encourage them to focus on what they can learn from their drawings.

- Show me the copper wire at first.
- In your model, show me what happened to the copper wire.
- In your model, show me what you know about the steel wire.
- What are you comparing? Where is that difference in your model?

Notice the number size is quite small here. The calculations are not the issue but rather the relationships. Students will eventually solve similar problems with larger numbers, but they will begin here at a simple level numerically.

NOTES

Debrief Questions

- How are your tape diagrams for Problem 1 and Problem 2 similar?
- How did your tape diagrams vary across all problems?
- In Problem 3, how did drawing a double tape diagram help you to visualize the problem?
- What was most challenging about drawing the tape diagram for Problem 4? What helped you find the best diagram to solve the problem?
- What different ways are there to draw a tape diagram to solve comparative problems?
- What does the word *compare* mean?
- What phrases do you notice repeated through many of today's problems that help you to see the problem as a comparative problem?

Multiple Means of Action and Expression

Students working below grade level may continue to need additional support in subtracting numbers using place value charts or disks.

Multiple Means of Action and Expression

For students who may find Problem 4 challenging, remind them of the work done earlier in this mission with multiples of 10. For example, 180 is ten times as much as 18. If 18 divided by 2 is 9, then 180 divided by 2 is 90.

Multiple Means of Action and Expression

Challenge students to think about how reasonableness can be associated with rounding. If the actual answer does not round to the estimate, does it mean that the answer is not reasonable?

Ask students to explain their thinking. (For example, 376 – 134 = 242. Rounding to the nearest hundred would result with an estimate of 400 – 100 = 300. The actual answer of 242 rounds to 200, not 300.)

YOUR NOTES

Lesson 18

Solve multi-step word problems modeled with tape diagrams, and assess the reasonableness of answers using rounding.

Note: Today's lesson uses the Problem Set. Solutions for each problem are included below.

Materials: (S) Problem Set (see Appendix)

Suggested delivery of instruction for solving Topic F's word problems

Note: In Lessons 17-19, the Problem Set comprises word problems from the lesson and is, therefore, to be used during the lesson itself.

1. Model the problem.

   Have two pairs of students (choose as models those students who are likely to successfully solve the problem) work at the board while the others work independently or in pairs at their seats. Review the following questions before solving the first problem.

   - Can you draw something?
   - What can you draw?
   - What conclusions can you make from your drawing?

   As students work, circulate. Reiterate the questions above. After two minutes, have the two pairs of students share only their labeled diagrams. For about one minute, have the demonstrating students receive and respond to feedback and questions from their peers.

2. Calculate to solve and write a statement.

   Give everyone two minutes to finish work on the problem, sharing their work and thinking with a peer. All should then write their equations and statements for the answer.

3. Assess the solution for reasonableness.

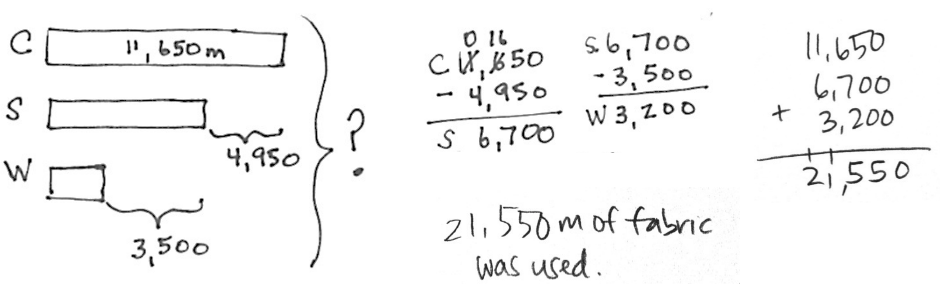
   Give students one to two minutes to assess and explain the reasonableness of their solutions.

YOUR NOTES

OPTIONAL FOR FLEX DAY: PROBLEM 1

Problem 1: Solve a multi-step word problem requiring addition and subtraction, modeled with a tape diagram, and check the reasonableness of the answer using estimation.

In one year, a factory used 11,650 meters of cotton, 4,950 fewer meters of silk than cotton, and 3,500 fewer meters of wool than silk. How many meters in all were used of the three fabrics?



This problem is a step forward for students as they subtract to find the amount of wool from the amount of silk. Students also might subtract the sum of 4,950 and 3,500 from 11,650 to find the meters of wool and add that to the amount of silk. It is a longer method but makes sense. Circulate and look for other alternate strategies, which can be quickly mentioned or explored more deeply as appropriate. Be advised, however, not to emphasize creativity but rather analysis and efficiency. Ingenious shortcuts might be highlighted.

After students have solved the problem, ask them to check their answers for reasonableness:

T: How can you know if 21,550 is a reasonable answer? Discuss with your partner.

S: Well, I can see by looking at the diagram that the amount of wool fits in the part where the amount of silk is unknown, so the answer is a little less than double 12,000. Our answer makes sense.

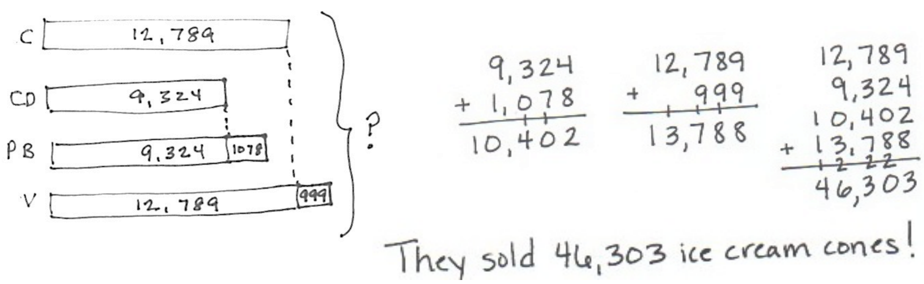
S: Another way to think about it is that 11,650 can be rounded to 12 thousands. 12 thousands plus 7 thousands for the silk, since 12 thousands minus 5 thousands is 7 thousands, plus about 4 thousands for the wool. That's 23 thousands.

YOUR NOTES

OPTIONAL FOR FLEX DAY: PROBLEM 2

Problem 2: Solve an additive multi-step word problem using a tape diagram, modeled with a tape diagram, and check the reasonableness of the answer using estimation.

The shop sold 12,789 chocolate and 9,324 cookie dough cones. It sold 1,078 more peanut butter cones than cookie dough cones and 999 more vanilla cones than chocolate cones. What was the total number of ice cream cones sold?



The solution above shows calculating the total number of cones of each flavor and then adding. Students may also add like units before adding the extra parts.

After students have solved the problem, ask them to check their answers for reasonableness.

T: How can you know if 46,303 is a reasonable answer? Discuss with your partner.

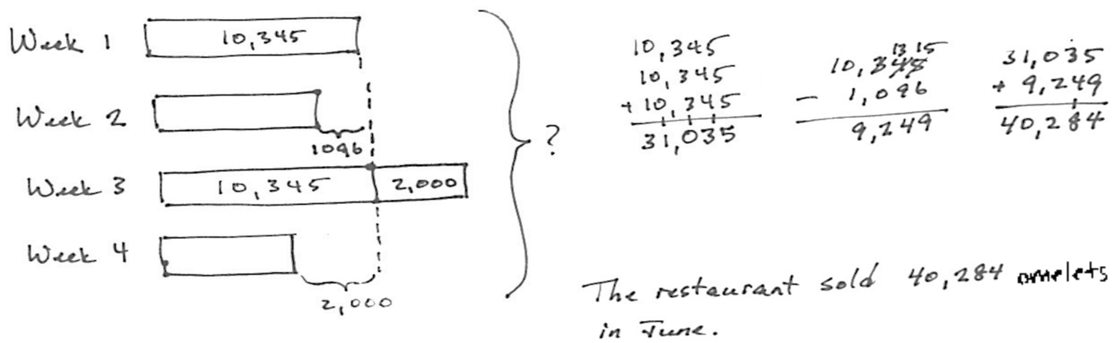
S: By looking at the tape diagram, I can see we have 2 thirteen thousands units. That's 26 thousands. We have 2 nine thousands units. So, 26 thousands and 18 thousands is 44 thousands. Plus about 2 thousands more. That's 46 thousands. That's close.

S: Another way to see it is that I can kind of see 2 thirteen thousands, and the little extra pieces with the peanut butter make 11 thousands. That is 37 thousands plus 9 thousands from cookie dough is 46 thousands. That's close.

Problem 3: Solve a multi-step word problem requiring addition and subtraction, modeled with a tape diagram, and check the reasonableness of the answer using estimation.

In the first week of June, a restaurant sold 10,345 omelets. In the second week, 1,096 fewer omelets were sold than in the first week. In the third week, 2 thousand more omelets were sold than in the first week. In the fourth week, 2 thousand fewer omelets were sold than in the first week. How many omelets were sold in all in June?

YOUR NOTES



This problem is interesting because 2 thousand more and 2 thousand less mean that there is one more unit of 10,345. We, therefore, simply add in the omelets from the second week to three units of 10,345.

T: How can you know if 40,284 is a reasonable answer? Discuss with your partner.

S: By looking at the tape diagram, it's easy to see it is like 3 ten thousands plus 9 thousands. That's 39 thousands. That is close to our answer.

S: Another way to see it is just rounding one week at a time starting at the first week; 10 thousands plus 9 thousands plus 12 thousands plus 8 thousands. That's 39 thousands.

NOTES

Debrief Questions

- How are the problems alike? How are they different?
- How was your solution the same and different from those that were demonstrated by your peers?
- Why is there more than one right way to solve, for example, Problem 3?
- Did you see other solutions that surprised you or made you see the problem differently?
- In Problem 1, was the part unknown or the total unknown? What about in Problems 2 and 3?
- Why is it helpful to assess for reasonableness after solving?
- How were the tape diagrams helpful in estimating to test for reasonableness? Why is that?

YOUR NOTES

Lesson 19

Create and solve multi-step word problems from given tape diagrams and equations.

Note: Today's lesson uses the Problem Set. Solutions for each problem are included below.

OPTIONAL FOR FLEX DAY: ALL OF LESSON 19

Enrichment

Materials: (S) Problem Set (see Appendix)

Suggested delivery of instruction for solving Topic F's word problems

Note: In Lessons 17-19, the Problem Set comprises word problems from the lesson and is, therefore, to be used during the lesson itself.

1. Model the problem.

   Have two pairs of students (choose as models those students who are likely to successfully solve the problem) work at the board while the others work independently or in pairs at their seats. Review the following questions before solving the first problem.

   - Can you draw something?
   - What can you draw?
   - What conclusions can you make from your drawing?

   As students work, circulate. Reiterate the questions above. After two minutes, have the two pairs of students share only their labeled diagrams. For about one minute, have the demonstrating students receive and respond to feedback and questions from their peers.

2. Calculate to solve and write a statement.

   Give everyone two minutes to finish work on the problem, sharing their work and thinking with a peer. All should then write their equations and statements for the answer.

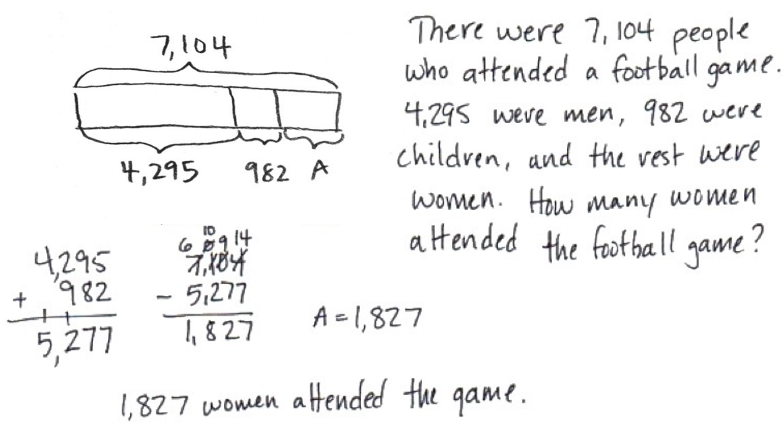
YOUR NOTES

3. Assess the solution for reasonableness.

   Give students one to two minutes to assess and explain the reasonableness of their solutions.

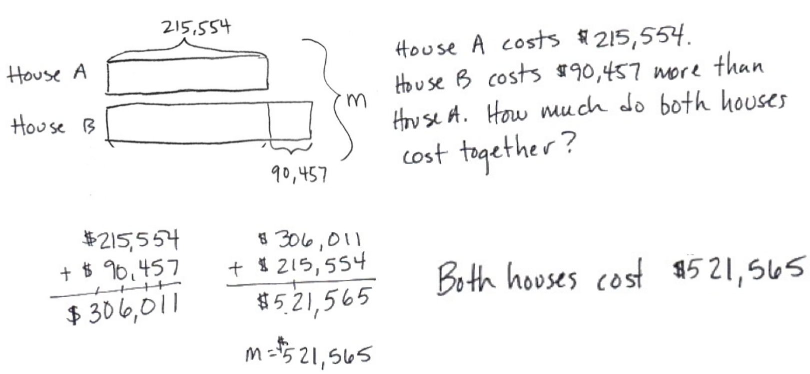
Problem 1: Create and solve a simple two-step word problem from the tape diagram below.

Suggested context: people at a football game.



Problem 2: Create and solve a two-step addition word problem from the tape diagram below.

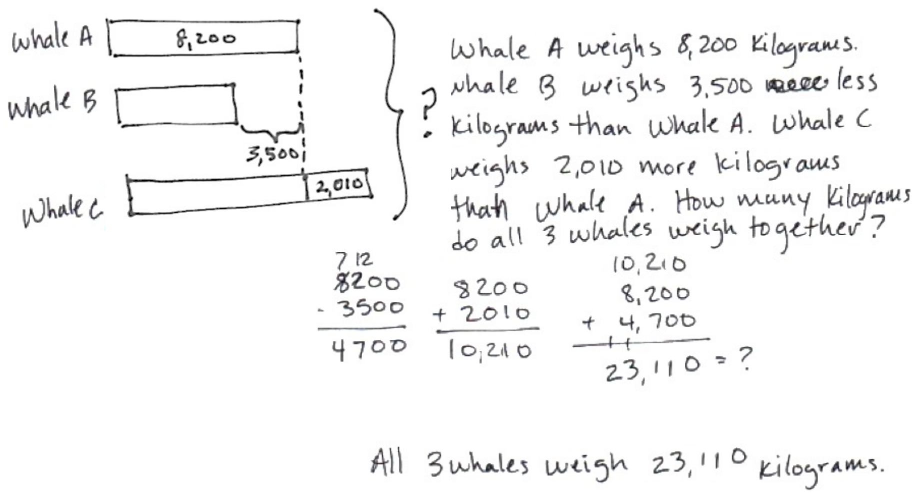
Suggested context: cost of two houses.



YOUR NOTES

Problem 3: Create and solve a three-step word problem involving addition and subtraction from the tape diagram below.

Suggested context: weight in kilograms of three different whales.

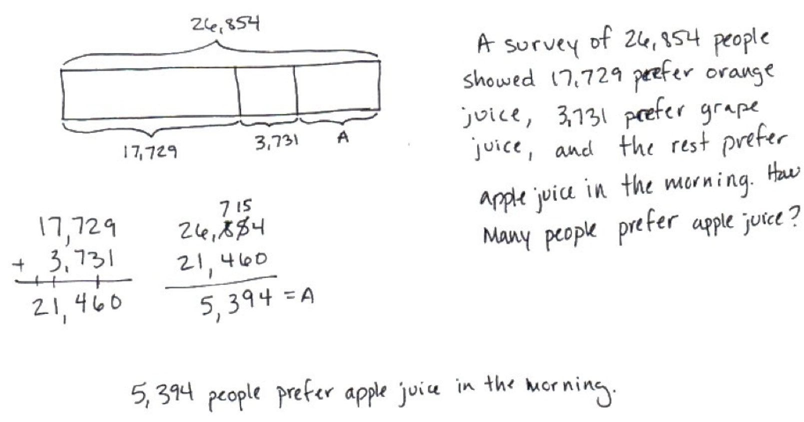


Problem 4: Students use equations to model and solve multi-step word problems.

Display the equation 26,854 = 17,729 + 3,731 + A.

T: Draw a tape diagram that models this equation.

T: Compare with your partner. Then, create a word problem that uses the numbers from the equation. Remember to first create a context. Then, write a statement about the total and a question about the unknown. Finally, tell the rest of the information.



Students work independently. Students can share problems in partners to solve or select word problems to solve as a class.

NOTES

Debrief Questions

- How does a tape diagram help when solving a problem?
- What is the hardest part about creating a context for a word problem?
- To write a word problem, what must you know?
- There are many different contexts for Problem 2, but everyone found the same answer. How is that possible?
- What have you learned about yourself as a mathematician over the past mission?
- How can you use this new understanding of addition, subtraction, and solving word problems in the future?

Multiple Means of Representation

Students who are English language learners may find it difficult to create their own problems. Work together with a small group of students to explain what the tape diagram is showing. Work with students to write information into the tape diagram. Discuss what is known and unknown. Together, build a question based on the discussion.

Multiple Means of Action and Representation

Students working below grade level may struggle with the task of creating their own problems. These students may benefit from working together in a partnership with another student. First, encourage them to design a tape diagram showing the known parts, the unknown part, and the whole. Second, encourage them to create a word problem based on the diagram.

Appendix

Topic A: Place Value of Multi-Digit Whole Numbers

Lesson 1

Unlabeled thousands place value chart (Template)

Lesson 2

Unlabeled millions place value chart (Template)

Topic B: Comparing Multi-Digit Whole Numbers

Lesson 5

Unlabeled hundred thousands place value chart (Template)

Topic D: Multi-Digit Whole Number Addition

Lesson 11

Millions place value chart (Template)

millions	hundred thousands	ten thousands	thousands	hundreds	tens	ones
,			,			

Topic F: Addition and Subtraction Word Problems

Lesson 17

Problem Set

Name ____ Date ____________________

Draw a tape diagram to represent each problem. Use numbers to solve, and write your answer as a statement.

Problem 1

Sean's school raised $32,587. Leslie's school raised $18,749. How much more money did Sean's school raise?

Problem 2

At a parade, 97,853 people sat in bleachers, and 388,547 people stood along the street. How many fewer people were in the bleachers than standing on the street?

Problem 3

A pair of hippos weighs 5,201 kilograms together. The female weighs 2,038 kilograms. How much more does the male weigh than the female?

Problem 4

A copper wire was 240 meters long. After 60 meters was cut off, it was double the length of a steel wire. How much longer was the copper wire than the steel wire at first?

Lesson 18

Problem Set

Name ______________________________ Date ______________

Draw a tape diagram to represent each problem. Use numbers to solve, and write your answer as a statement.

Problem 1

In one year, the factory used 11,650 meters of cotton, 4,950 fewer meters of silk than cotton, and 3,500 fewer meters of wool than silk. How many meters in all were used of the three fabrics?

Problem 2

The shop sold 12,789 chocolate and 9,324 cookie dough cones. It sold 1,078 more peanut butter cones than cookie dough cones and 999 more vanilla cones than chocolate cones. What was the total number of ice cream cones sold?

Problem 3

In the first week of June, a restaurant sold 10,345 omelets. In the second week, 1,096 fewer omelets were sold than in the first week. In the third week, 2 thousand more omelets were sold than in the first week. In the fourth week, 2 thousand fewer omelets were sold than in the first week. How many omelets were sold in all in June?

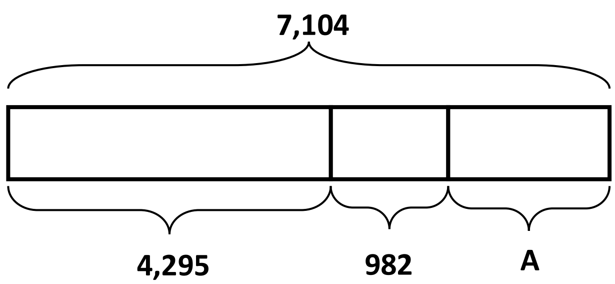
Lesson 19

Problem Set

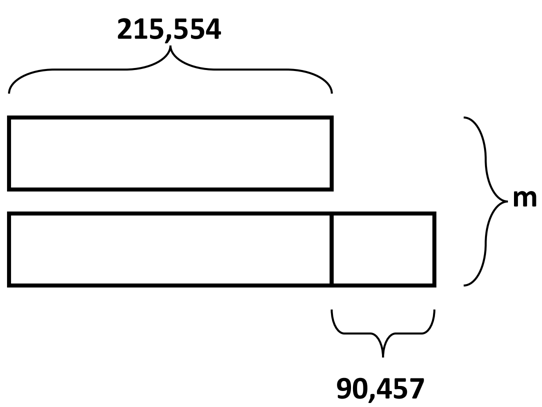
Name ______________________________ Date ________________

Using the diagrams below, create your own word problem. Solve for the value of the variable.

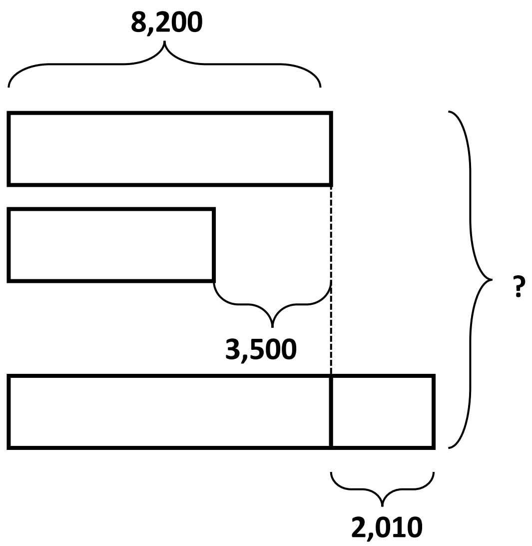
Problem 1



Problem 2



Problem 3



Problem 4

Draw a tape diagram to model the following equation. Create a word problem. Solve for the value of the variable.

$$26{,}854 = 17{,}729 + 3{,}731 + A$$